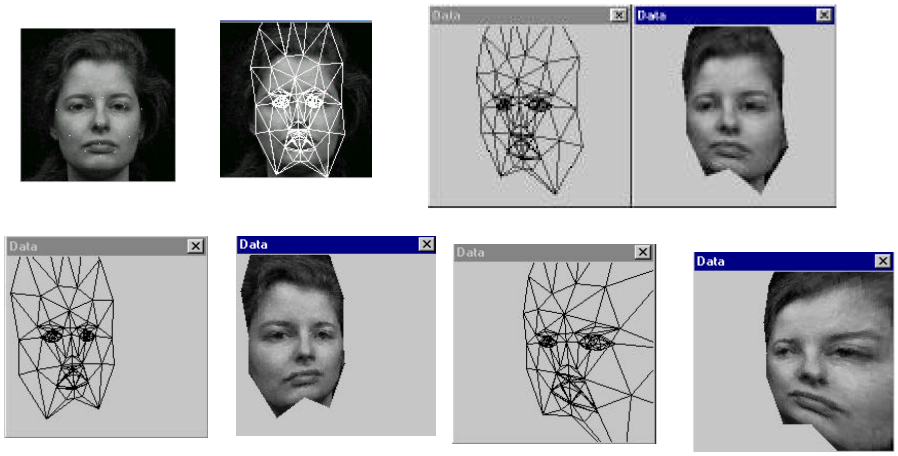
Lecture-2

Imaging Geometry

Transformations

- Translation
- Scaling
- Rotation
- Perspective
- Homogenous

Pose Estimation/Image Synthesis



Motion Estimation

Motion Estimation

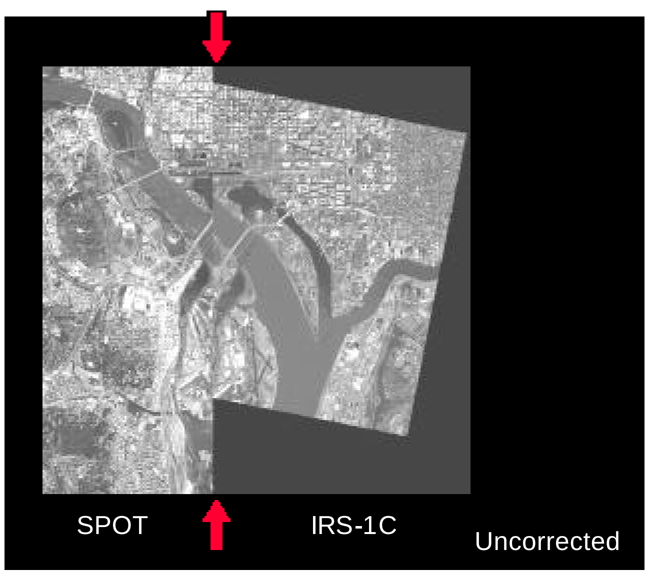
Object Recognition

- Robotics
- Image Registration

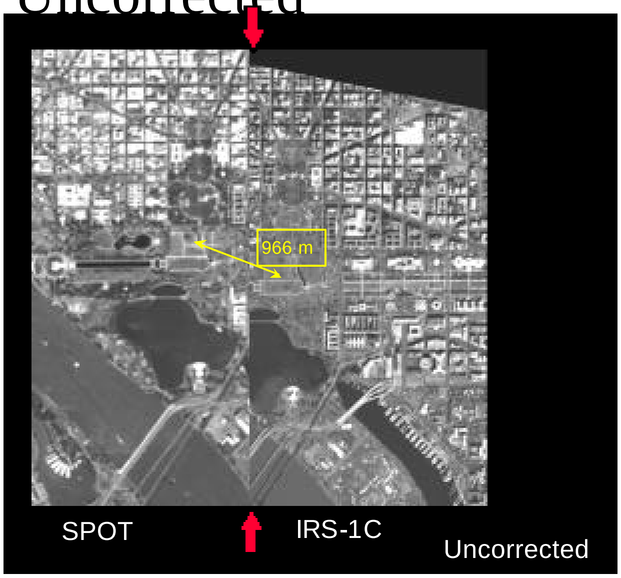
IRS-1C - Washington, DC

SPOT - Washington, DC

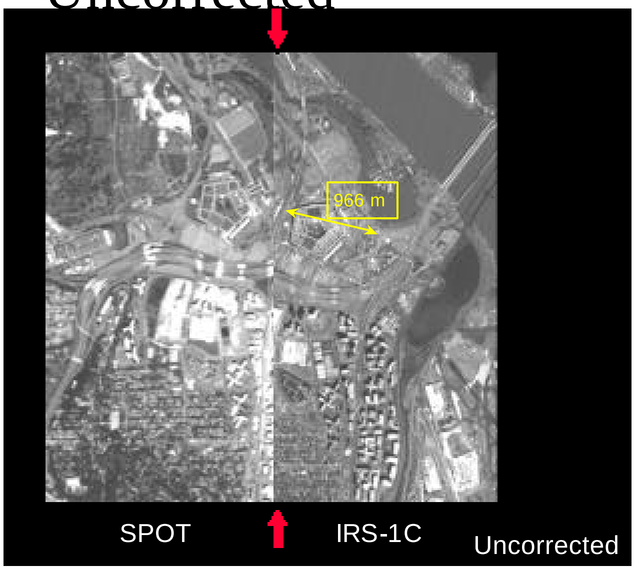
SPOT/IRS-1C
Uncorrected



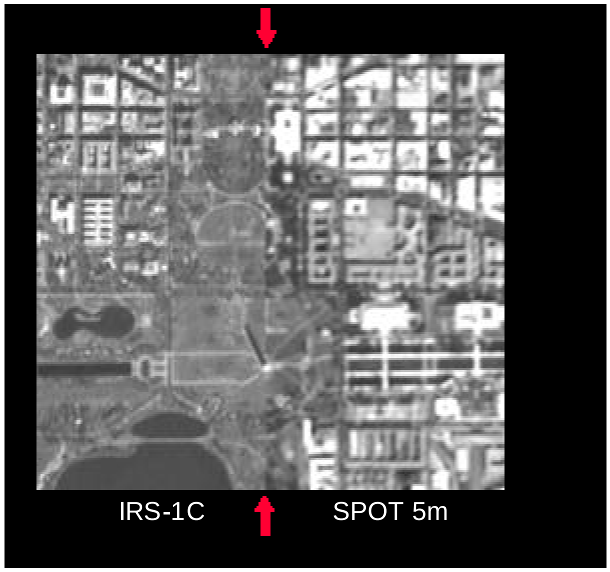
SPOT/IRS-1C Uncorrected



SPOT/IRS-1C Uncorrected



IRS-1C/SPOT Registered



Registered IRS-1C to SPOT

Translation

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = T \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Translation Matrix}$$

$$\mathrm{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$TT^{-1} = T^{-1}T = I$$

$$\begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \times S_x \\ Y_1 \times S_y \\ Z_1 \times S_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = S \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Scaling Matrix}$$

$$\mathrm{S}^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 & 0 \\ 0 & 1/S_y & 0 & 0 \\ 0 & 0 & 1/S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

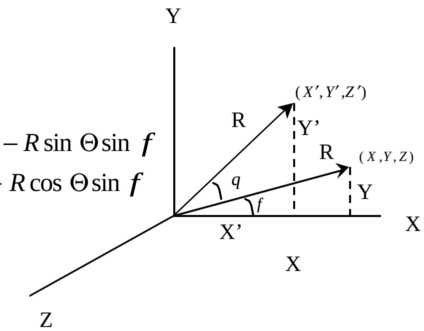
$$SS^{-1} = S^{-1}S = I$$

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{bmatrix} 1/S_x & 0 & 0 & 0 \\ 0 & 1/S_y & 0 & 0 \\ 0 & 0 & 1/S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation

