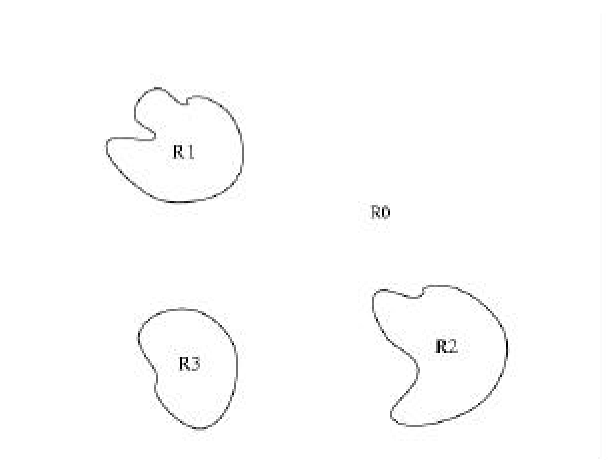
Lecture-10

Region Merging

&

Region Properties

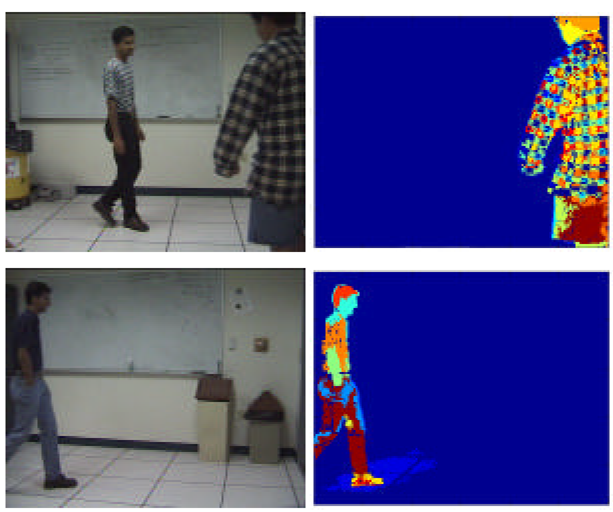
Segmentation



Segmentation

- Partition $f(x,y)$ into sub-images: R_1, R_2, ….,R_n such that the following constraints are satisfied:
  - $\bigcup_{i=1}^{n} R_i = f(x,y)$
  - $R_i \cap R_j = \phi, i \neq j$
  - Each sub-mage satisfies a predicate or set of predicates
    - All pixels in any sub-image musts have the same gray levels.
    - All pixels in any sub-image must not differ more than some threshold
    - All pixels in any sub-image may not differ more than some threshold from the mean of the gray of the region
    - The standard deviation of gray levels in any sub-image must be small.

Initial Segmentation



Applications of Segmentation

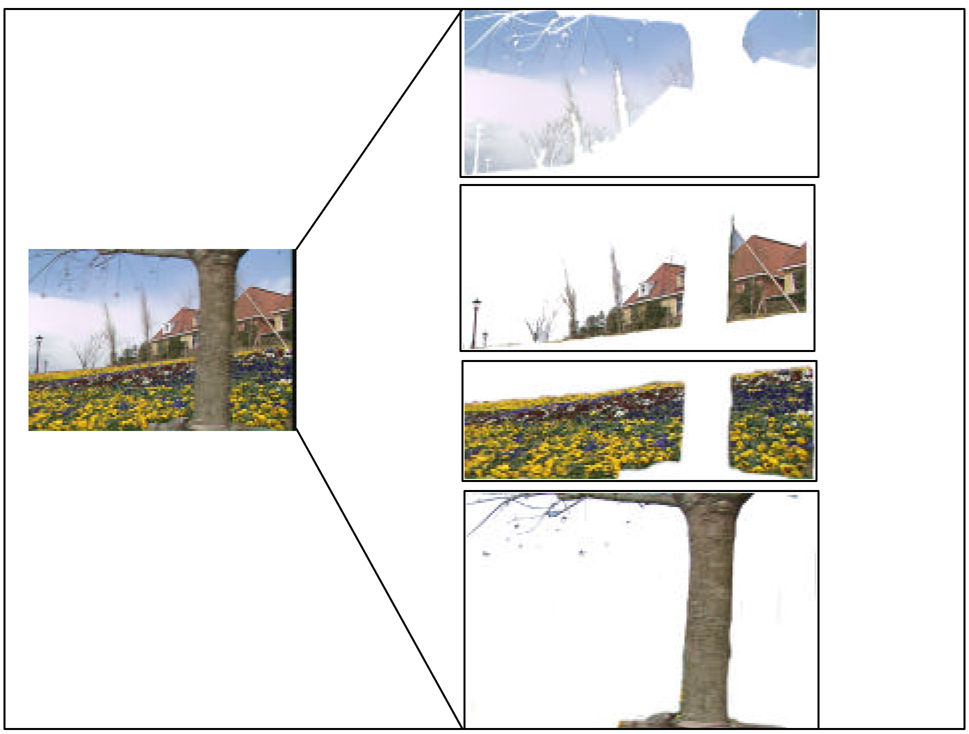
- Object recognition
- MPEG-4 video compression

