# Lecture-7

Edge Detection: LG, Canny

# Edge Detection


(a) (b)

Figure 2.20: (a). The bottle image. (b). The edge map from the Canny edge detector.

## Edge Detectors

- Gradient operators: Sobel, Prewit, Robert
- Laplacian of Gaussian (Marr-Hildreth)
- Gradient of Gaussian (Canny)
- Facet Model Based Edge Detector (Haralick)

## Laplacian of Gaussian Edge Detector

- Generate a mask for LG for a given $s$
- Apply mask to the image
- Detect zerocrossings
  - Scan along each row, record an edge point at the location of zerocrossing.
  - Repeat above step along each column

# Laplacian of Gaussian

$$g(x,y) = e^{\frac{-(x^2+y^2)}{2o^2}}$$

$$\frac{\partial}{\partial x} g(x,y) = (-\frac{x}{o^2})e^{\frac{-(x^2+y^2)}{2o^2}}$$

$$\frac{\partial^2}{\partial x^2} g(x,y) = (-\frac{x}{o^2})(-\frac{x}{o^2})e^{\frac{-(x^2+y^2)}{2o^2}} + (-\frac{1}{o^2})e^{\frac{-(x^2+y^2)}{2o^2}}$$

$$\frac{\partial^2}{\partial x^2} g(x,y) = -\frac{1}{o^2}(1-\frac{x^2}{o^2})^{\frac{-(x^2+y^2)}{2o^2}}$$

# Laplacian of Gaussian

$$g_{xx} = \frac{\partial^2}{\partial x^2} g(x,y) = -\frac{1}{o^2}(1-\frac{x^2}{o^2})^{\frac{-(x^2+y^2)}{2o^2}}$$

$$g_{yy} = \frac{\partial^2}{\partial y^2} g(x,y) = -\frac{1}{o^2}(1-\frac{y^2}{o^2})^{\frac{-(x^2+y^2)}{2o^2}}$$

$$\Delta^2 g(x,y) = \frac{\partial^2}{\partial x^2} g(x,y) + \frac{\partial^2}{\partial y^2} g(x,y)$$

$$= -\frac{1}{o^2}(1-\frac{x^2}{o^2})^{\frac{-(x^2+y^2)}{2o^2}} - \frac{1}{o^2}(1-\frac{y^2}{o^2})^{\frac{-(x^2+y^2)}{2o^2}}$$

$$\Delta^2 g(x,y) = -\frac{1}{o^2}(2-\frac{x^2+y^2}{o^2})^{\frac{-(x^2+y^2)}{2o^2}}$$

# Zerocrossings

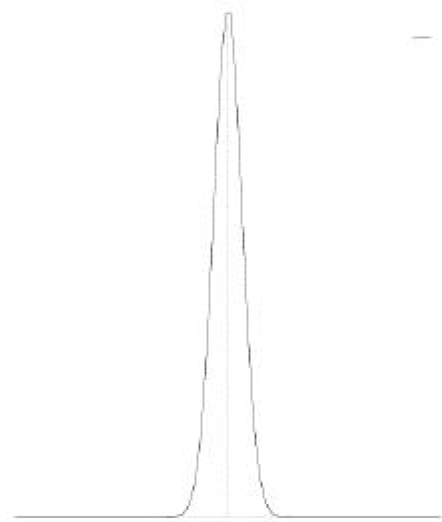
- Four cases of zerocrossings :{+,-}, {+,0,-}, {-,+}, {-,0,+}
- Slope of zerocrossing {a, -b} is |a-b|.
- To detect zerocrossing apply threshold to the slope. If the slope is above some threshold, then that point is an edge point.

# Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

Standard deviation

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *x* | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| g(x) | .011 | .13 | .6 | 1 | .6 | .13 | .011 |