$$X = R\cos f$$

$$Y = R\sin f$$



$$X' = R\cos(\Theta + f) = R\cos\Theta\cos f - R\sin\Theta\sin f$$

$$Y' = R\sin(\Theta + f) = R\sin\Theta\cos f + R\cos\Theta\sin f$$

$$X' = X\cos\Theta - Y\sin\Theta$$

$$Y' = X\sin\Theta + Y\cos\Theta$$

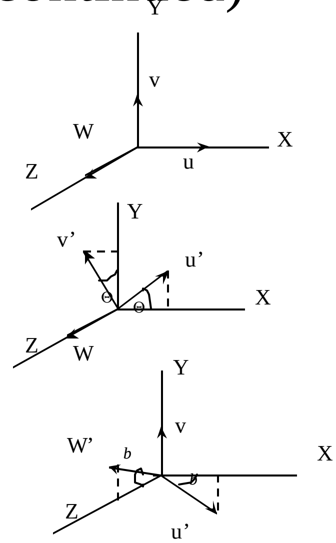
$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Rotation (continued)

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_q^Z = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{-b}^Y = \begin{bmatrix} \cos b & 0 & -\sin b \\ 0 & 1 & 0 \\ \sin b & 0 & \cos b \end{bmatrix}$$



$$(R_q^Z)^{-1} = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(R_q^Z)^{-1} = (R_q^Z)^T$$

$$(R_q^Z)(R_q^Z)^T = I$$

Rotation matrices are orthonormal matrices

$$r_i . r_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwis} \end{cases}$$

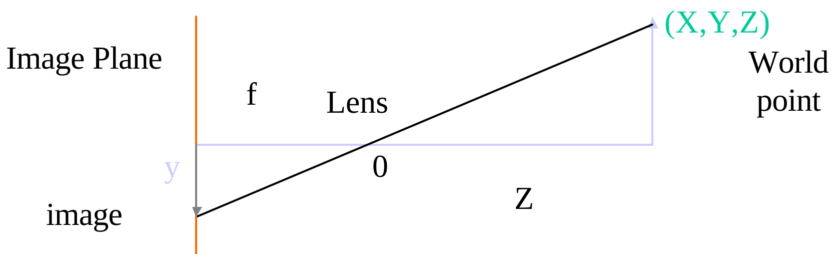
Euler Angles

$$R = R_Z^a R_Y^b R_X^g = \begin{bmatrix} \cos a \cos b & \cos a \sin b \sin g - \sin a \cos g & \cos a \sin b \cos g + \sin a \sin g \\ \sin a \cos b & \sin a \sin b \sin g + \cos a \cos g & \sin a \sin b \cos g - \cos a \sin g \\ -\sin b & \cos b \sin g & \cos b \cos g \end{bmatrix}$$

if angles are small(

$$R = \begin{bmatrix} 1 & -a & b \\ a & 1 & -g \\ -b & g & 1 \end{bmatrix}$$

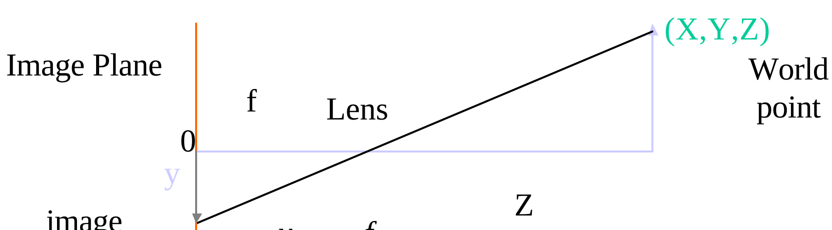
Perspective Projection (origin at the lens center)



$$\frac{-y}{Y} = \frac{f}{Z}$$

$$y = -\frac{fY}{Z} \qquad x = -\frac{fX}{Z}$$

Perspective Projection (origin at image center)



$$\frac{-y}{Y} = \frac{f}{Z-f}$$

$$y = -\frac{fY}{Z-f} \qquad x = -\frac{fX}{Z-f}$$

$$y = \frac{fY}{f-Z} \qquad x = \frac{fX}{f-Z}$$

Perspective

$$\begin{bmatrix} x \\ y \\ \\ \end{bmatrix} = \begin{bmatrix} & ? & \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Image coordinates → $\begin{bmatrix} x \\ y \end{bmatrix}$; World coordinates → $\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

