

Lecture-12

Face Recognition

Simple Approach

- Recognize faces (mug shots) using gray levels (appearance)
- Each image is mapped to a long vector of gray levels
- Several views of each person are collected in the model-base during training
- During recognition a vector corresponding to an unknown face is compared with all vectors in the model-base
- The face from model-base, which is closest to the unknown face is declared as a recognized face.

Problems and Solution

- Problems :
 - Dimensionality of each face vector will be very large (250,000 for a 512X512 image!)
 - Raw gray levels are sensitive to noise, and lighting conditions.
- Solution:
 - Reduce dimensionality of face space by finding principal components (eigen vectors) to span the face space
 - Only a few most significant eigen vectors can be used to represent a face, thus reducing the dimensionality

Eigen Vectors and Eigen Values

The eigen vector, x , of a matrix A is a special vector, with the following property

$$Ax = \lambda x \quad \text{Where } \lambda \text{ is called eigen value}$$

To find eigen values of a matrix A first find the roots of:

$$\det(A - \lambda I) = 0$$

Then solve the following linear system for each eigen value to find corresponding eigen vector

$$(A - \lambda I)x = 0$$

Example

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix}$$

Eigen Values

$$\lambda_1 = 7, \lambda_2 = 3, \lambda_3 = -1$$

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Eigen Vectors

Eigen Values

$$\det(A - \lambda I) = 0$$

$$\det \left(\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) = 0$$

$$\det \begin{bmatrix} -1-\lambda & 2 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & 7-\lambda \end{bmatrix} = 0$$

$$(-1-\lambda)(3-\lambda)(7-\lambda) = 0$$

$$(-1-\lambda)(3-\lambda)(7-\lambda) = 0$$

$$\lambda = -1, \quad \lambda = 3, \quad \lambda = 7$$

Eigen Vectors

$$\lambda = -1 \quad (A - \lambda I)x = 0$$

$$\left(\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 \\ 0 & 4 & 4 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0 + 2x_2 + 0 = 0$$

$$0 + 4x_2 + 4x_3 = 0$$

$$0 + 0 + 8x_3 = 0$$

$$x_1 = 1, \quad x_2 = 0, \quad x_3 = 0$$

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Face Recognition

Collect all gray levels in a long vector u :

$$u = (I(1,1), \dots, I(1,N), I(2,1), \dots, I(2,N), \dots, I(M,1), \dots, I(M,N))^T$$

Collect n samples (views) of each of p persons in matrix A ($MN \times pn$):

$$A = [u_1^1, \dots, u_n^1, u_1^2, \dots, u_n^2, \dots, u_1^p, \dots, u_n^p]$$

Form a correlation matrix L ($MN \times MN$):

$$L = AA^T$$

Compute eigen vectors, $f_1, f_2, f_3, \dots, f_{n_1}$, of L , which form a bases for whole face space

Face Recognition

Each face, u , can now be represented as a linear combination of eigen vectors

$$u = \sum_{i=1}^{n_1} a_i f_i$$

Eigen vectors for a symmetric matrix are orthonormal:

$$f_i^T f_j = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

Face Recognition

$$u_x^T \mathbf{f}_i = \left(\sum_{i=1}^n a_i \mathbf{f}_i \right)^T \mathbf{f}_i$$

$$= (a_1 \mathbf{f}_1^T + a_2 \mathbf{f}_2^T + \dots + a_i \mathbf{f}_i^T + \dots + a_n \mathbf{f}_n^T) \mathbf{f}_i$$

$$u_x^T \mathbf{f}_i = (a_1 \mathbf{f}_1^T \mathbf{f}_i + a_2 \mathbf{f}_2^T \mathbf{f}_i + \dots + a_i \mathbf{f}_i^T \mathbf{f}_i + \dots + a_n \mathbf{f}_n^T \mathbf{f}_i)$$

$$u_x^T \mathbf{f}_i = a_i$$

Therefore: $a_i = u_x^T \mathbf{f}_i$

Face Recognition

L is a large matrix, computing eigen vectors of a large matrix is time consuming. Therefore compute eigen vectors of a smaller matrix, C:

$$C = A^T A$$

Let \mathbf{a}_i be eigen vectors of C, then $A\mathbf{a}_i$ are the eigen vectors of A:

$$C\mathbf{a}_i = \lambda_i \mathbf{a}_i$$

$$A^T A\mathbf{a}_i = \lambda_i \mathbf{a}_i$$

$$AA^T(A\mathbf{a}_i) = \lambda_i(A\mathbf{a}_i)$$

$$L(A\mathbf{a}_i) = \lambda_i(A\mathbf{a}_i)$$

Training

- Create A matrix from training images
- Compute C matrix from A .
- Compute eigenvectors of C .
- Compute eigenvectors of L from eigenvectors of C .
- Select few most significant eigenvectors of L for face recognition.
- Compute coefficient vectors corresponding to each training image.
- For each person, coefficients will form a cluster, compute the mean of cluster.

Recognition

- Create a vector u for the image to be recognized.
- Compute coefficient vector for this u .
- Decide which person this image belongs to, based on the distance from the cluster mean for each person.

```

load faces.mat
C=A'*A;
[vectorC,valueC]=eig(C);
ss=diag(valueC);
[ss,iii]=sort(-ss);
vectorC=vectorC(:,iii);
vectorL=A*vectorC(:,1:5);
Coeff=A'*vectorL;
for I=1:30
    model(i, :)=mean(coeff((5*(i-1)+1):5*I,:));
end
while (1)
    imagename=input('Enter the filename of the image to
Recognize(0 stop):');
    if (imagename <1)
        break;
    end;
    imageco=A(:,imagename)*vectorL;
    disp ('');
    disp ('The coefficients for this image are:');

```

```

    mess1=sprintf('%0.2f %0.2f %0.2f %0.2f %0.2f',
imageco(1),imageco(2), imageco(3),imageco(4),
imageco(5));
    disp(mess1);
    top=1;
    for I=2:30
        if (norm(model(i,:)-imageco,1)<norm(model
(top, :)-imageco,1))
            top=i
        end
    end
    mess1=sprintf('The image input was a image of person
number %d',top);
    disp(mess1);
end
b=A(:,81);
b=reshape(b,34,51);
imshow(b,gray(255));

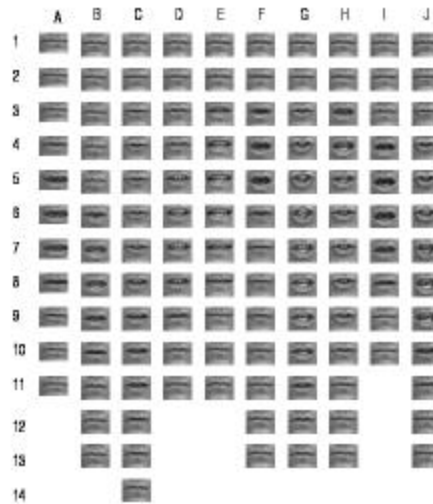
```


Webpage

<http://vismod.www.media.mit.edu/vismod/demos/>

Visual Lipreading

Image Sequences of “A” to “J”



Particulars

- **Problem:** Pattern differ spatially
- **Solution:** Spatial registration using SSD
- **Problem :** Articulations vary in length, and thus, in number of frames.
- **Solution:** Dynamic programming for temporal warping of sequences.
- **Problem:** Features should have compact representation.
- **Solution:** Principle Component Analysis.