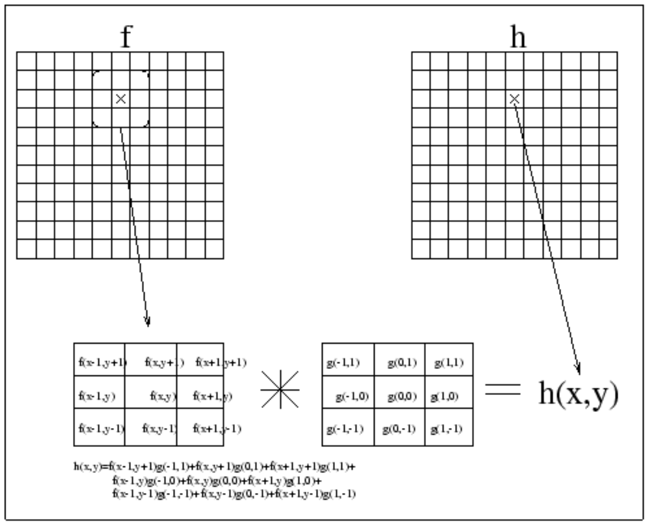
# 2-D Gaussian

$$g(x, y) = e^{\frac{-(x^2+y^2)}{2o^2}}$$

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 3 | 6 | 9 | 11 | 9 | 6 | 3 | 1 | 0 | 0 |
| 0 | 1 | 4 | 11 | 20 | 30 | 34 | 30 | 20 | 11 | 4 | 1 | 0 |
| 0 | 3 | 11 | 26 | 50 | 73 | 82 | 73 | 50 | 26 | 11 | 3 | 0 |
| 1 | 6 | 20 | 50 | 93 | 136 | 154 | 136 | 93 | 50 | 20 | 6 | 1 |
| 2 | 9 | 30 | 73 | 136 | 198 | 225 | 198 | 136 | 73 | 30 | 9 | 2 |
| 2 | 11 | 34 | 82 | 154 | 225 | 255 | 225 | 154 | 82 | 34 | 11 | 2 |
| 2 | 9 | 30 | 73 | 136 | 198 | 225 | 198 | 136 | 73 | 30 | 9 | 2 |
| 1 | 6 | 20 | 50 | 93 | 136 | 154 | 136 | 93 | 50 | 20 | 6 | 1 |
| 0 | 3 | 11 | 26 | 50 | 73 | 82 | 73 | 50 | 26 | 11 | 3 | 0 |
| 0 | 1 | 4 | 11 | 20 | 30 | 34 | 30 | 20 | 11 | 4 | 1 | 0 |
| 0 | 0 | 1 | 3 | 6 | 9 | 11 | 9 | 6 | 3 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 1 | 0 | 0 | 0 | 0 |

$s = 2$

# Convolution




# Convolution (contd)

$$h(x,y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} f(x+i, y+j)g(i,j)$$

$$h(x,y) = f(x,y) * g(x,y)$$

## Separability of Gaussian

$$h(x,y) = f(x,y) * g(x,y)$$

Requires $n^2$ multiplications for a $n$ by $n$ mask, for each pixel.

$$h(x,y) = (f(x,y) * g(x)) * g(y)$$

This requires $2n$ multiplications for a $n$ by $n$ mask, for each pixel.

## Separability of Laplacian of Gaussian

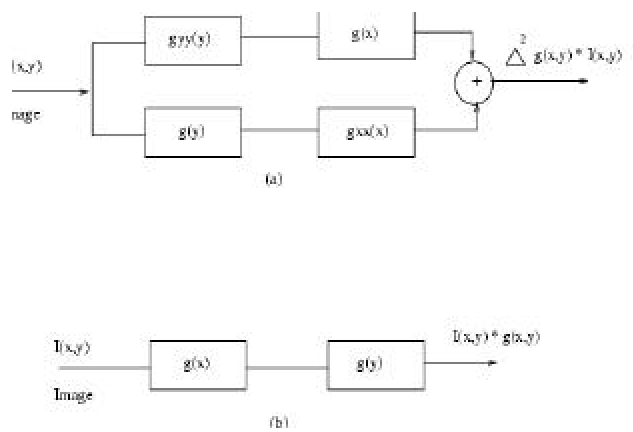
$$h(x,y) = f(x,y) * \Delta^2 g(x,y)$$

Requires $n^2$ multiplications for a $n$ by $n$ mask, for each pixel.

$$h(x,y) = (f(x,y) * g_{xx}(x)) * g(y) + (f(x,y) * g_{yy}(y)) * g(x)$$

This requires $4n$ multiplications for a $n$ by $n$ mask, for each pixel.

# Separability




# Decomposition of LG into four 1-D convolutions

- Convolve the image with a second derivative of Gaussian mask $g_{yy}(y)$ along each column.
- Convolve the resultant image from step (1) by a Gaussian mask $g(x)$ along each row. Call the resultant image $I^x$.
- Convolve the original image with a Gaussian mask, $g(y)$ along each column.
- Convolve the resultant image from step (3) by a second derivative of Gaussian mask $g_{xx}(x)$ along each row. Call the resultant image $I^y$.
- Add $I^x$ and $I^y$.

## Canny Edge Detector

- Compute the gradient of image *f(x,y)* by convolving it with the first derivative of Gaussian masks in *x* and *y* directions.
- Perform non-maxima suppression on the gradient magnitude.
- Apply hysteresis thresholding to the non-maxima suppressed magnitude.

