CAP5415 Computer Vision
Spring 2003

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Image Filtering

- Modifying the pixels in an image based on some function of a local neighborhood of the pixels

p

N(p)

10	30	10
20	11	20
11	9	1

$\xrightarrow{f(p)}$

	5.7	

Linear Filtering

- The output is the linear combination of the neighborhood pixels

$$f(p) = \sum_{q_i \in N(p)} a_i q_i$$

- The coefficients of this linear combination combine to form the "filter-kernel"

Image

1	3	0
2	10	2
4	1	1

$\otimes$

Kernel

1	0	-1
1	0.1	-1
1	0	-1

=

Filter Output

	5	

Convolution

$$f(i,j) = I * H = \sum_k \sum_l I(k,l)H(i-k, j-l)$$

$I =$ Image

$H =$ Kernel

H

H_1	H_2	H_3
H_4	H_5	H_6
H_7	H_8	H_9

$X - flip$

H_7	H_8	H_9
H_4	H_5	H_6
H_1	H_2	H_3

$Y - flip$

I

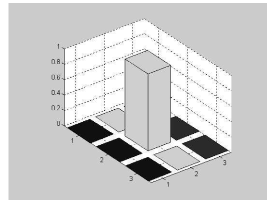
I_1	I_2	I_3
I_4	I_5	I_6
I_7	I_8	I_9

$\otimes$

H_9	H_8	H_7
H_6	H_5	H_4
H_3	H_2	H_1

$$\begin{aligned} I * H = {} & I_1 H_9 + I_2 H_8 + I_3 H_7 \\ & + I_4 H_6 + I_5 H_5 + I_6 H_4 \\ & + I_7 H_3 + I_8 H_2 + I_9 H_1 \end{aligned}$$

Linear Filtering

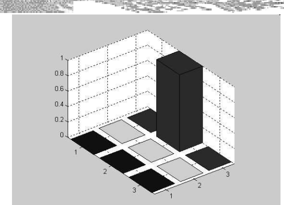


*

0	0	0
0	1	0
0	0	0

=

Linear Filtering

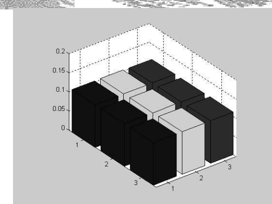


*

0	0	0
0	0	1
0	0	0

=

Linear Filtering

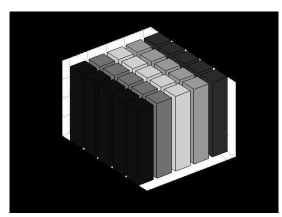


$* \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$=$

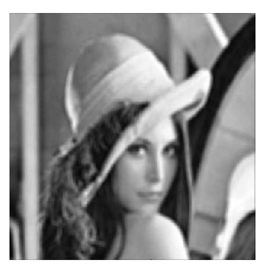
Linear Filtering



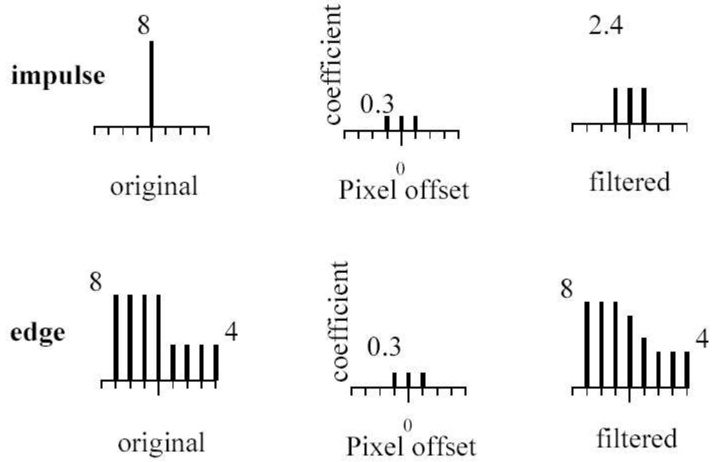
$* \frac{1}{25}$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