Feature Subspace Generation

- Generate a lower dimension subspace onto which image sequences are projected to produce a vector of coefficients.
- Components
 - Sample Matrix
 - Most Expressive Features

Generating the Sample Matrix

- Consider e letters, each of which has a training set of K sequences. Each sequence is composed of images:

$$I_1, I_2, \dots, I_P$$

- Collect all gray-level pixels from all images in a sequence into a vector:

$$u = (I_1(1,1), \dots, I_1(M, N), I_2(1,1), \dots, I_2(M, N), \dots, I_p(1,1), \dots, I_p(M, N))$$

. Generating the Sample Matrix

- For letter \mathbf{W} , collect vectors into matrix T

$$T_w = [u^1, u^2, \dots, u^K]$$

- Create sample matrix A :

$$A = [T_1, T_2, \dots, T_e]$$

- The eigenvectors of a matrix $L = AA^T$ are defined as:

$$L\mathbf{f}_i = \lambda_i \mathbf{f}_i$$

The Most Expressive Features

- \mathbf{f} is an orthonormal basis of the sample matrix.

- Any image sequence, u , can be represented as:

$$u = \sum_{n=1}^Q a_n \mathbf{f}_n = \mathbf{f}a$$

- Use Q most significant eigenvectors.

- The linear coefficients can be computed as:

$$a_n = u^T \mathbf{f}_n$$

Training Process

- Model Generation
 - Warp all the training sequences to a fixed length.
 - Perform spatial registration (SSD).
 - Perform PCA.
 - Select Q most significant eigensequences, and compute coefficient vectors “a”.
 - Compute mean coefficient vector for each letter.

Warping

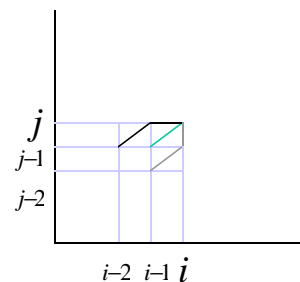
$$A = [a_1, a_2, \dots, a_i, a_I]$$

$$B = [b_1, b_2, \dots, b_j, b_J]$$

$$d_{ij} = |a_i - b_j|$$

$$g_{11} = 2d_{11}$$

$$g(i, j) = \min \begin{bmatrix} g(i-1, j-2) + 2d(i, j-1) + d(i, j) \\ g(i-1, j-1) + 2d(i, j) \\ g(i-2, j-1) + 2d(i-1, j) + d(i, j) \end{bmatrix}$$



Recognition

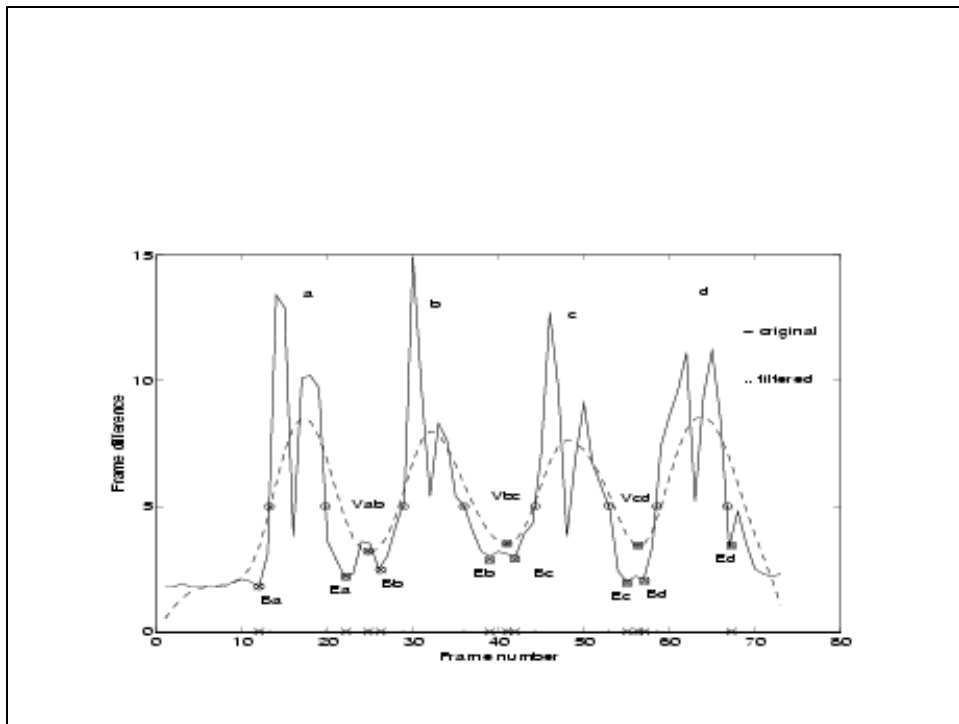
- Warp the unknown sequence.
- Perform spatial registration.
- Compute:
$$a_i^x = u_x^T \cdot \mathbf{f}_i$$
$$d^w = \| a^w - a^x \|$$
- Determine best match by $\min_w (d^w)$