$(X, Y, Z) \rightarrow \quad \rightarrow \rightarrow \qquad (kX, kY, kZ, k)$, Homogenous transformation

$(C_{h1}, C_{h2}, C_{h3}, C_{h4}) \rightarrow \qquad (\frac{C_{h1}}{C_{h4}}, \frac{C_{h2}}{C_{h4}}, \frac{C_{h3}}{C_{h4}})$, Inverse homogenous

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}$$

Perspective

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix}$$

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix} = \begin{bmatrix} kX \\ kY \\ kZ \\ k - \frac{kZ}{f} \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}} = \frac{kX}{k - \frac{kZ}{f}} = \frac{fX}{f - Z}$$

$$y = \frac{C_{h2}}{C_{h4}} = \frac{kY}{k - \frac{kZ}{f}} = \frac{fY}{f - Z}$$

Camera Model

- Camera is at the origin of the world coordinates first
- Then translated (G),
- then rotated around Z axis in counter clockwise direction,
- then rotated again around X in counter clockwise direction, and
- then translated by C.

$$C_h = PCR_{-f}^{X} R_{-q}^{Z} G W_h$$

Camera Model

$$C_h = PCR_{-f}^{X} R_{-q}^{Z} G W_h$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}, R_{-q}^{Z} = \begin{bmatrix} \cos q & \sin q & 0 & 0 \\ -\sin q & \cos q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{-f}^{X} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos f & \sin f & 0 \\ 0 & -\sin f & \cos f & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & -r_1 \\ 0 & 1 & 0 & -r_2 \\ 0 & 0 & 1 & -R_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Model

$$C_h = PCR^X_{-f} R^Z_{-q} GW_h$$

$$x = f \frac{(X - X_0)\cos q + (Y - Y_0)\sin q - r_1}{-(X - X_0)\sin q \sin f + (Y - Y_0)\cos q \sin f - (Z - Z_0)\cos f + r_3 + f}$$

$$f \frac{(X - X_0)\sin q \cos f + (Y - Y_0)\cos q \cos f + (Z - Z_0)\sin f - r_2}{-(X - X_0)\sin q \sin f + (Y - Y_0)\cos q \sin f - (Z - Z_0)\cos f + r_3 + f}$$

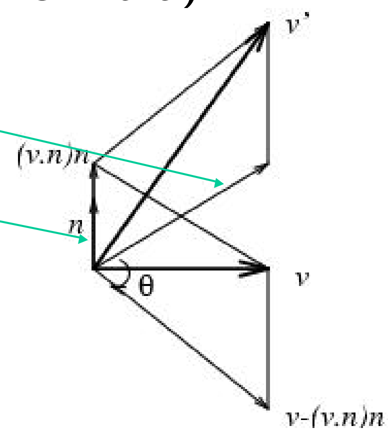
Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$V = (V.n)n + (V - (V.n)n)$$

$$V'_{\perp} = \cos q (V - (V.n)n) + \sin q (n \times (V - (V.n)n))$$

$$V'_{II} = (V.n)n$$

$$V' = V'_{\perp} + V'_{II}$$



$$V' = \cos q\, V + \sin q\; n \times V + (1 - \cos q)(V.n)n$$

$$V' = V + \sin q\; n \times V + (1 - \cos q)\, n \times (n \times V)$$

$$n \times (n \times V) = (V.n)n - V$$

$$n \times (n \times V) + V = (V.n)n$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$V' = V + \sin q \; n \times V + (1 - \cos q)\; n \times (n \times V)$$

$$V' = R(n, q)V$$

$$R(n, q) = I + \sin q \; X(n) + (1 - \cos q) X^2(n)$$

$$X(n) = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$r = \| r \| \frac{r}{\| r |} = qn$$

$$R(r, q) = I + \sin q \frac{X(r)}{\| r \|} + (1 - \cos q) \frac{X^2(r)}{\| r \|^2}$$

$$X(r) = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$$