Lecture-5

Computing Camera Parameters

Computing Camera Parameters

- Using known 3-D points and corresponding image points, estimate camera matrix employing pseudo inverse method of section 1.6 (Fundamental of Computer Vision).
- Compute camera parameters by relating camera matrix with estimated camera matrix.
  - Extrinsic
    - Translation
    - Rotation
  - Intrinsic
    - Horizontal and vertical focal lengths
    - Translation o_x and o_y

Comparison

$$\hat{M} = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix}$$

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Computing Camera Parameters: Estimating scale

estimated → $\hat{M} = \gamma M$ Since M is defined up to a scale factor

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$\sqrt{\hat{m}_{31}^2 + \hat{m}_{32}^2 + \hat{m}_{33}^2} = |\gamma| \sqrt{r_{31}^2 + r_{32}^2 + r_{33}^2} = |\gamma|$$

Divide each entry of $\hat{M}$ by $|\gamma|$.

Computing Camera Parameters: estimating third row of rotation matrix and translation in depth

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = g \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$T_z = s\hat{m}_{34}, \quad s = \pm 1$$
$$r_{3i} = s\hat{m}_{3i}, \qquad i = 1,2,3$$

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = g \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Let

$$q_1 = [\hat{m}_{11} \quad \hat{m}_{12} \quad \hat{m}_{13}]$$
$$q_2 = [\hat{m}_{21} \quad \hat{m}_{22} \quad \hat{m}_{23}]$$
$$q_3 = [\hat{m}_{31} \quad \hat{m}_{32} \quad \hat{m}_{33}]$$

Computing Camera Parameters: origin of image

$$q_1^T q_3 = \hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33}$$

$$\begin{aligned}
\hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33} = \\
&(-f_x r_{11} + r_{31}o_x \quad -f_x r_{12} + r_{32}o_x \quad -f_x r_{13} + r_{33}o_x).(r_{31} \quad r_{32} \quad r_{33}) \\
&= (-f_x r_{11} \quad -f_x r_{12} \quad -f_x r_{13}).(r_{31} \quad r_{32} \quad r_{33}) + \\
&(r_{31}o_x \quad r_{32}o_x \quad r_{32}o_x).(r_{31} \quad r_{32} \quad r_{33}) \\
&= (r_{31}o_x \quad r_{32}o_x \quad r_{32}o_x).(r_{31} \quad r_{32} \quad r_{33}) \\
&= (r_{31}^2 o_x + r_{32}^2 o_x + r_{33}^2 o_x) \\
&= o_x (r_{31}^2 + r_{32}^2 + r_{33}^2) \\
q_1^T q_3 &= o_x
\end{aligned}$$

Therefore:

$$o_x = q_1^T q_3$$
$$o_y = q_2^T q_3$$

Computing Camera Parameters: vertical and horizontal focal lengths

$$q_1^T q_1 = \hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13}$$

$$\begin{aligned}
\hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13} = \\
&(-f_x r_{11} + r_{31}o_x \quad -f_x r_{12} + r_{32}o_x \quad -f_x r_{13} + r_{33}o_x).(-f_x r_{11} + r_{31}o_x \quad -f_x r_{12} + r_{32}o_x \quad -f_x r_{13} + r_{33}o_x) \\
&= (-f_x r_{11} + r_{31}o_x)^2 + (-f_x r_{12} + r_{32}o_x)^2 + (-f_x r_{13} + r_{33}o_x)^2 \\
&= (f_x^2 r_{11}^2 + r_{31}^2 o_x^2) + (f_x^2 r_{12}^2 + r_{32}^2 o_x^2) + (f_x^2 r_{13}^2 + r_{33}^2 o_x^2) \\
&= f_x^2 (r_{11}^2 + r_{12}^2 + r_{13}^2) + o_x^2 (r_{31}^2 + r_{32}^2 + r_{33}^2) \\
&= f_x^2 + o_x^2 \\
q_1^T q_1 &= f_x^2 + o_x^2 \\
\sqrt{q_1^T q_1 - o_x^2} &= f_x
\end{aligned}$$