Extracting letters from Connected Sequences

- Average absolute intensity difference function

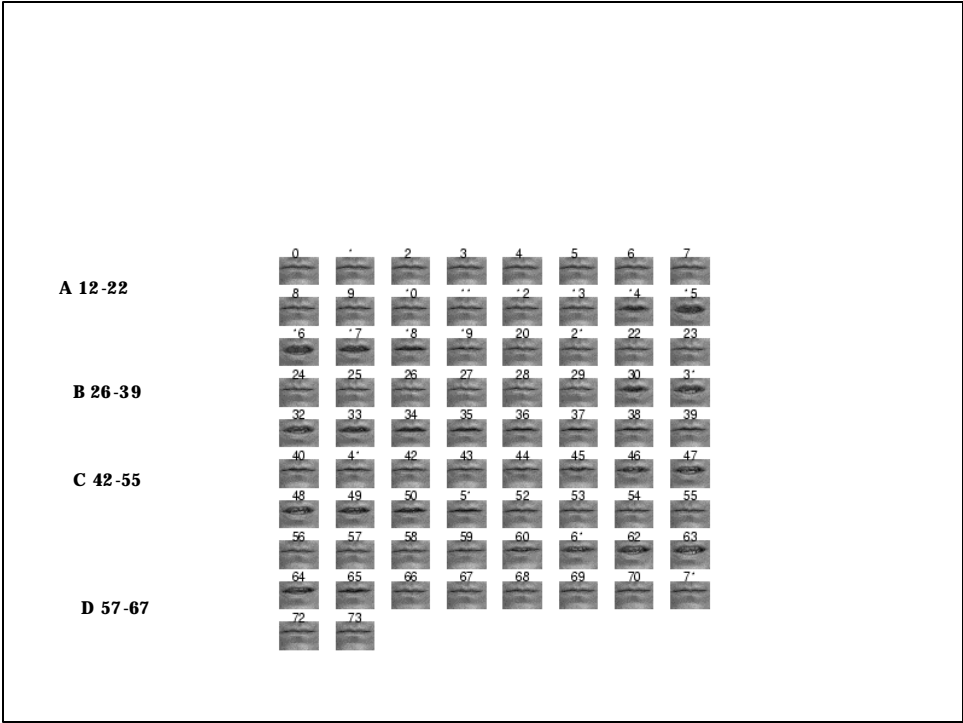
$$f(n) = \frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N \| I_n(x, y) - I_{n-1}(x, y) \|$$

- f is smoothed to obtain g .
- Articulation intervals correspond to peaks and non-articulation intervals correspond to valleys in “ g ”.

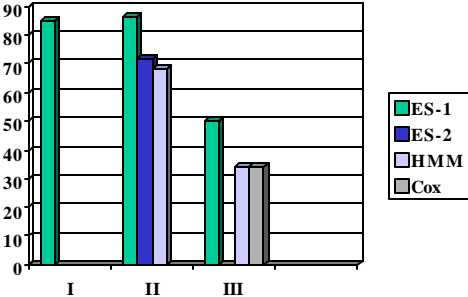


Extracting letters from Connected Sequences

- Detect valleys in g.
- From valley locations in g, find locations where f crosses high threshold.
- Locate beginning and ending frames.

