Lecture-17

Theorems 5.8 and 5.9

&

Levenberg-Marquadet

Convergence of Algorithms with restarts

We can use Theorem 5.7 to prove global convergence for algorithms, which are periodically started by setting $b_k = 0$

If restarts occur at $k_1, k_2, \ldots$

Since at the restarts

$$\cos q_k = -1$$

$$\sum_{k \geq 0} \cos^2 q_k \|\nabla f_k\|^2 < \infty$$

$$\sum_{k=k_1,k_2,\ldots} \|\nabla f_k\|^2 < \infty$$

If the restarts are done after every *n* iterations, the sequence $\{k_j\}_{j=1}^{\infty}$ is infinite

$$\lim_{j \to \infty} \|\nabla f_{k_j}\| = 0$$

Therefore a subsequence of gradients will approach to zero:

$$\liminf_{k \to \infty} \|\nabla f_k\| = 0$$

Theorem 5.8

Suppose that the function is Lipschitz continuously differentiable, $\|\nabla f_k\| \le \bar{g}$, and Algorithm 5.4 is implemented with a line search that satisfies strong Wolfe conditions, with $0<c_2<1/2$. Then

$$\liminf_{k\to\infty} \|\nabla f_k\| = 0$$

Proof

$$\liminf_{k\to\infty} \|\nabla f_k\| = 0$$

Proof by contradiction:
Assume that:

$$\|\nabla f_k\| \ge g \quad \text{for all } k \text{ sufficiently large}, g > 0$$

$$c_1 \frac{\|\nabla f_k\|}{\|p_k\|} \le \quad \cos q_k \quad \le c_2 \frac{\|\nabla f_k\|}{\|p_k\|}, \ \forall k = 0,1\ldots$$

We have proved this (Lecture 16)

Now $\sum_{k\ge 0} \cos^2 q_k \|\nabla f_k\|^2 < \infty \Rightarrow \sum_{k=1}^{\infty} \frac{\|\nabla f_k\|^4}{\|p_k\|^2} < \infty$ (D)

Proof

$$p_{k+1} \leftarrow -\nabla f_{k+1} + b_{k+1}^{FR} p_k;$$
$$p_k = -\nabla f_k + b_k^{FR} p_{k-1}$$

$$p_k^T p_k = (-\nabla f_k + b_k^{FR} p_{k-1})^T (-\nabla f_k + b_k^{FR} p_{k-1})$$
$$p_k^T p_k = \nabla f_k^T \nabla f_k - b_k^{FR} \nabla f_k^T p_{k-1} - b_k^{FR} p_{k-1}^T \nabla f + (b_k^{FR})^2 p_{k-1}^T p_{k-1}$$
$$p_k^T p_k = \nabla f_k^T \nabla f_k - 2 b_k^{FR} \nabla f_k^T p_{k-1} + (b_k^{FR})^2 p_{k-1}^T p_{k-1}$$
$$\| p_k \|^2 \le \| \nabla f_k \|^2 + 2 b_k^{FR} | \nabla f_k^T p_{k-1} | + (b_k^{FR})^2 \| p_{k-1} \|^2$$

From (C)

$$\le \| \nabla f_k \|^2 + \frac{2c_2}{1-c_2} b_k^{FR} \| \nabla f_{k-1} \|^2 + (b_k^{FR})^2 \| p_{k-1} \|^2 \qquad | \nabla f_k^T p_{k-1} | \le -c_2 \nabla f_{k-1}^T p_{k-1} \le \frac{c_2}{1-c_2} \| \nabla f_{k-1} \|^2$$

$$= \| \nabla f_k \|^2 + \frac{2c_2}{1-c_2} \frac{\nabla f_k^T \nabla f_k}{\nabla f_{k-1}^T \nabla f_{k-1}} \| \nabla f_{k-1} \|^2 + (b_k^{FR})^2 \| p_{k-1} \|^2 \qquad b_{k+1}^{FR} = \frac{\nabla f_{k+1}^T \nabla f_{k+1}}{\nabla f_k^T \nabla f_k}$$

$$\| p_k \|^2 \le \left(\frac{1+c_2}{1-c_2} \right) \| \nabla f_k \|^2 + (b_k^{FR})^2 \| p_{k-1} \|^2$$

