Lecture-4

Camera Model Revisited

- Section 2.4, and 6.3 from "Introductory Techniques for 3-D Computer Vision"

Camera Parameters

- Extrinsic parameters
  - Parameters that define the location and orientation of the camera reference frame with respect to a known world reference frame
    - 3-D translation vector
    - A 3 by 3 rotation matrix
- Intrinsic parameters
  - Parameters necessary to link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame
    - Perspective projection
    - Transformation between camera frame coordinates and pixel coordinates

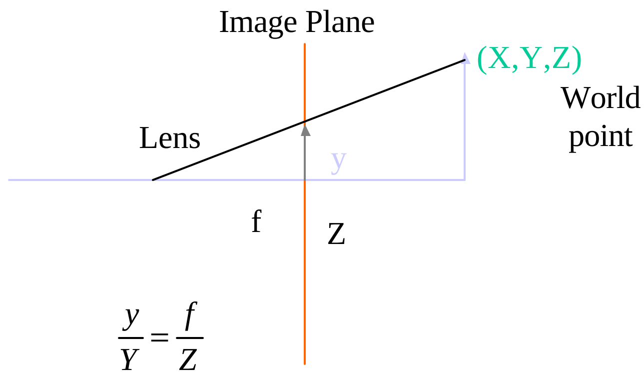
Camera Model Revisited: Rotation & Translation

$$P_c = TRP_w = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Perspective Projection: Revisited



$$\frac{y}{Y} = \frac{f}{Z}$$

$$y = \frac{fY}{Z} \qquad x = \frac{fX}{Z}$$

Camera Model Revisited: Perspective

$$C_h = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

Origin at the lens
Image plane in front of the lens

Camera Model Revisited: Image and Camera coordinates

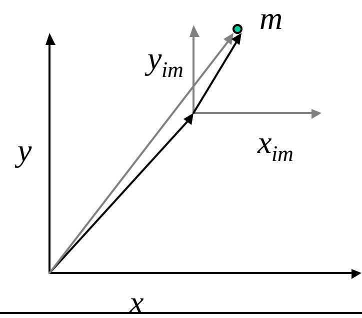
$$x = -(x_{im} - o_x)s_x$$
$$y = -(y_{im} - o_y)s_y$$

$$x_{im} = -\frac{x}{s_x} + o_x$$

$$y_{im} = -\frac{y}{s_y} + o_y$$

$$\begin{bmatrix} x_{im} \\ y_{im} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_x} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(x_{im}, y_{im}) image coordinates
(x, y) camera coordinates
(o_x, o_y) image center (in pixels)
(s_x, s_y) effective size of pixels (in millimeters) in the horizontal and vertical directions.



Camera Model Revisited

$$C_h = C'P'T'R'W_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_x} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M_{int} M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

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f_x effective focal length expressed in effective horizontal pixel size

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Equation 6.18, pp 134

Computing Camera Parameters

- Using known 3-D points and corresponding image points, estimate camera matrix employing pseudo inverse method of section 1.6 (Fundamental of Computer Vision).
- Compute camera parameters by relating camera matrix with estimated camera matrix.
  - Extrinsic
    - Translation
    - Rotation
  - Intrinsic
    - Horizontal f_x and f_y vertical focal lengths
    - Translation o_x and o_y

Comparison

$$\hat{M} = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \\ \hat{m}_{41} & \hat{m}_{42} & \hat{m}_{43} & \hat{m}_{44} \end{bmatrix}$$

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Computing Camera Parameters

estimated

$$\hat{M} = gM$$

Since M is defined up to a scale factor

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \\ \hat{m}_{41} & \hat{m}_{42} & \hat{m}_{43} & \hat{m}_{44} \end{bmatrix} = g \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$\sqrt{\hat{m}_{31}^2 + \hat{m}_{32}^2 + \hat{m}_{33}^2} = |g| \sqrt{r_{31}^2 + r_{32}^2 + r_{33}^2} = |g|$$

Because rotation matr
is orthonormal

Divide each entry of $\hat{M}$ by $|g|$.

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \\ \hat{m}_{41} & \hat{m}_{42} & \hat{m}_{43} & \hat{m}_{44} \end{bmatrix} = g \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$r_{3i} = s\hat{m}_{3i}, \quad i = 1,2,3$$

$$T_z = s\hat{m}_{34}, \quad s = \pm 1$$

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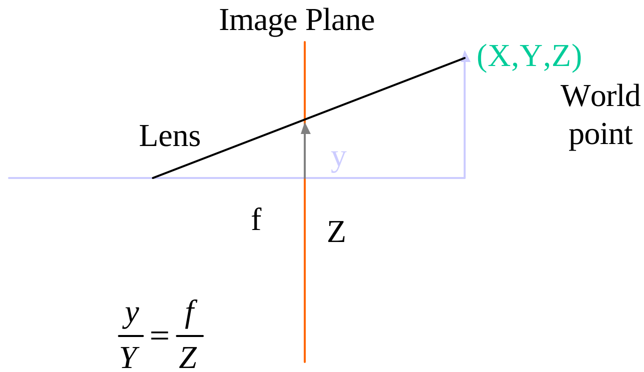

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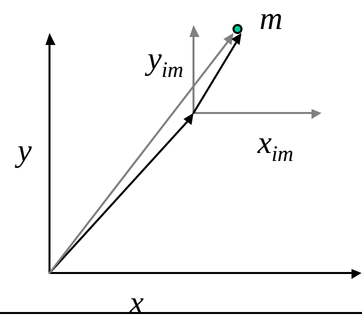

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Computing Camera Parameters

estimated

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