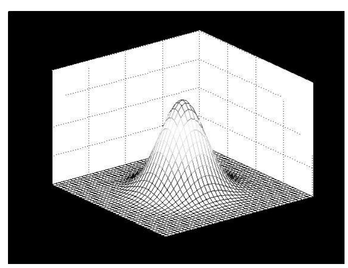
$=$



Blur examples



Gaussian Filter



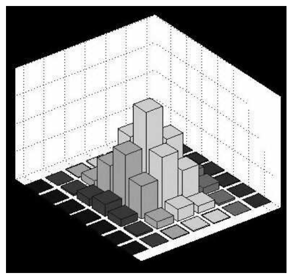
$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2 + y^2)}{2\sigma^2}\right)$$

$$H(i, j) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\left((i-k-1)^2 + (j-k-1)^2\right)}{2\sigma^2}\right)$$

where $H(i, j)$ is $(2k+1) \times (2k+1)$ array

Linear Filtering(Gaussian Filter)

*



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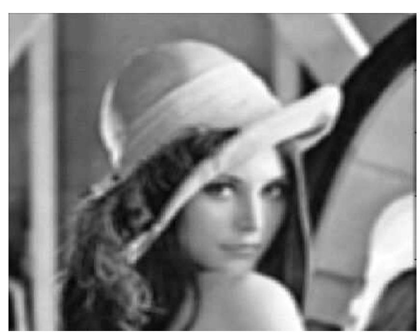
Gaussian Vs Average

Gaussian Smoothing



Smoothing by Averaging

Noise Filtering

After Averaging

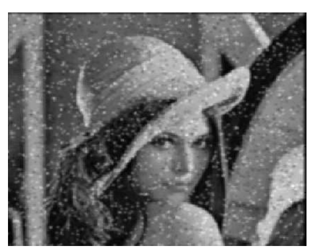
Gaussian Noise

After Gaussian Smoothing

Noise Filtering

After Averaging

Salt & Pepper Noise



After Gaussian Smoothing

Shift Invariant Linear Systems

- Superposition

$$R(f+g) = R(f) + R(g)$$

- Scaling

$$R(kf) = kR(f)$$

- Shift Invariance

Linear image transformations

- In analyzing images, it's often useful to make a change of basis.

transformed image

$$\vec{F} = U\vec{f}$$

Vectorized image

Fourier transform, or
Wavelet transform, or
Steerable pyramid transform

Self-inverting transforms

Same basis functions are used for the inverse transform

$$\vec{f} = U^{-1}\vec{F}$$

$$= U^{+}\vec{F}$$

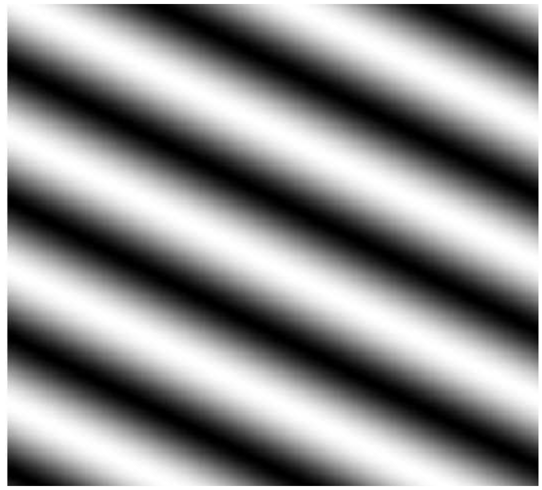
U transpose and complex conjugate

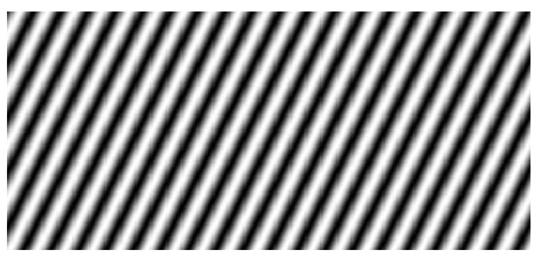
Fourier Transform

Continuous : $F(g(x,y))(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-i2\pi(ux+vy)}dxdy$

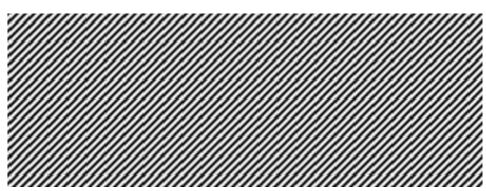
Discrete $F[m,n] = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} f[k,l]e^{-\pi i\left(\frac{km}{M}+\frac{ln}{N}\right)}$

To get some sense of what basis elements look like, we plot a basis element --- or rather, its real part --- as a function of x,y for some fixed u, v. We get a function that is constant when (ux+vy) is constant. The magnitude of the vector (u, v) gives a frequency, and its direction gives an orientation. The function is a sinusoid with this frequency along the direction, and constant perpendicular to the direction.