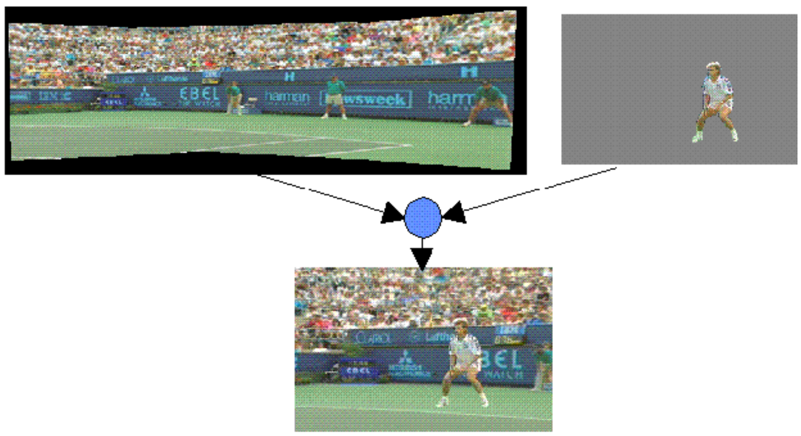
Object Recognition Using Region Properties

- Training
  - For all training samples of each model object
    - Segment the image
    - Compute region properties (features)
  - Compute mean feature vector for each model object
- Recognition
  - Given an image of unknown object,
    - segment the image
    - compute its feature vector
    - match the vector to all possible models to determine its identity.

Object-Based Compression (MPEG-4)

- Advantages of OBC
  - large increase in compression ratio
  - allows manipulation of compressed video (inserting, deleting and modifying objects)
- How does it work?
  - Find objects (*Object Segmentation*)
  - code objects and their locations separately
    - through masks or splines
  - Build mosaics of globally static objects
  - Render scene at receiver





Steps in Seed Segmentation Using Histogram

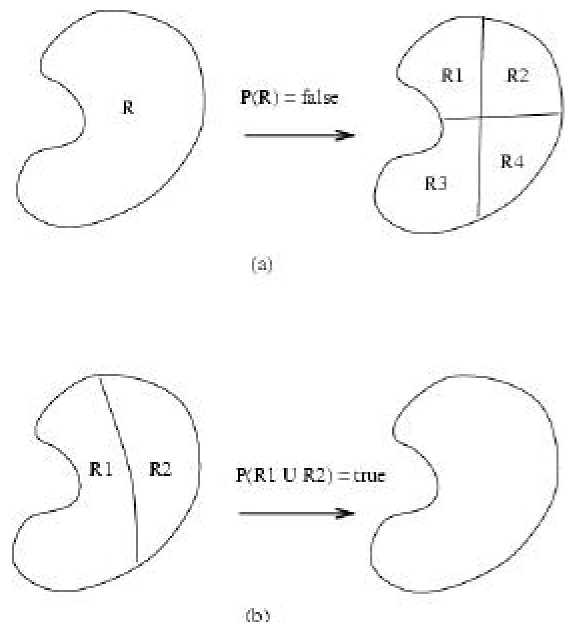
1. Compute the histogram of a given image.
2. Smooth the histogram by averaging peaks and valleys in the histogram.
3. Detect good peaks by applying thresholds at the valleys.
4. Segment the image into several binary images using thresholds at the valleys.
5. Apply connected component algorithm to each binary image find connected regions.