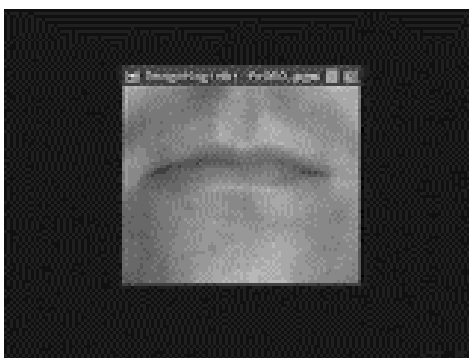


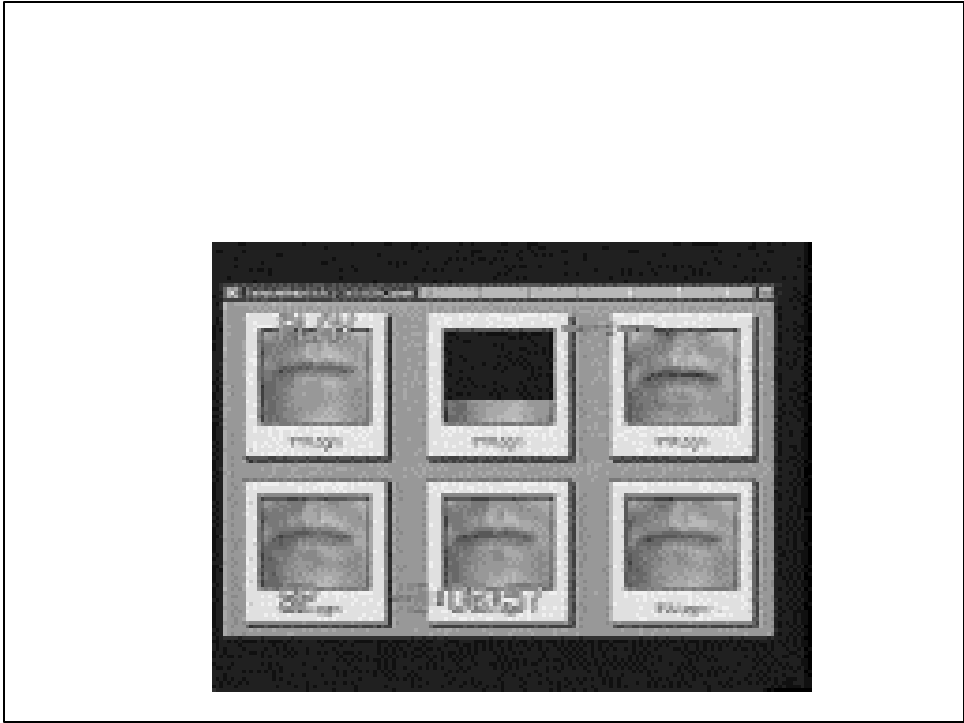
Results



- I: "A" to "J" one speaker, 10 training seqs
- II. "A" to "M", one speaker, 10 training seqs
- III. "A" to "Z", ten speakers, two training seqs/letter/person

Show Video Clip





paper

<http://www.cs.ucf.edu/~vision/papers/shah/97/NDS97.pdf>

Program-2 & 3

- For the program-2 you will implement “Synthesizing Realistic Facial Expressions from Photographs” method (Lecture-11).
 - You will assume one view of face is available, the aim is to estimate a pose of camera, translation, rotation, scaling, etc.
 - Do not estimate the changes in “p”, vertices.
 - If you have a better face model, like Alias, use it, otherwise use Candide model from the class webpage.
 - Select 13 feature points manually
 - Synthesize a face image from a novel view, once the pose is correctly estimated.
 - Due Nov 7

Program-2 & 3

- For Program-3 implement “Motion Estimation Using Flexible Wireframe Model” (Lecture-9).
 - Use the output of Program-2, conformed wireframe model
 - Assume simple optical flow constraint equation, no need to use generalized optical flow constraint equation
 - Using estimated motion and changes in wireframe mode, synthesize image sequence, and compare it with the original sequence for video compression (MPEG-4).
 - Due Nov 30