# Lecture-3

Camera Model

# Homework (Due 1/25/01)

- Exercises in section 1.11 in "Fundamentals of Computer Vision".
  - 1 to 5
  - 8 to 9
  - 15

# Thursday’s class Room 103 CSB Computer Vision Lab

- Digitization
- Image tools in C
  - Reading an image
  - Writing an image
  - Displaying an image

# Rotation Around an Arbitrary Axis (Rodriguez’s Formula)

$$V = (V.n)n + (V - (V.n)n)$$

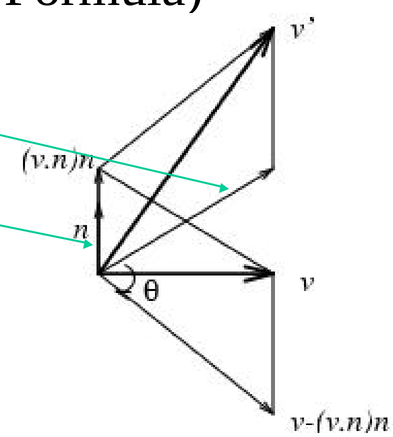
$$V'_{\perp} = \cos\theta(V - (V.n)n) + \sin\theta(n \times (V - (V.n)n)$$

$$V'_{\parallel} = (V.n)n$$

$$V' = V'_{\perp} + V'_{\parallel}$$

$$V' = \cos\theta\, V + \sin\theta\; n \times V + (1 - \cos\theta)(V.n)n$$

$$V' = V + \sin\theta\; n \times V + (1 - \cos\theta)n \times (n \times V)$$




$$n \times (n \times V) = (V.n)n - V$$

$$n \times (n \times V) + V = (V.n)n$$

# Dot and Cross Products

$$V.n = (V_x, V_y, V_z).(n_x, n_y, n_z) = V_x n_x + V_y n_y + V_z n_z$$

$$V \times n = (V_x, V_y, V_z) \times (n_x, n_y, n_z)$$

$$= \begin{vmatrix} i & j & k \\ V_x & V_y & V_z \\ n_x & n_y & n_z \end{vmatrix} = (V_y n_z - V_z n_y, V_z n_x - V_x n_z, V_x n_y - V_y n_x)$$

# Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$V' = V + \sin q \; n \times V + (1 - \cos q) \; n \times (n \times V)$$

$$V' = R(n, q)V$$

$$R(n, q) = I + \sin q \; X(n) + (1 - \cos q) X^2(n)$$

$$X(n) = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

# Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$r = \| r \| \frac{r}{\| r \|} = qn$$

$$R(r, \boldsymbol{q}) = I + \sin \boldsymbol{q} \frac{X(r)}{\| r \|} + (1 - \cos \boldsymbol{q}) \frac{X^2(r)}{\| r \|^2}$$

$$X(r) = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$$

# Camera Model

$$C_h = PCR^X_{-f} R^Z_{-q} GW_h$$

$$C_h = AW_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

# Camera Model

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

# Camera Model

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

One point

$$a_{11}X_1 + a_{12}Y_1 + a_{13}Z_1 + a_{14} - a_{41}X_1x_1 - a_{42}Y_1x_1 - a_{43}Z_1x_1 - a_{44}x_1 = 0$$

$$a_{11}X_2 + a_{12}Y_2 + a_{13}Z_2 + a_{14} - a_{41}X_2x_2 - a_{42}Y_2x_2 - a_{43}Z_2x_2 - a_{44}x_2 = 0$$

$$\vdots$$

$$a_{11}X_n + a_{12}Y_n + a_{13}Z_n + a_{14} - a_{41}X_nx_n - a_{42}Y_nx_n - a_{43}Z_nx_n - a_{44}x_n = 0$$

$$a_{21}X_1 + a_{22}Y_1 + a_{23}Z_1 + a_{24} - a_{41}X_1y_1 - a_{42}Y_1y_1 - a_{43}Z_1y_1 - a_{44}y_1 = 0$$

$$a_{21}X_2 + a_{22}Y_2 + a_{23}Z_2 + a_{24} - a_{41}X_2y_2 - a_{42}Y_2y_2 - a_{43}Z_2y_2 - a_{44}y_2 = 0$$

$$\vdots$$

$$a_{21}X_n + a_{22}Y_n + a_{23}Z_n + a_{24} - a_{41}X_ny_n - a_{42}Y_ny_n - a_{43}Z_ny_n - a_{44}y_n = 0$$

n points
2n equations,
12 unknowns

# Camera Model

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\
X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 & -x_2 \\
 & & & & & & \vdots & & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 & -y_1 \\
0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 & -y_2 \\
 & & & & & & \vdots & & & & & \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n & -y_n
\end{bmatrix}
\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ a_{44} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$CP = 0$$

# Camera Model

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 \\
X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 \\
 & & & & & & \vdots & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 \\
0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 \\
 & & & & & & \vdots & & & & \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n
\end{bmatrix}
\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \end{bmatrix}
=
\begin{bmatrix} x_1 \\ x_2 \\ \\ \\ \\ x_n \\ y_1 \\ y_2 \\ \\ \\ y_n \end{bmatrix}
$$

$$DQ = R$$

$$D^T DQ = D^T R$$

$$Q = (D^T D)^{-1} D^T R$$

## Camera Model Revisited

$$P_c = RTP_w = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 & -T_x \\ 0 & 1 & 0 & -T_y \\ 0 & 0 & 1 & -T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = R(P_w - T) = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{bmatrix} X - T_x \\ Y - T_y \\ Z - T_z \end{bmatrix}$$

$$P_c = R(P_w - T) = \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} [P_w - T]$$

$$x_c = f \frac{R_1^T (P_w - T)}{R_3^T (P_w - T)}$$

$$y_c = f \frac{R_2^T (P_w - T)}{R_3^T (P_w - T)}$$

## Camera Model Revisited

$$x = -(x_{im} - o_x) s_x$$

$$y = -(y_{im} - o_y) s_y$$

$(x_{im}, y_{im})$ image coordinates

$(x, y)$ camera coordinates

$(o_x, o_y)$ image center (in pixels)

$(s_x, s_y)$ effective size of pixels (in millimeters) in the horizontal and vertical directions.

$$-(x_{im} - o_x) s_x = f \frac{R_1^T (P_w - T)}{R_3^T (P_w - T)}$$

$$-(y_{im} - o_y) s_y = f \frac{R_2^T (P_w - T)}{R_3^T (P_w - T)}$$

# Camera Model Revisited

$$M_{\text{int}} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_x} & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -R_1^T T \\ r_{21} & r_{22} & r_{23} & -R_2^T T \\ r_{31} & r_{32} & r_{33} & -R_3^T T \end{bmatrix}$$

$$M = \begin{bmatrix} -fr_{11} & -fr_{12} & -fr_{13} & fR_1^T T \\ -fr_{21} & -fr_{22} & -fr_{23} & fR_2^T T \\ r_{31} & r_{32} & r_{33} & -R_3^T T \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \end{bmatrix} = M_{\text{int}} M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$