Homework 1: Haskell and Functional Programming

Due: 1, 3-8, 12, and 13, Thursday, September 9, 1999; 15-17, 18(parts a and b), 20, and 21 September 16, 1999; the rest on October 5;

In this homework you will learn about the basics of Haskell, how Haskell can be considered as a domain-specific language for working with lists, basic techniques of recursive programming over various types of data, and abstracting from patterns, higher-order functions, currying, and infinite data. Many of the problems below exhibit polymorphism. Finally, the problems as a whole illustrate how functional languages work without statements (including assigment statements).

Since the purpose of this homework is to ensure skills in functional programming, this is an individual homework. That is, for this homework, you are to do the work on your own, not in groups.

For all Haskell programs, you must run your code with Haskell 98 (for example, using `hugs`). You must also provide evidence that your program is correct (for example, test cases). Hand in a printout of your code and your testing, for questions that require code.

Be sure to clearly label what problem each function solves with a comment.

Read Thompson's book (the second edition), chapters 1–7, 9–10, 11-14 and 16–18. You may also want to read a tutorial on the concepts of functional programming languages, such as Hudak's computing survey article mentioned in the "Introduction to the Literature" handout, or the "Gentle Introduction to Haskell" for a different introduction to the language.

Some of the problems build on each other. Don't hesitate to contact the staff if you are stuck at some point.

Acknowledgement: many of these problems are due to John Hughes.

1. (30 points) Write a function

```
> delete_all :: (Eq a) => a -> ([a] -> [a])
```

that takes an item (of a type that is an instance of the `Eq` class) and a list, and returns a list just like the argument list, but with the each occurrence of the item (if any) removed. For example.

```
delete_all 3 ([]::[Int]) = [] :: [Int]
delete_all 1 [1, 2, 3, 2, 1, 2, 3, 2, 1] = [2, 3, 2, 2, 3, 2]
delete_all 4 [1, 2, 3, 2, 1, 2, 3, 2, 1] = [1, 2, 3, 2, 1, 2, 3, 2, 1]
delete_all 3 [1, 2, 3] = [1, 2]
```

Do this (a) by writing out the recursion yourself, (b) using a list comprehension, and (c) by just using functions in the Haskell Prelude, which is in the following file.

```
/usr/unsup/hugs/share/hugs/lib/Prelude.hs
```

2. (suggested practice) Write a function

```
delete_second :: (Eq a) => a -> ([a] -> [a])
```

that takes an item (of a type that has an == function defined for it) and a list, and returns a list just like the argument list, but with the second occurrence of the item (if any) removed. For example.

```
delete_second 3 ([]::[Int]) = [] :: [Int]
delete_second 1 [1, 2, 3, 2, 1, 2, 3, 2, 1] = [1, 2, 3, 2, 2, 3, 2, 1]
delete_second 4 [1, 2, 3, 2, 1, 2, 3, 2, 1] = [1, 2, 3, 2, 1, 2, 3, 2, 1]
delete_second 3 [1, 2, 3] = [1, 2, 3]
```

Do this both (a) by writing out the recursion yourself, and (b) by just using functions in the Haskell Prelude. (Can this be done using a list comprehension?)

Hint: for part (a), you may need a helping function.

3. (10 points) Write a function

```
merge :: (Ord a) => [[a]] -> [a]
```

that takes a finite list of sorted finite lists and merges them into a single sorted list. A "sorted list" means a list sorted in increasing order (using `<`); you may assume that the sorted lists are finite. For example

```
merge ([[]]::[[Int]]) = [] :: [Int]
merge [[1, 2, 3]] = [1, 2, 3]
merge [[1, 3, 5, 7],  [2, 4, 6]] = [1, 2, 3, 4, 5, 6, 7]
merge [[1,3,5,7], [2,4,6], [3,5,9,10,11,12]] = [1,2,3,3,4,5,5,6,7,9,10,11,12]
take 8 (merge [[1, 3, 5, 7], [1,2,3,4,5,6,7,8]]) = [1, 1, 2, 3, 3, 4, 5, 5]
```

(For 30 points extra credit, make your solution work when the sorted lists are not necessarily finite; you can still assume that there are a finite number of sorted lists.)

