Spring, 1992 Name: ______________________________

CS 541 — Programming Languages 1

Functional Programming and Lambda Calculus Test (Features, Version 1)

Before you begin, please take a moment to look over the entire test so that you can budget your time.

If you get an incomplete on this test, you can get at most a 'B' on the makeup test.

Short answer questions will be graded on the basis of completeness (did you list all the relevant parts of the answer?) soundness (did you make any errors of fact or contradict yourself?) and clarity. You can help ensure clarity by giving examples where possible.

For essays, critical justifications, and the like there may be no one right answer; so the criteria for judgement will be whether your essay is:

- clear (you use examples where appropriate, you write clear sentences, diagrams, and you avoid excess verbosity),
- sound (you start from facts or reasonable assumptions, your logic is convincing), and
- complete (you consider relevant aspects of the problem and alternatives).

The highest grade will be earned by an answer that is, in addition, concise and elegant; that is, answers that say the most with the least amount of excess. Extra consideration will be given to answers that are especially creative.

Don't feel obligated to fill up all the space on the page. Conversely, don't feel limited by the space on the page, since you can use the back if you explicitly tell us on the front when you do so.

1. (5 points) Give a formal definition of a bound variable in the lambda calculus.

2. (5 points) What is currying?

3. (7 points) In simply typed lambda calculus and an empty type environment, what is the type, if any, of the following term. If it does not have a type, write "no type" and say why. You don't have to give a proof of its type, although you may do so if you wish.

   $\lambda z : \mathbf{o} \rightarrow \mathbf{o} \,.\, \lambda g : \mathbf{o} \rightarrow \mathbf{o} \,.\, \lambda y : \mathbf{o} \,.\, g(z(g(y)))$

4. (24 points) Reduce each of the following lambda expressions to β,η-normal form if possible, or show (briefly) that the expression has no normal form. If the term is already in normal form, write "already in normal form". You must show each of the reduction steps involved, if any are needed. (Each is worth 4 points)

   (a) $\lambda f\,.\,(\lambda x\,.\,f\ x)\,(\lambda x\,.\,f\ x)$

   (b) $(\lambda w\,.\,(\lambda x\,.\,x\ x\ x))\ f\ \lambda y\,.\,y\ y$

   (c) $\lambda z\,.\,y\ \lambda y\,.\,(y\ \lambda x\,.\,x\ y)$

   (d) $(\lambda x\,.\,\lambda y\,.\,z\ x\ y)\ \lambda x\,.\,x\ x$

   (e) $\lambda b\,.\,(\lambda c\,.\,(c\ x\ y))\,(\lambda n\,.\,\lambda e\,.\,\lambda k\,.\,k(f\ n))$

   (f) $(\lambda y\,.\,\lambda t\,.\,(\lambda e\,.\,\lambda c\,.\,y\ e\,(t\ e\ c)))(\lambda e\,.\,\lambda c\,.\,\lambda s\,.\,s)((\lambda x\,.\,x\ x)\,(\lambda x\,.\,x\ x))$