Therefore:

$$f_x = \sqrt{q_1^T q_1 - o_x^2}$$
$$f_y = \sqrt{q_2^T q_2 - o_y^2}$$

Computing Camera Parameters: remaining rotation and translation parameters

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$r_{1i} = \mathbf{s}(0_x \hat{m}_{3i} - \hat{m}_{1i}) / f_x, \quad i = 1,2,3$$
$$r_{2i} = \mathbf{s}(0_y \hat{m}_{3i} - \hat{m}_{2i}) / f_y, \quad i = 1,2,3$$
$$T_x = \mathbf{s}(0_x T_z - \hat{m}_{14}) / f_x$$
$$T_y = \mathbf{s}(0_y T_z - \hat{m}_{24}) / f_y$$

Application

$$M = \begin{bmatrix} .17237 & -.15879 & .01879 & 274.943 \\ .131132 & .112747 & .2914 & 258.686 \\ .000346 & .0003 & .00006 & 1 \end{bmatrix}$$

Camera location: intersection of California And Mason streets, at an elevation of 435 feet Above sea level. The camera was oriented At an angle of 8^0 above the horizon. $fs_x=495$, $fs_y=560$.

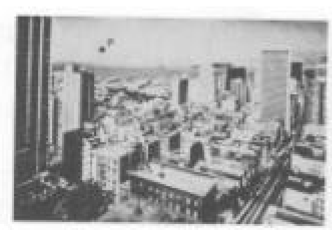


FIGURE 8 PHOTOGRAPH OF SAN FRANCISCO

FIGURE 9 MAP OF SAN FRANCISCO

Application

$$M = \begin{bmatrix} -.175451 & -.10520 & .00435 & 297.83 \\ .02698 & -.09635 & .2303 & 249.574 \\ .00015 & -.00016 & .00001 & 1.0 \end{bmatrix}$$

Camera location: at an elevation of 1200 feet
Above sea level. The camera was oriented
At an angle of 4^0 above the horizon. $fs_x=876$,
$fs_y=999$.

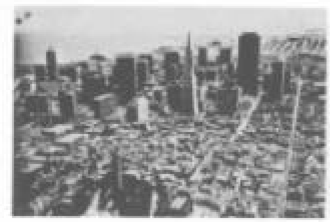


FIGURE 10 ANOTHER PHOTOGRAPH OF SAN FRANCISCO

FIGURE 11 MAP OF SAN FRANCISCO

Pose estimation: Object Recognition

- To determine the orientation and position of an object which would result in the projection of given 3-D points into a given set of image points.
  - 3-D points are given form the model
  - 2-D points are given from the image
  - Determine 3-D rotation(orientation), and 3-D translation (position)

Pose Estimation

$$(X',Y',Z') \xrightarrow{\text{Rotation}} (X,Y,Z) \xrightarrow{\text{trans}} (X+T_X, Y+T_Y, Z+T_Z) \xrightarrow{\text{perspect}} (x',y')$$

$$(x',y') = \left(\frac{f(X+T_X)}{Z+T_Z}, \frac{f(Y+T_Y)}{Z+T_Z} \right)$$

Image displacements instead of
3-D translations

$$(x',y') = \left(\frac{fX}{Z+D_Z} + D_X, \frac{fY}{Z+D_Z} + D_Y \right) = (fXc + D_X,\ fYc + D_Y) \qquad c = \frac{1}{Z+D_Z}$$

Error in x coordinates:

$$E_{x'} = \frac{\partial x'}{\partial D_X}\Delta D_X + \frac{\partial x'}{\partial D_Y}\Delta D_Y + \frac{\partial x'}{\partial D_Z}\Delta D_Z + \frac{\partial x'}{\partial f_X}\Delta f_X + \frac{\partial x'}{\partial f_Y}\Delta f_Y + \frac{\partial x'}{\partial f_Z}\Delta f_Z$$

Pose Estimation

Derivatives

$$(x',y') = \left(\frac{fX}{Z+D_Z} + D_X, \frac{fY}{Z+D_Z} + D_Y \right) = (fXc + D_X,\ fYc + D_Y)$$

$$\frac{\partial x'}{\partial D_X} = 1$$

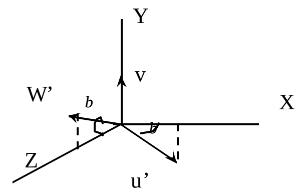
$$\frac{\partial x'}{\partial f_Y} = \frac{\partial x'}{\partial X}\frac{\partial X}{\partial f_Y} + \frac{\partial x'}{\partial Z}\frac{\partial Z}{\partial f_Y}$$

Chain rule

$$\frac{\partial x'}{\partial f_Y} = \frac{\partial X}{\partial f_Y}\frac{f}{Z+D_Z} - \frac{fX}{(Z+D_Z)^2}\frac{\partial Z}{\partial f_Y}$$