4. (10 points) Define the function

```
> matches :: (Eq a) => a -> [a] -> [a]
```

that picks out all occurrences of its first argument in a list. For example,

```
matches 1 [1,2,1,4,5,1,7] = [1, 1, 1]
matches 1 [5, 4, 2] = []
```

Do this (a) using a list comprehension, and (b) using functions in the Haskell prelude.

5. (15 points) Do problem 5.13 in the second edition on Thompson's book (library database functions).

6. The following relate to modularization of numeric code using functional techniques and lazy evaluation (you should read chapter 17 in Thompson's book about lazyness). In particular, we will explore the Newton-Raphson algorithm. This algorithm computes better and better approximations to the square root of a number `n` from a previous approximation `x` by using the following function.

```
> next :: (Real a, Fractional a) => a -> a -> a
> next n x = (x + n / x) / 2
```

   (a) (10 points) Using the `iterate` function in the Haskell Prelude, write a function

```
> approximations :: (Real a, Fractional a) => a -> a -> [a]
```

such that `approximations n a0` returns the infinite list of approximations to the square root of `n`, starting with `a0`. For example,

```
approximations 1.0 1.0 = [1.0, 1.0 ..]
take 5 (approximations 2.0 1.0) = [1.0, 1.5, 1.41667, 1.41422, 1.41421]
take 5 (approximations 64.0 1.0) = [1.0, 32.5, 17.2346, 10.474, 8.29219]
```

(b) (20 points) Define a function `within`

```
> within :: (Ord a, Num a) => a -> [a] -> a
```

that takes a tolerance, that is, a number `epsilon`, and an infinite list of numbers, and looks down the list to find two consecutive numbers in the list that differ by no more than `epsilon`; it returns the second of these. (It might never return if there is no such pair of consecutive elements.) For example,

```
within 1.0 [1.0 ..] = 2.0
within 0.5 ([1.0, 32.5, 17.2346, 10.474, 8.29219, 8.00515]
           ++ [8.0, 8.0 ..])
      = 8.00515
```

(c) (10 points) Using the two pieces above, make a function `squareRoot`

```
> squareRoot :: (Real a, Fractional a) => a -> a -> a -> a
```

that takes an initial guess, a tolerance `epsilon`, and a number, `n` and returns an approximation to the square root of `n` that is within `epsilon`. For example,

```
squareRoot 1.0 0.0000001 2.0 = 1.41421
squareRoot 1.0 0.0000001 64.0 = 8.0
```

(d) (15 points) Write a function `relativeSquareRoot`

```
> relativeSquareRoot :: (Real a, Fractional a) => a -> a -> a -> a
```

which keeps iterating until the ratio of the difference between the last and the previous approximation to the last approximation approaches 0, instead of waiting for the differences between the approximations themselves to approach zero. (This is equivalent to iterating until the ratio of the last two approximations approaches 1.) This is better for square roots of very large numbers, and for square roots of very small numbers. The function `relativeSquareRoot` takes an initial approximation, a tolerance `epsilon` (for how closely the ratio between the last two approximations must approach 1), and the number `n`. (Hint, define a function `relative` that plays the role of `within`; use absolute values.) For example:

```
relativeSquareRoot 1.0 0.1e-5 9.0e+10 = 3.0e+5
relativeSquareRoot 1.0 0.1e-5 9.0e-40 = 3.0e-20
```

7. (15 points) You may recall that the derivative of a function `f` at a point `x` can be approximated by the following function.

```
> easydiff :: (Real a, Fractional a) => (a -> a) -> a -> a -> a
> easydiff f x delta = (f(x+delta) - f(x)) / delta
```