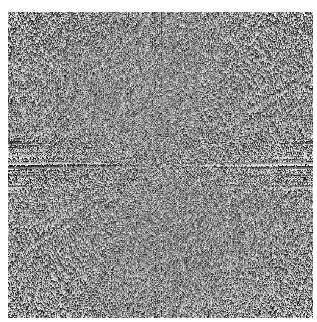
Here u & v are larger than the previous slide



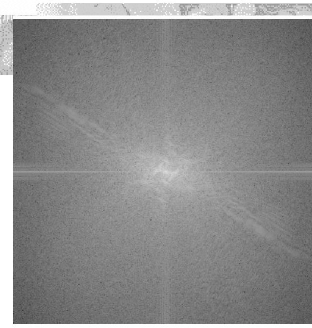
Larger than the upper example

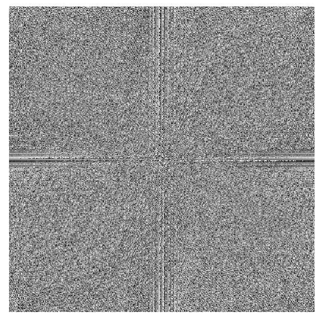
Phase and Magnitude

- Fourier transform of a real function is complex
  - difficult to plot, visualize
  - instead, we can think of the phase and magnitude of the transform
- Phase is the phase of the complex transform
- Magnitude is the magnitude of the complex transform
- Curious fact
  - all natural images have about the same magnitude transform
  - hence, phase seems to matter, but magnitude largely doesn't
- Demonstration
  - Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?



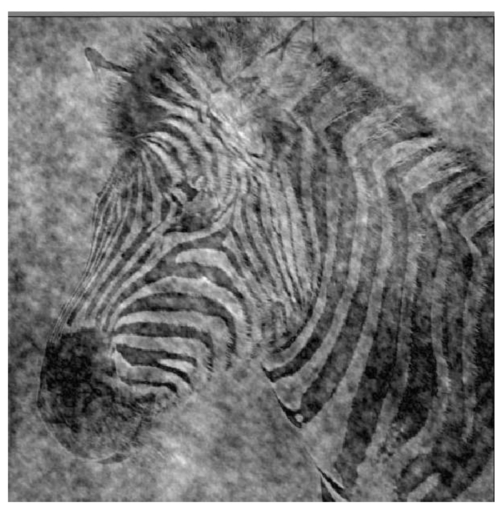
Cheetah Image
Fourier Magnitude (above)
Fourier Phase (below)



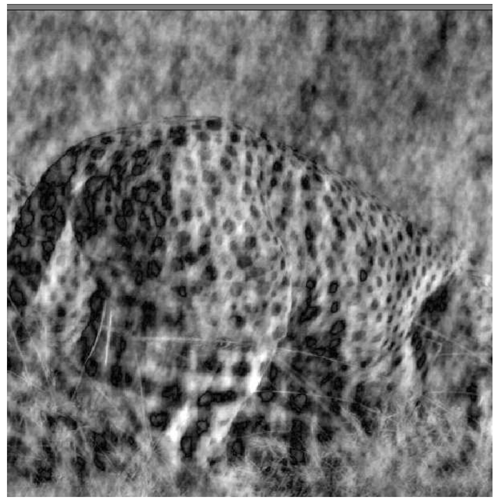


Zebra Image
Fourier Magnitude (above)
Fourier Phase (below)

Reconstruction with
Zebra phase,
Cheetah Magnitude



Reconstruction with
Cheetah phase,
Zebra Magnitude



Suggested Reading

- Chapter 7, David A. Forsyth and Jean Ponce, "Computer Vision: A Modern Approach"