Proof

$$| \nabla f_{k+1}^T p_k | \le -c_2 \nabla f_k^T p_k$$ Wolf's condition

$$| \nabla f_k^T p_{k-1} | \le -c_2 \nabla f_{k-1}^T p_{k-1} \qquad \text{(A)}$$

$$-\frac{1}{1-c_2} \le \frac{\nabla f_k^T p_k}{\| \nabla f_k \|^2} \le \frac{2c_2 - 1}{1-c_2}, \ \forall k = 0,1\ldots$$ Lemma 5.6

$$-\frac{1}{1-c_2} \le \frac{\nabla f_{k-1}^T p_{k-1}}{\| \nabla f_{k-1} \|^2} \le \frac{2c_2 - 1}{1-c_2} \qquad \text{(B)}$$

Combining (A) and (B)

$$| \nabla f_k^T p_{k-1} | \le -c_2 \nabla f_{k-1}^T p_{k-1} \le \frac{c_2}{1-c_2} \| \nabla f_{k-1} \|^2 \qquad \text{(C)}$$

Proof

$$\| p_k \|^2 \le \left(\frac{1+c_2}{1-c_2}\right) \| \nabla f_k \|^2 + (\beta_k^{FR})^2 \| p_{k-1} \|^2$$

$$c_3 = (1+c_2)/(1-c_2) \ge 1$$

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^2 + (\beta_k^{FR})^2 \| p_{k-1} \|^2$$

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^2 + (\beta_k^{FR})^2 [c_3 \| \nabla f_{k-1} \|^2 + (\beta_{k-1}^{FR})^2 \| p_{k-2} \|^2]$$

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^2 + (\beta_k^{FR})^2 c_3 \| \nabla f_{k-1} \|^2 + (\beta_k^{FR})^2 (\beta_{k-1}^{FR})^2 \| p_{k-2} \|^2$$

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^2 + \frac{\| \nabla f_k \|^4}{\| \nabla f_{k-1} \|^4} c_3 \| \nabla f_{k-1} \|^2 + \frac{\| \nabla f_k \|^4}{\| \nabla f_{k-1} \|^4} \frac{\| \nabla f_{k-1} \|^4}{\| \nabla f_{k-2} \|^4} \| p_{k-2} \|^2$$

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^2 + \frac{\| \nabla f_k \|^4}{\| \nabla f_{k-1} \|^2} c_3 + \frac{\| \nabla f_k \|^4}{\| \nabla f_{k-2} \|^4} \| p_{k-2} \|^2$$

$$= c_3 \| \nabla f_k \|^4 \left[\frac{1}{\| \nabla f_k \|^2} + \frac{1}{\| \nabla f_{k-1} \|^2}\right] + \frac{\| \nabla f_k \|^4}{\| \nabla f_{k-2} \|^4} \| p_{k-2} \|^2$$

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^4 \sum_{j=1}^{k} \| \nabla f_j \|^{-2}$$

$$(\beta_k^{FR})^2 (\beta_{k-1}^{FR})^2 \cdots (\beta_{k-i}^{FR})^2 = \frac{\| \nabla f_k \|^4}{\| \nabla f_{k-i-1} \|^4}$$

Proof

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^4 \sum_{j=1}^{k} \| \nabla f_j \|^{-2}$$

$$\| p_k \|^2 \le c_3 \| \nabla f_k \|^4 \sum_{j=1}^{k} \frac{1}{\| \nabla f_j \|^2}$$

$$\| p_k \|^2 \le c_3 \bar{\gamma}^4 k \frac{1}{\gamma^2}$$

$$\| \nabla f_k \| \le \bar{\gamma} \qquad \| \nabla f_k \| \ge \gamma$$

$$\sum_{k=1}^{\infty} \frac{1}{\| p_k \|^2} \ge \gamma_4 \sum_{k=1}^{\infty} \frac{1}{k}$$

Proof

$$\sum_{k=1}^{\infty} \frac{1}{\| p_k \|^2} \geq g_4 \sum_{k=1}^{\infty} \frac{1}{k}$$