Good approximations are given by small values of `delta`, but if `delta` is too small, then rounding errors may swamp the result. One way to choose `delta` is to compute a sequence of approximations, starting with a reasonably large one. If `(within epsilon)` is used to select the first approximation that is accurate enough, this can reduce the risk of a rounding error affecting the result. Write function

```
> diffApproxims :: (Real a, Fractional a) => a -> (a -> a) -> a -> [a]
```

that takes an initial value for `delta`, and the function `f`, and a point `x`, and returns an infinite list of approximations to the derivative of `f` at `x`, where at each step, the current `delta` is halved. For example

```
take 9 (diffApproxims 500.0 (\x -> x*x) 20)
    = [540.0, 290.0, 165.0, 102.5, 71.25, 55.625, 47.8125, 43.9062, 41.9531]
take 8 (diffApproxims 100.0 (\x -> x*x*x) 10)
    = [13300.0, 4300.0, 1675.0, 831.25, 526.562, 403.516, 349.316, 324.048]
```

8. (15 points) Write a function

```
> differentiate :: (Real a, Fractional a) => a -> a -> (a -> a) -> a -> a
```

that takes a tolerance, `epsilon`, an initial value for `delta`, and the function `f`, and a point `x`, and returns an approximation to the derivative of `f` at `x`. For example.

```
differentiate 0.1e-6 500.0 (\x -> x*x) 20 = 40.0
differentiate 0.1e-6 100.0 (\x -> x*x*x) 10 = 300
```

9. (30 points; extra credit) Write a function in Haskell to do numerical integration, using the ideas above.

10. (15 points) Write a function

```
> compose :: [(a -> a)] -> (a -> a)
```

that takes a list of functions, and returns a function which is their composition. For example.

```
compose [] [1, 2, 3] = [1, 2, 3]
compose [(\ x -> x + 1), (\ x -> x + 2)] 4 = 7
compose [tail, tail, tail] [1, 2, 3, 4, 5] = [4, 5]
compose [(\ x -> 3 : x), (\ y -> 4 : y)] [] = 3 : (4 : [])
```

Hint: note that `compose []` is the identity function.

11. (extra credit) Consider the following type as a representation of binary relations.

```
> type BinaryRel a b = [(a, b)]
```

(a) (10 points, extra credit) Write a function

```
> isFunction :: (Eq a, Eq b) => (BinaryRel a b) -> Bool
```

that returns True just when its argument satisfies the standard definition of a function; that is, `isFunction r` is True just when for each pair (x, y) in the list `r`, there is no pair (x, z) in `r` such that $y \neq z$.

(b) (10 points, extra credit) Write a function

```
> brelCompose :: (Eq a, Eq b, Eq c) =>
>                 (BinaryRel a b) -> (BinaryRel b c) -> (BinaryRel a c)
```

that returns the relational composition of its arguments. That is, a pair (x, y) is in the result if and only if there is a pair (x, z) in the first relation argument of the pair of arguments, and a pair (z, y) in the second argument. For example,

```
brelCompose ([]::[(Int, Int)]) [(2,'b'), (3, 'c')] = []
brelCompose []::[(Int, Int)] ([]::[(Int, Int)]) = []
brelCompose [(1,2),(2,3)]  [(2,'b'), (3, 'c')] = [(1, 'b'), (2, 'c')]
brelCompose [(1,2),(1,3)]  [(2,'b'), (3, 'c')] = [(1, 'b'), (1, 'c')]
brelCompose [(1,3), (2,3)] [(3,'b'), (3,'c')]
             = [(1,'b'), (1,'c'), (2,'b'), (2,'c')]
```

12. (5 points) Define a function

```
> commaSeparate :: [String] -> String
```

that takes a list of strings and returns a single string that, contains the given strings in order, separated by `", "`. For example,

```
commaSeparate [] = ""
commaSeparate ["a", "b"] = "a, b"
commaSeparate ["Monday", "Tuesday", "Wednesday"]
      = "Monday, Tuesday, Wednesday"
```

13. (10 points) Define a function

```
> onSeparateLines :: [String] -> String
```

that takes a list of strings and returns a single string that, when printed, shows the strings on separate lines. Do this both (a) using functions in the prelude, and (b) defining it explicitly using recursion.

14. (10 points) Define a function

```
> separatedBy :: String -> [String] -> String
```

That is a generalization of `onSeparateLines` and `commaSeparated`. Test it by using it to define these other functions.