Pose Estimation

$$R^{Y}_{-b} = \begin{bmatrix} \cos b & 0 & -\sin b \\ 0 & 1 & 0 \\ \sin b & 0 & \cos b \end{bmatrix}$$



$$R^{Y}_{f_Y} = \begin{bmatrix} \cos f_Y & 0 & \sin f_Y \\ 0 & 1 & 0 \\ -\sin f_Y & 0 & \cos f_Y \end{bmatrix}$$

$$X = X'\cos f_Y + Z'\sin f_Y$$
$$Y = Y'$$
$$Z = -X'\sin f_Y + Z'\cos f_Y$$

$$\frac{\partial X}{\partial f_Y} = -X'\sin f_Y + Z'\cos f_Y = Z$$

$$\frac{\partial Y}{\partial f_Y} = 0$$

$$\frac{\partial Z}{\partial f_Y} = -X'\cos f_Y - Z'\sin f_Y = -X$$

Pose Estimation

$$E_{x'} = \frac{\partial x'}{\partial D_X}\Delta D_X + \frac{\partial x'}{\partial D_Y}\Delta D_Y + \frac{\partial x'}{\partial D_Z}\Delta D_Z + \frac{\partial x'}{\partial f_X}\Delta f_X + \frac{\partial x'}{\partial f_Y}\Delta f_Y + \frac{\partial x'}{\partial f_Z}\Delta f_Z$$

$$E_{y'} = \frac{\partial y'}{\partial D_X}\Delta D_X + \frac{\partial y'}{\partial D_Y}\Delta D_Y + \frac{\partial y'}{\partial D_Z}\Delta D_Z + \frac{\partial y'}{\partial f_X}\Delta f_X + \frac{\partial y'}{\partial f_Y}\Delta f_Y + \frac{\partial y'}{\partial f_Z}\Delta f_Z$$

$$\begin{bmatrix} \frac{\partial x'}{\partial D_X} & \frac{\partial x'}{\partial D_Y} & \frac{\partial x'}{\partial D_Z} & \frac{\partial x'}{\partial f_X} & \frac{\partial x'}{\partial f_Y} & \frac{\partial x'}{\partial f_Z} \\ & & & & \vdots & \\ \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} \\ & & & & \vdots & \end{bmatrix} \begin{bmatrix} \Delta D_X \\ \Delta D_Y \\ \Delta D_Z \\ \Delta f_X \\ \Delta f_Y \\ \Delta f_Z \end{bmatrix} = \begin{bmatrix} E_{x'} \\ \\ E_{y'} \\ \\ \end{bmatrix}$$

$$A\Delta = E$$
$$\Delta = (AA^T)^{-1}A^T E$$

Least squares fit

Derivatives and Averages

Derivatives and Averages

- Derivative: Rate of change of some quantity
  - Speed is a rate of change of a distance
  - Acceleration is a rate of change of speed
- Average (Mean)
  - The numerical result obtained by dividing the sum of two or more quantities by the number of quantities

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

$$v = \frac{ds}{dt} \qquad \text{speed}$$

$$a = \frac{dv}{dt} \qquad \text{acceleration}$$

Examples

$$y = x^2 + x^4 \qquad y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = 2x + 4x^3 \qquad \frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Second Derivative

$$\frac{df_x}{dx} = f''(x) = f_{xx}$$

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$\frac{d^2y}{dx^2} = 2 + 12x^2$$

Discrete Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$$

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

(Finite Difference)

Discrete Derivative

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$ Left difference

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$ Right difference

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$$ Center difference

Example

	F(x)=10	10	10	10	20	20	20
Left difference	F'(x)=0	0	0	0	10	0	0
	F''(x)=0	0	0	0	10	-10	0

-1 1 left difference
1 -1 right difference
-1 0 1 center difference

Derivatives in Two Dimensions

$f(x,y)$

(partial Derivatives)

$$\frac{\partial f}{\partial x} = f_x = \lim_{\Delta x \to 0} \frac{f(x,y) - f(x-\Delta x, y)}{\Delta x}$$

$$\frac{\partial f}{\partial y} = f_y = \lim_{\Delta y \to 0} \frac{f(x,y) - f(x, y-\Delta y)}{\Delta y}$$