Then from (D)

$$\sum_{k=1}^{\infty} \frac{\| \nabla f_k \|^4}{\| p_k \|^2} < \infty$$

Assume $\| \nabla f_k \| \geq g \quad \text{for all } k \text{ sufficiently large}$

$$\sum_{k=1}^{\infty} \frac{1}{\| p_k \|^2} < \infty$$

$$\sum_{k=1}^{\infty} \frac{1}{k} < \infty$$

Which is not true

General Result

In general, if we can show that there is a constant c_4 such that:

$$\cos q_k \geq c_4 \frac{\| \nabla f_k \|}{\| p_k \|} \quad k = 1,2\ldots$$

And another constant c_5 such that:

$$\frac{\| \nabla f_k \|}{\| p_k \|} \geq c_5 > 0 \quad k = 1,2\ldots$$

Then using Theorem 5.7 $\sum_{k\geq 0} \cos^2 q_k \| \nabla f_k \|^2 < \infty$

We can show: $\lim_{k\to\infty} \| \nabla f_k \| = 0$

General Result

This result can be established for PR assuming the function is strongly convex and then an exact line search is used.

For general functions, it is not possible to prove this for PR.

Theorem 5.9

Consider PR-GC with an ideal line search. There exists a twice continuously differentiable function *f*: $\mathbf{R}^3 \rightarrow \mathbf{R}^2$ and a starting point x_0, such that the sequence of gradients $\{\| \nabla f_k \|\}$ is bounded away from zero.

Proof of this requires that the consecutive search directions become almost negatives of each others. In the ideal line search this can only happen when $\beta_k < 0$ so that suggest restart of PR.

$$\beta_{k+1}^{+} = \max(\beta_{k+1}^{PR}, 0)$$

Lecture-18

Levenberg-Marquadet

Weighted Non-linear least squares fit

Consider a set of non-linear equations:

$$y_i = f(x_i, a)$$

Our aim is to determine *a* vector such that the following is minimized:

$$\Psi(a) = \sum_{i=1}^{N} \left(\frac{y_i - f(x_i, a)}{s_i} \right)^2$$

weights

Function to be minimized $$\Psi(a)=\sum_{i=1}^{N}\left(\frac{y_i - f(x_i,a)}{s_i}\right)^2 \qquad \text{(D)}$$

Newton's (Inverse Hessian) method:

$$a_{next} = a_{current} + D^{-1}[-\nabla\Psi(a_{current})] \qquad \text{(A)}$$

Where D is a Hessian matrix

The gradient is given by:

$$\frac{\partial\Psi}{\partial a_k} = -2\sum_i\left[\frac{y_i - f(x_i,a)}{s_i^2}\right]\frac{\partial f(x_i,a)}{\partial a_k} \quad k=1,\ldots,m$$

The Hessian is given by:

$$\frac{\partial^2\Psi}{\partial a_k \partial a_l} = 2\sum_{i=1}^{N}\frac{1}{s_i^2}\left[\frac{\partial f(x_i,y)}{\partial a_k}\frac{\partial f(x_i,y)}{\partial a_l} - [y_i - f(x_i,a)]\frac{\partial f^2(x_i,y)}{\partial a_k \partial a_l}\right] \qquad \text{(B)}$$

Let us define:

$$b_k \equiv -\frac{1}{2}\frac{\partial\Psi}{\partial \mathrm{a_k}} \qquad a_{kl} \equiv \frac{1}{2}\frac{\partial^2\Psi}{\partial \mathrm{a_k}\partial a_l}$$

Now the Hessian is given by:

$$[a] = \frac{1}{2}D$$

Newton's method (A) can be written as

$$a_{next} = a_{current} + D^{-1}[-\nabla\Psi(a_{current})] \qquad \text{(A)}$$

$$\sum_{l=1}^{M} a_{kl}da_l = b_k \qquad \text{(E)}$$ Where $$da_l = a_{next} - a_{current}$$

The gradient descent is given by:

$$a_{next} = a_{current} + const[-\nabla\Psi(a_{current})]$$

$$da_l = const\, \mathbf{b}_l \qquad \text{(C)} \qquad da_l = a_{next} - a_{current}$$

$$b_k \equiv -\frac{1}{2}\frac{\partial\Psi}{\partial a_k}$$

Assume the second term in (B) is zero:

$$\frac{\partial^2\Psi}{\partial a_k \partial a_l} = 2\sum_{i=1}^{N}\frac{1}{s_i^2}\left[\frac{\partial f(x_i,y)}{\partial a_k}\frac{\partial f(x_i,y)}{\partial a_l} - [y_i - f(x_i,a)]\frac{\partial f^2(x_i,y)}{\partial a_k \partial a_l}\right] \qquad \text{(B)}$$