15. (5 points) Redefine the `++` operator on lists using `foldr`, by completing the following module by adding arguments to `foldr`. You'll find the code for `foldr` in the Haskell prelude. (Hint: you can pass the "`:`" constructor as a function by writing `(:)`.)

```
module MyAppend where
import Prelude hiding ((++))
(++) :: [a] -> [a] -> [a]
xs ++ ys = foldr _________________________
```

16. (10 points) Define the function

```
> doubleAll :: [Integer] -> [Integer]
```

that takes a list of Integers, and returns a list with each of its elements doubled. Do this (a) using a list comprehension, and (b) using `foldr` in a way similar to the previous problem. (Hint: use a `where` to define the function you need to pass to `foldr`. You might want to use function composition, written with an infix dot `(.)` in Haskell.) The following are examples.

```
doubleAll [] = []
doubleAll [8,10] = [16,20]
doubleAll [1, 2 .. 500] = [2, 4 .. 1000]
```

17. (15 points) (a) Define the `map` functional using `foldr`. As part of your testing, use `map` to (a) define `doubleAll`, and (b) to add 1 to all the elements of a list of Integers. (Hint: import the Prelude hiding `map` in the module for this answer; see problem 15 above.)

18. Consider the following data type for trees, which represents a Tree of type `a` as a Node, which contains an item of type `a` and a list of Trees of type `a`.

```
> data Tree a = Node a [Tree a]
```

(a) (10 points) Define a function `sumTree`

```
> sumTree :: Tree Integer -> Integer
```

which adds together all the Integers in a Tree of Integers. For example,

```
sumTree (Node 4 []) = 4
sumTree (Node 3 [Node 4 [], Node 7 []]) = 14
sumTree (Node 10 [Node 3 [Node 4 [], Node 7 []],
                  Node 10 [Node 20 [], Node 30 [], Node 40 []]]) = 124
```

(b) (10 points) Define a function `preorderTree`

```
> preorderTree :: Tree a -> [a]
```

which takes a Tree and returns a list of the elements in its node in a preorder traversal. For example,

```
preorderTree (Node True []) = [True]
preorderTree (Node 5 [Node 6 [], Node 7 []]) = [5, 6, 7]
preorderTree (Node 10 [Node 3 [Node 4 [], Node 7 []],
                       Node 10 [Node 20 [], Node 30 [], Node 40 []]])
     = [10, 3, 4, 7, 10, 20, 30, 40]
```

(c) (15 points) Give a Haskell `instance` declaration (see chapter 12) that makes `Tree` an instance of the type class `Functor`. (See the Prelude for the definition of the `Functor` class.) Your code should start as follows.

```
instance Functor Tree where
    fmap f ...
```

(d) (30 points) By generalizing your answers to the above problems, define a Haskell function `foldTree`

```
> foldTree :: (a -> b -> c) -> (c -> b -> b) -> b -> Tree a -> c
```

that is analogous to `foldr` for lists. This should take a function to replace the Node constructor, one to replace the `(:)` constructor for lists, and a value to replace the empty list. You should, for example, be able to define `sumTree`, `preorderTree`, and the Functor instance for `fmap` on Trees as follows.

```
> sumTree = foldTree (+) (+) 0
> preorderTree = foldTree (:) (++) []
> instance Functor Tree where
>    fmap f = foldTree (Node . f) (:) []
```

19. (30 points) A set can be described by a "characteristic function" (whose range is the booleans) that determines if an element occurs in the bag. For example, the function ϕ such that $\phi($"`coke`"$) = \phi($"`pepsi`"$) =$ `True` and for all other arguments x, $\phi(x) =$ `False` is the characteristic function for a set containing the strings `"coke"`, `"pepsi"` and nothing else. Allowing the user to construct a set from a characteristic function gives one the power to construct sets that may "contain" an infinite number of elements (such as the set of all prime numbers).

Let the polymorphic type constructor `Set` be some polymorphic type that you decide on (you can declare this with something like the following).

```
type Set a = ...
-- or perhaps something like --
data Set a = ...
```

Hint: think about using a function type.

The operations on sets are described informally as follows.