## Canny Edge Detector

$$f_x(x,y) = f(x,y) * g_x(x,y) = (f(x,y) * g_x(x)) * g(y)$$

$$f_y(x,y) = f(x,y) * g_y(x,y) = (f(x,y) * g_y(y)) * g(x)$$

$(f_x, f_y)$ Gradient Vector

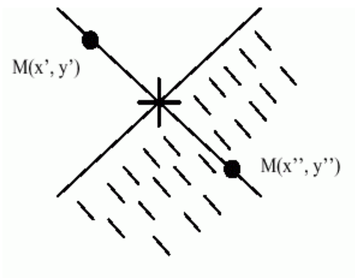
$$\text{magnitude} = \sqrt{(f_x^2 + f_y^2)}$$

$$\text{direction} = q = \tan^{-1} \frac{f_y}{f_x}$$

# Non-maxima Suppression

- Suppress the pixels which are not local maxima.

$$M(x,y) = \begin{cases} M(x,y) & \text{if } M(x,y) > M(x',y') \ \& \\ & \text{if } M(x,y) > M(x'',y'') \\ 0 & \text{otherwise} \end{cases}$$

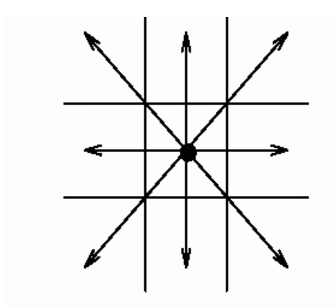



# Quantization in Eight Possible Directions

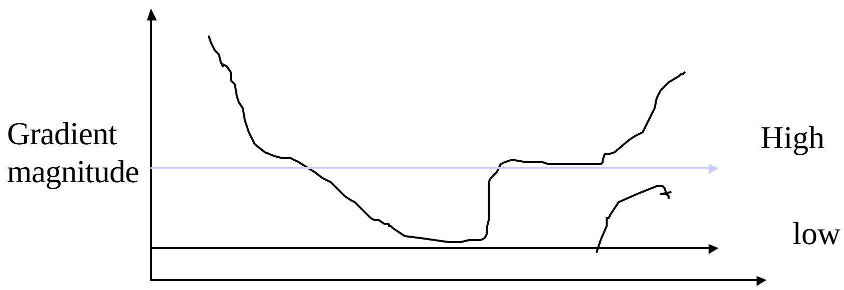
$(f_x, f_y)$ Gradient Vector

magnitude $= \sqrt{(f_x^2 + f_y^2)}$

direction $= q = \tan^{-1} \frac{f_y}{f_x}$



# Hysteresis Thresholding




# Hysteresis Thresholding

- Scan the image from left to right, top-bottom. If
  - The gradient magnitude at a pixel is above a high threshold declare that as an edge point
  - Then recursively consider the *neighbors* of this pixel.
    - If the gradient magnitude is above the low threshold declare that as an edge pixel.

# Connectedness

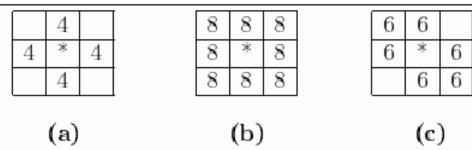



Figure 3.6: Pixel connectedness. (a) 4-connected. (b) 8-connected. (c) 6-connected.

# Connected Component

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & d & 0 & c & 0 \end{bmatrix}$$

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# Connected Component

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & c & 0 & c & 0 \end{bmatrix}$$

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1. Scan the binary image left to right, top to bottom.
2. If there is an unlabeled pixel with a value of '1' assign a new label to it.
3. Recursively check the neighbors of the pixel in step 2 and assign the same label if they are unlabeled with a value of '1'.
4. Stop when all the pixels of value '1' have been labeled.

Figure 3.7: Recursive Connected Component Algorithm.

# Sequential

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & d & c & c & 0 \end{bmatrix} \quad d=c$$

1. Scan the binary image left to right, top to bottom.
2. If an unlabeled pixel has a value of '1', assign a new label to it according to the following rules:

$$\begin{matrix} & 0 \\ 0 & 1 \end{matrix} \rightarrow \begin{matrix} & 0 \\ 0 & L \end{matrix} \qquad \begin{matrix} & 0 \\ L & 1 \end{matrix} \rightarrow \begin{matrix} & 0 \\ L & L \end{matrix}$$

$$\begin{matrix} & L \\ 0 & 1 \end{matrix} \rightarrow \begin{matrix} & L \\ 0 & L \end{matrix} \qquad \begin{matrix} & L \\ M & 1 \end{matrix} \rightarrow \begin{matrix} & L \\ M & L \end{matrix} \text{ (Set } L = M\text{).}$$

3. Determine equivalence classes of labels.
4. In the second pass, assign the same label to all elements in an equivalence class.

Figure 3.8: Sequential Connected Component Algorithm.

# Recursive

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & a & 0 \\ b & b & 0 & a & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c & c & 0 \\ 0 & c & c & c & 0 \end{bmatrix}$$