Improving Seed Segmentation

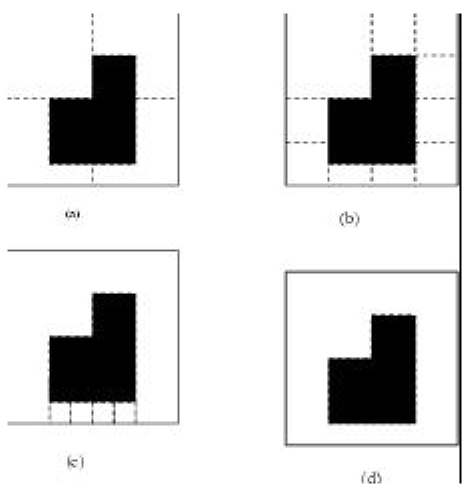
- Merge small neighboring regions
- Split large regions
- Remove weak boundaries between adjacent regions

Split and Merge

1. Split region *R* into four adjacent regions (quadrants) if *Predicate*(*R*) = *false.*
2. Merge any two adjacent regions R_1 and R_2 if $R_1 \, U \, R_2$= *true.*
3. Stop when no further merging and splitting are possible.



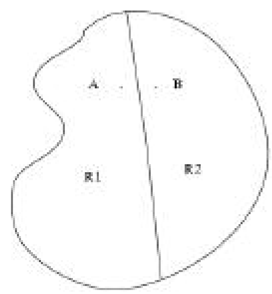
Split and Merge



Phagocyte Algorithm: Weakness of Boundaries

$$W(A,B) = \begin{cases} 1 & \text{if } S(A,B) < T_1 \\ 0 & \text{Otherwise} \end{cases}$$

$$W(Boundary) = \sum_{\forall A,B} W(A,B)$$



Phagocyte Algorithm

1. Merge two regions if

$$\frac{W(Boundary)}{\min(P_1, P_2)} > T_2, \quad 0 \le T_2 \le 1$$

Phagocyte

Where P_1 and P_2 are the perimeters of regions R_1 and R_2.

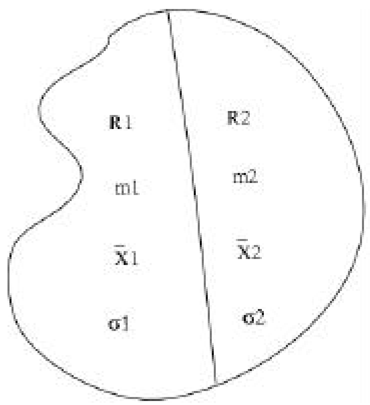
if threshold T_2 > 1/2 then the resulting boundary must shrink, and If threshold T_2 <1/2 then the boundary may grow

2. Merge regions if

$$\frac{W(Boundary)}{\text{Total number of points on the border}} > T_3, \quad 0 < T_3 \le 1$$

Weakness

Merging Using Likelihood Ratio Test



Merging Using Likelihood Ratio Test

H_1: There are two regions
H_2: There is one region

$$p(x) = \frac{1}{\sqrt{2\hat{\lambda} s}} e^{-\frac{(x-\bar{x})^2}{2s^2}}$$

$$p(x_1,\ldots,x_{m_1}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_1}}\right)^2 e^{-\frac{m_1}{2}} \qquad p(x_{m_1+1}, x_{m_1+2},\ldots,x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_2}}\right)^{m_2} e^{-\frac{m_2}{2}}$$

$$p(x_1, x_{2,\ldots}, x_{m_1}, x_{m_1+1}, x_{m_1+2},\ldots,x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_0}}\right)^{m_1+m_2} e^{-\frac{m_1+m_2}{2}}$$

$$P(H_2) = P(x_1, x_{2,\ldots}, x_{m_1}, x_{m_1+1}, x_{m_1+2},\ldots,x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_0}}\right)^{m_1+m_2} e^{-\frac{m_1+m_2}{2}}$$

$$P(H_1) = p(x_1,\ldots,x_{m_1}).p(x_{m_1+1}, x_{m_1+2},\ldots,x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_1}}\right)^{m_1} e^{-\frac{m_1}{2}} . \left(\frac{1}{\sqrt{2\hat{\lambda} s_2}}\right)^{m_2} e^{-\frac{m_2}{2}}$$

Merging Using Likelihood Ratio Test

$$P(H_2) = P(x_1, x_{2,\dots}, x_{m_1}, x_{m_1+1}, x_{m_1+2}, \dots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda}}\, s_0}\right)^{m_1+m_2} e^{-\frac{m_1+m_2}{2}}$$