(f_x, f_y) Gradient Vector

magnitude $= \sqrt{(f_x^2 + f_y^2)}$

direction $= q = \tan^{-1} \frac{f_y}{f_x}$

$$\Delta^2 f = f_{xx} + f_{yy} = \text{Laplacian}$$

Derivatives of an Image

Derivative & average

−1	0	1		−1	−1	−1	Prewit
−1	0	1		0	0	0	
−1	0	1		1	1	1	
	f_x				f_y		

$$I(x,y) = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \quad I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 30 & 30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Derivatives of an Image

$$I(x,y) = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \quad I_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Average

$$I = \frac{I_1 + I_2 + \ldots I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

Weighted Average

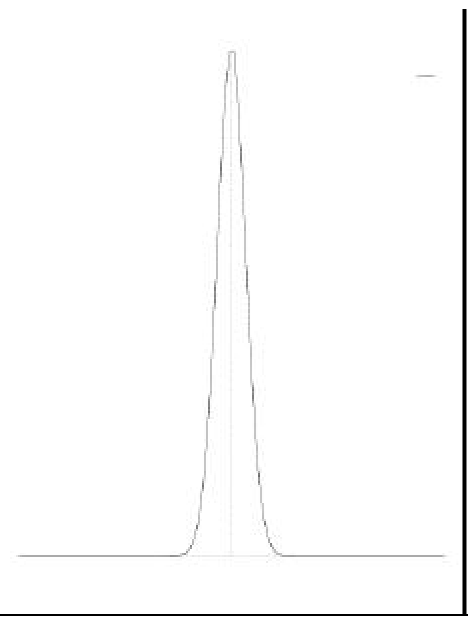
$$I = \frac{w_1 I_1 + w_2 I_2 + \ldots + w_n I_n}{n} = \frac{\sum_{i=1}^{n} w_i I_i}{n}$$

Gaussian

$$g(x) = e^{\frac{-x^2}{2o^2}}$$

Standard deviation

x	-3	-2	-1	0	1	2	3
g(x)	.011	.13	.6	1	.6	.13	.011



Gaussian

- Most natural phenomenon can be modeled by Gaussian.
- Take a bunch of random variables of any distribution, find the mean, the mean will approach to Gaussian distribution.
- Gaussian is very smooth function, it has infinite no of derivatives.

Gaussian

- Fourier Transform of Gaussian is Gaussian.
- If you convolve Gaussian with itself, it is again Gaussian.
- There are cells in human brain which perform Gaussian filtering.
  - Laplacian of Gaussian edge detector

Carl F. Gauss

- Born to a peasant family in a small town in Germany.
- Learned counting before he could talk.
- Contributed to Physics, Mathematics, Astronomy,…
- Discovered most methods in modern mathematics, when he was a teenager.

Carl F. Gauss

- Some contributions
  - Gaussian elimination for solving linear systems
  - Gauss-Seidel method for solving sparse systems
  - Gaussian curvature
  - Gaussian qudrature

Noise

- Image contains noise due to
  - Lighting variations
  - Lens de-focus
  - Camera electronics
  - Surface reflectance
- Remove noise
  - Averaging
  - Weighted averaging

Example

F(x)=10	10	10	10	20	20	20
n(x)= 0	5	0	0	3	0	0
F~(x)=10	15	10	10	23	20	20
H(x)= 10	12	12	14	17	21	20

Edge Detection

- Find edges in the image
- Edges are locations where intensity changes the most
- Edges can be used to represent a shape of an object

Edge Detection

- Images contain noise, need to remove noise by averaging, or weighted averaging
- To detect edged compute derivative of an image (gradient)
- If gradient magnitude is high at pixel, intensity change is maximum, that is an edge pixel
- If Laplacian (second derivative) is zero then at that point the first derivative is maximum, that point is an edge pixel.

Edge Detectors

- Prewit
- Sobel
- Roberts
- Marr-Hildreth (Laplacian of Gaussian)
- Canny (Gradient of Gaussian)

Derivatives of an Image

P
Sobel

−1	0	1
−2	0	2
−1	0	1

f_x

−1	−2	−1
0	0	0
1	2	1

f_y

Roberts

0	1
−1	0

f_x

1	0
0	−1

f_y

Laplacian of Gaussian

- Filter the image by weighted averaging (Gaussian)
- Find Laplacian of image
- Detect zero-crossings

$$\Delta^2 f = f_{xx} + f_{yy} = \text{Laplacian}$$

Canny Edge Detector

- Filter the image with Gaussian
- Find the gradient magnitude
- Edges are maxima of gradient magnitude