$$\frac{\partial^2\Psi}{\partial a_k \partial a_l} = 2\sum_{i=1}^{N}\frac{1}{s_i^2}\left[\frac{\partial f(x_i,y)}{\partial a_k}\frac{\partial f(x_i,y)}{\partial a_l}\right]$$

$$\text{constant} = \frac{1}{l a_{ll}}$$

$$da_l = \frac{1}{l a_{ll}} b_l$$

Now $$a_{kl} = \sum_{i=1}^{N}\frac{1}{s_i^2}\left[\frac{\partial y(x_i,y)}{\partial a_k}\frac{\partial y(x_i,y)}{\partial a_l}\right]$$

$$da_l = const\, \mathbf{b}_l \qquad \text{(C)}$$

Let the constant be given by

$$const = \frac{1}{\mathbf{l}\,\mathbf{a}_{ll}} \qquad da_l = \frac{1}{\mathbf{l}\,\mathbf{a}_{ll}}\mathbf{b}_l \quad \text{(G)}$$

Gradient descent

Now define:

$$\mathbf{a}'_{jj} \equiv \mathbf{a}_{jj}(1+\mathbf{l}) \quad \text{for } i = j$$

$$\mathbf{a}'_{kj} \equiv \mathbf{a}_{kj} \quad \text{when } j \neq k \qquad \text{(F)}$$

Newton from (E) $$\sum_{l=1}^{M} a_{kl}\, da_l = b_k$$

Combining (E) and (G) and using (F) $$\sum_{l=1}^{m} a'_{kl}\, da_l = b_k \qquad \text{(H)}$$

Algorithm

1.Start with some initial estimate of *a*.

2. Compute $\Psi(x_i ,a)$ (equation D).

3. Pick a modest value of λ=.001.

4. Solve linear system (H) for δa and evaluate $\Psi(x_i ,a+\delta a)$. $\sum_{l=1}^{m} a'_{kl}\, da_l = b_k$

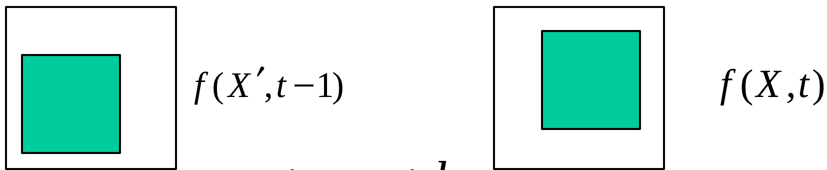
5. If $\Psi(x_i ,a+\delta a) \geq \Psi(x_i,a)$, increase λ by a factor of 10, and go to step (4)

6. If $\Psi(x_i ,a+\delta a) \leq \Psi(x_i,a)$ decrease λ by a factor of 10, update the trial solution: $a \leftarrow a+\delta a$, and go back to step 4.

Szeliski

Projective

Projective



$$x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}$$

$$y' = \frac{a_3 x + a_4 y + b_1}{c_1 x + c_2 y + 1}$$

Szeliski

$$x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}$$

Projective

$$y' = \frac{a_3 x + a_4 y + b_2}{c_1 x + c_2 y + 1}$$

$$E = \sum [f(x', y') - f(x, y)]^2 = \sum e^2$$

min

Szeliski

Motion Vector:

$$\mathbf{m} = [a_1 \quad a_2 \quad a_3 \quad a_4 \quad b_1 \quad b_2 \quad c_1 \quad c_2]^T$$

Szeliski (Levenberg-Marquadet)

Hessian $a_{kl} = \sum \frac{\partial e}{\partial m_k} \frac{\partial e}{\partial m_l}$

$b_k = -\sum e \frac{\partial e}{\partial m_k}$ **gradient**

$$\Delta m = (A + l I)^{-1} b$$

$$\frac{\partial e}{\partial a_1} = \frac{\partial e}{\partial x'}\frac{\partial x'}{\partial a_1} + \frac{\partial e}{\partial y'}\frac{\partial y'}{\partial a_1}$$