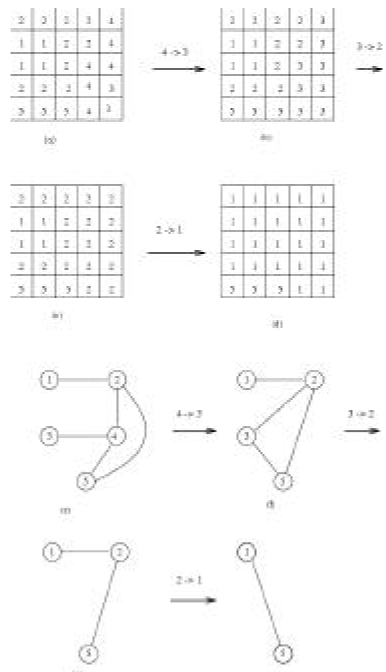
$$P(H_1) = p(x_1, \dots, x_{m_1}).p(x_{m_1+1}, x_{m_1+2}, \dots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda}}\, s_1}\right)^{m_1} e^{-\frac{m_1}{2}} . \left(\frac{1}{\sqrt{2\hat{\lambda}}\, s_2}\right)^{m_2} e^{-\frac{m_2}{2}}$$

$$LH = \frac{P(H_1)}{P(H_2)} = \frac{(s_0)^{m_1+m_2}}{(s_1)^{m_1}(s_2)^{m_2}}$$

Merge regions if $LH<T$.

Region Adjacency Graph

- Regions are nodes
- Adjacent regions are connected by an arc



Issues in Region Growing

- The number of thresholds used in the algorithm.
- The order of merging is very important.
- Seed segmentation is important.

Edge Detection Vs Region Segmentation

- Region segmentation results in closed boundaries, while the boundaries obtained by edge detection are not necessarily closed.
- Region segmentation can be improved by using multi-spectral images (e.g. color images), however there is not much an advantage in using multi-spectral images in edge detection.
- The position of a boundary is localized in edge detection, but not necessarily in region segmentation.

Geometrical Properties

Area

$$A = \sum_{x=0}^{m} \sum_{y=0}^{n} B(x,y)$$

Centroid

$$\bar{x} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} xB(x,y)}{A}, \quad \bar{y} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} yB(x,y)}{A}$$

Moments

$$M_x^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} xB(x,y), \quad M_x^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} xB(x,y), \quad M_y^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} yB(x,y)$$

$$M_x^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} x^2 B(x,y), \quad M_y^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} xB(x,y), \quad M_y^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} y^2 B(x,y)$$

Compactness

$$C = 4\lambda \frac{A}{\_^2}$$

Perimeter: The sum of its border points of the region. A pixel which has at least one pixel in its neighborhood from the background is called a border pixel.

Moments

Binary image

General Moments

$$m_{pq} = \int \int x^p y^q r(x, y) dxdy$$

Central Moments (Translation Invariant)

$$m_{pq} = \int\int (x - \bar{x})^p (y - \bar{y})^q \, r(x, y) \; d(x - \bar{x}) d(y - \bar{y})$$

$$\bar{x} = \frac{m_{10}}{m_{00}}, \bar{y} = \frac{m_{01}}{m_{00}}$$

centroid

Central Moments

$$m_{00} = m_{00} \equiv m$$
$$m_{01} = 0$$
$$m_{10} = 0$$
$$m_{20} = m_{20} - m\bar{x}^2$$
$$m_{11} = m_{11} - m\bar{x}\bar{y}$$
$$m_{02} = m_{02} - m\bar{y}^2$$
$$m_{30} = m_{30} - 3m_{20}\bar{x} + 2m\bar{x}^3$$
$$m_{21} = m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2m\bar{x}^2 y$$
$$m_{12} = m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2m\bar{x}y^2$$
$$m_{03} = m_{03} - 3m_{02}\bar{y} + 2m\bar{y}^3$$

Moments

Hu Moments: translation, scaling and rotation invariant

$$u_1 = m_{20} + m_{02}$$

$$u2 = (m_{20} - m_{02})^2 + m_{11}{}^2$$

$$u_3 = (m_{30} - 3m_{12})^2 + (3m_{12} - m_{03})^2$$

$$u_4 = (m_{30} + m_{12})^2 + (m_{21} + m_{03})^2$$

$\vdots$

Orientation of the Region

$$E = \iint (x \sin q - y \cos q)^2 B(x, y) dx dy$$

$$\sin 2q = \pm \frac{b}{\sqrt{b^2 + (a - c)^2}}$$

$$\cos 2q = \pm \frac{a - c}{\sqrt{b^2 + \ldots}}$$

$$a = \iint x'^2 B(x,y) dx' dy'$$

$$b = \iint x' y' B(x, y) dx' dy'$$

$$c = \iint y'^2 B(x, y) dx' dy'$$

$$a = \sum \sum x^2 B(x,y) - A\bar{x}^2$$

$$b = 2 \sum \sum xyB(x, y) - A\bar{x}\bar{y}$$

$$c = \sum \sum y^2 B(x, y) - A\bar{y}^2$$