(a) The function

```
setSuchThat :: (a -> Bool) -> (Set a)
```

takes a characteristic function, f and returns a set such that each value x (of appropriate type) is in the set just when fx is `True`.

(b) The function

```
unionSet :: Set a -> Set a -> Set a
```

takes two sets, with characteristic functions f and g, and returns a set such that each value x (of appropriate type) is in the set just when (fx) or (gx) is true.

(c) The function

```
intersectSet :: Set a -> Set a -> Set a
```

takes two sets, with characteristic functions f and g, and returns a set such that each value x (of appropriate type) is in the set just when both (fx) and (gx) are true.

(d) The function

```
memberSet :: Set a -> a -> Bool
```

tells whether the second argument is a member of the first argument.

(e) The function

```
complementSet :: Set a -> Set a
```

which returns a set that contains everything (of the appropriate type) not in the original set.

As examples, consider the following.

```
memberSet (setSuchThat (\ x -> x == "coke")) "coke" = True
memberSet (setSuchThat (\ x -> x == "coke")) "pepsi" = False
memberSet (complementSet (setSuchThat(\ x -> x == "coke"))) "coke" = False
memberSet (unionSet (setSuchThat (\ x -> x == "coke"))
                    (setSuchThat (\ x -> x == "pepsi")))
         "pepsi" = True
memberSet (unionSet (setSuchThat(\ x -> x == "coke"))
                    (setSuchThat(\ x -> x == "pepsi")))
         "coke" = True
memberSet (unionSet (setSuchThat(\ x -> x == "coke"))
                    (setSuchThat(\ x -> x == "pepsi")))
         "sprite" = False
memberSet (intersectSet (setSuchThat(\ x -> x == "coke"))
                        (setSuchThat(\ x -> x == "pepsi")))
         "coke" = False
```

Note (hint, hint) that the following equations must hold, for all `f`, `g`, and `x` of appropriate types.

```
memberSet (unionSet (setSuchThat f) (setSuchThat g)) x = (f x) || (g x)
memberSet (intersectSet (setSuchThat f) (setSuchThat g)) x = (f x) && (g x)
memberSet (setSuchThat f) x = f x
memberSet (complementSet (setSuchThat f)) x = not (f x)
```

20. (25 points) Consider the following data definitions.

```
> data Exp = BoolLit Bool | IntLit Integer | CharLit Char
>          | Sub Exp Exp
>          | Equal Exp Exp
```

```
>            | If Exp Exp Exp
> data OType = OBoolean | OInt | OChar | OWrong
>              deriving Eq
```

Write a function

```
typeOf :: Exp -> OType
```

that takes an `Exp` and returns its `OType`. For example.

```
typeOf (Equal (IntLit 3) (IntLit 4)) = OBoolean
typeOf (Sub (IntLit 3) (IntLit 4)) = OInt
typeOf (Sub (CharLit 'a') (IntLit 4)) = OWrong
typeOf (If (BoolLit True) (IntLit 4) (IntLit 5)) = OInt
typeOf (If (BoolLit True) (IntLit 4) (BoolLit True)) = OWrong
typeOf (If (IntLit 3) (IntLit 4) (IntLit 5)) = OWrong
```

Your program should incorporate a reasonable notion of what the exact type rules are. (Exactly what "reasonable" is left up to you; explain any decisions you feel the need to make.)

21. (50 points total) Read either one of the following two articles:

- Paul Hudak and Tom Makucevich and Syam Gadde and Bo Whong. Haskore music notation–An Algebra of Music, *Journal of Functional Programming*, 6(3):465–483, May 1996.
- Conal Elliott. An Embedded Modeling Language Approach to Interactive 3D and Multimedia Animation. *IEEE Transactions on Software Engineering*, 25(3):291–308, May 1999.

a published research article in a journal or conference proceedings

Write a short (1 or 2 page maximum) review of the article, stating:

- (10 points) what the problem was that the article was claiming to solve,
- (20 points) the main points made in the article and what you learned from it,
- (20 points) what contribution it make vs. any related work mentioned in the article.

In your writing, be sure to digest the material; that is, don't just select various quotes from the article and string them together, instead, really summarize it.