$$E = \sum [\, f(x', y') - f(x, y)]^2 = \sum e^2$$

$$x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}$$

$$y' = \frac{a_3 x + a_4 y + b_2}{c_1 x + c_2 y + 1}$$

Hessian

$$A = \begin{bmatrix}
\sum_x \frac{\partial^2 e}{\partial a_1 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial a_1 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial a_1 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial a_1 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial a_1 \partial b_1} & \sum_x \frac{\partial^2 e}{\partial a_1 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial a_1 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial a_1 \partial c_2} \\
\sum_x \frac{\partial^2 e}{\partial a_2 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial a_2 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial a_2 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial a_2 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial a_2 \partial b_1} & \sum_x \frac{\partial^2 e}{\partial a_2 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial a_2 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial a_2 \partial c_2} \\
\sum_x \frac{\partial^2 e}{\partial a_3 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial a_3 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial a_3 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial a_3 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial a_3 \partial b_1} & \sum_x \frac{\partial^2 e}{\partial a_3 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial a_3 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial a_3 \partial c_2} \\
\sum_x \frac{\partial^2 e}{\partial a_4 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial a_4 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial a_4 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial a_4 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial a_4 \partial b_1} & \sum_x \frac{\partial^2 e}{\partial a_4 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial a_4 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial a_4 \partial c_2} \\
\sum_x \frac{\partial^2 e}{\partial b_1 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial b_1 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial b_1 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial b_1 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial b_1 \partial b_1} & \sum_x \frac{\partial^2 e}{\partial b_1 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial b_1 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial b_1 \partial c_2} \\
\sum_x \frac{\partial^2 e}{\partial b_2 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial b_2 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial b_2 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial b_2 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial b_2 \partial b_1} & \frac{\partial^2 e}{\partial b_2 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial b_2 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial b_2 \partial c_2} \\
\sum_x \frac{\partial^2 e}{\partial c_1 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial c_1 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial c_1 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial c_1 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial c_1 \partial b_1} & \sum_x \frac{\partial^2 e}{\partial c_1 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial c_1 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial c_1 \partial c_2} \\
\sum_x \frac{\partial^2 e}{\partial c_2 \partial a_1} & \sum_x \frac{\partial^2 e}{\partial c_2 \partial a_2} & \sum_x \frac{\partial^2 e}{\partial c_2 \partial a_3} & \sum_x \frac{\partial^2 e}{\partial c_2 \partial a_4} & \sum_x \frac{\partial^2 e}{\partial c_2 \partial b_1} & \sum_x \frac{\partial^2 e}{\partial c_2 \partial b_2} & \sum_x \frac{\partial^2 e}{\partial c_2 \partial c_1} & \sum_x \frac{\partial^2 e}{\partial c_2 \partial c_2}
\end{bmatrix}$$

$$b = \begin{bmatrix} -\sum_x e\frac{\partial e}{\partial a_1} \\ -\sum_x e\frac{\partial e}{\partial a_2} \\ -\sum_x e\frac{\partial e}{\partial a_3} \\ -\sum_x e\frac{\partial e}{\partial a_4} \\ -\sum_x e\frac{\partial e}{\partial b_1} \\ -\sum_x e\frac{\partial e}{\partial b_2} \\ -\sum_x e\frac{\partial e}{\partial c_1} \\ -\sum_x e\frac{\partial e}{\partial c_2} \end{bmatrix}$$

$$x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}, \quad y' = \frac{a_3 x + a_4 y + b_2}{c_1 x + c_2 y + 1}$$

$$\frac{\partial x'}{\partial a_1} = \frac{x}{c_1 x + c_2 y + 1} \qquad \frac{\partial y'}{\partial a_1} = 0$$

$$\frac{\partial x'}{\partial a_2} = \frac{y}{c_1 x + c_2 y + 1} \qquad \frac{\partial y'}{\partial a_2} = 0$$

$$\frac{\partial x'}{\partial a_3} = 0 \qquad \frac{\partial y'}{\partial a_3} = \frac{x}{c_1 x + c_2 y + 1}$$

$$\frac{\partial x'}{\partial a_4} = 0 \qquad \frac{\partial y'}{\partial a_4} = \frac{y}{c_1 x + c_2 y + 1}$$

$$\frac{\partial x'}{\partial b_1} = \frac{1}{c_1 x + c_2 y + 1} \qquad \frac{\partial y'}{\partial b_1} = 0$$

$$\frac{\partial x'}{\partial b_2} = 0 \qquad \frac{\partial y'}{\partial b_2} = \frac{1}{c_1 x + c_2 y + 1}$$

$$\frac{\partial x'}{\partial c_1} = \frac{-x(a_1 x + a_2 y + b_1)}{(c_1 x + c_2 y + 1)^2} \qquad \frac{\partial y'}{\partial c_1} = \frac{-x(a_3 x + a_4 y + b_2)}{(c_1 x + c_2 y + 1)^2}$$

$$\frac{\partial x'}{\partial c_2} = \frac{-y(a_1 x + a_2 y + b_1)}{(c_1 x + c_2 y + 1)^2} \qquad \frac{\partial y'}{\partial c_2} = \frac{-y(a_3 x + a_4 y + b_2)}{(c_1 x + c_2 y + 1)^2}$$

$$\frac{\partial e}{\partial a_1}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial a_1}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial a_1}= f_{x'}\frac{x}{c_1x+c_2y+1}$$

$$\frac{\partial e}{\partial a_2}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial a_2}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial a_2}= f_{x'}\frac{y}{c_1x+c_2y+1}$$

$$\frac{\partial e}{\partial a_3}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial a_3}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial a_3}= f_{y'}\frac{x}{c_1x+c_2y+1}$$

$$\frac{\partial e}{\partial a_4}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial a_4}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial a_4}= f_{y'}\frac{y}{c_1x+c_2y+1}$$

$$\frac{\partial e}{\partial b_1}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial b_1}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial b_1}= f_{x'}\frac{1}{c_1x+c_2y+1}$$

$$\frac{\partial e}{\partial b_2}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial b_2}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial b_2}= f_{y'}\frac{1}{c_1x+c_2y+1}$$

$$\frac{\partial e}{\partial c_1}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial c_1}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial c_1}= f_{x'}\frac{-x(a_1x+a_2y+b_1)}{(c_1x+c_2y+1)^2}+ f_{y'}\frac{-x(a_3x+a_4y+b_2)}{(c_1x+c_2y+1)^2}$$

$$\frac{\partial e}{\partial c_2}=\frac{\partial e}{\partial x'}\frac{\partial x'}{\partial c_2}+\frac{\partial e}{\partial y'}\frac{\partial y'}{\partial c_2}= f_{x'}\frac{-y(a_1x+a_2y+b_1)}{(c_1x+c_2y+1)^2}+ f_{y'}\frac{-y(a_3x+a_4y+b_2)}{(c_1x+c_2y+1)^2}$$

$$\mathbf{b}=\begin{bmatrix} -\sum ef_{x'}\frac{x}{c_1x+c_2y+1} \\ -\sum ef_{x'}\frac{y}{c_1x+c_2y+1} \\ -\sum ef_{y'}\frac{x}{c_1x+c_2y+1} \\ -\sum ef_{y'}\frac{y}{c_1x+c_2y+1} \\ -\sum ef_{x'}\frac{1}{c_1x+c_2y+1} \\ -\sum ef_{y'}\frac{1}{c_1x+c_2y+1} \\ \sum ex\left[\frac{f_{x'}(a_1x+a_2y+b_1)+ f_{y'}(a_3x+a_4y+b_2)}{(c_1x+c_2y+1)^2}\right] \\ \sum ey\left[\frac{f_{x'}(a_1x+a_2y+b_1)+ f_{y'}(a_3x+a_4y+b_2)}{(c_1x+c_2y+1)^2}\right] \end{bmatrix}$$

Szeliski (Levenberg-Marquadet)

- Start with some initial value of m, and ë=.001
- For each pixel I at (x_i, y_i)
  - Compute (x', y') using projective transform.
  - Compute $e = f(x', y') - f(x, y)$
  - Compute $\frac{\partial e}{\partial m_k} = \frac{\partial e}{\partial x'}\frac{\partial x'}{\partial m_k} + \frac{\partial e}{\partial y'}\frac{\partial y'}{\partial m_k}$

Szeliski (Levenberg-Marquadet)

-Compute A **and** b

-Solve system

$$(A - lI)\Delta m = b$$

-Update

$$m^{t+1} = m^t + \Delta m$$

Szeliski (Levenberg-Marquadet)

- check if error has decreased, if not increase l by a factor of 10 and compute a new Δm

- If error has decreased, decrease by a factor of 10 and compute a new Δm

- Continue iteration until error is below threshold.

Video Mosaic

Video Mosaic

Video Mosaic