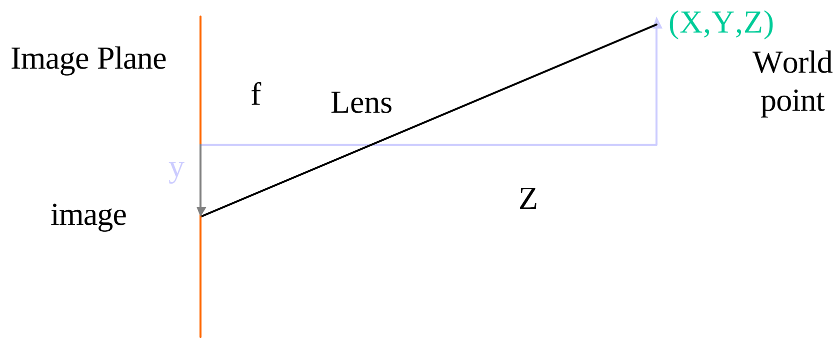
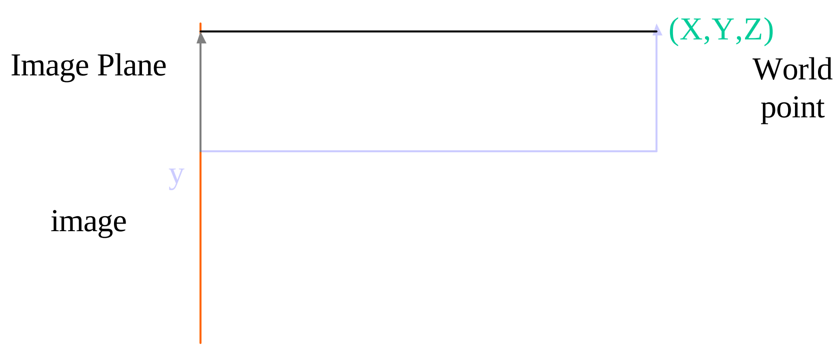
Perspective Projection



Orthographic Projection



Plane+Perspective(projective)

equation of a plane

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 1$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + T$$

3d rigid motion

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + T \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = A \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad A = R + T \begin{bmatrix} a & b & c \end{bmatrix}$$

Plane+perspective (contd.)

$$x' = \frac{a_1 x + a_2 y + a_3}{a_7 x + a_8 y + 1}$$

scale ambiguity

$$y' = \frac{a_4 x + a_5 y + a_6}{a_7 x + a_8 y + 1}$$

find a's by least squares

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -xx' & -yx' \\ 0 & 0 & 0 & x & y & 1 & -xy' & -yy' \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Displacement Models

Translation
$$x' = x + b_1$$
$$y' = y + b_2$$

Rigid
$$x' = x\cos q - y\sin q + b_1$$
$$y' = x\sin q + y\cos q + b_2$$

Affine
$$x' = a_1 x + a_2 y + b_1$$
$$y' = a_3 x + a_4 y + b_2$$

Projective
$$x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}$$
$$y' = \frac{a_3 x + a_4 y + b_1}{c_1 x + c_2 y + 1}$$

Biquadratic
$$x' = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 y^2 + a_6 xy$$
$$y' = a_7 + a_8 x + a_9 y + a_{10} x^2 + a_{11} y^2 + a_{12} xy$$

Bilinear
$$x' = a_1 + a_2 x + a_3 y + a_4 xy$$
$$y' = a_5 + a_6 x + a_7 y + a_8 xy$$

Pseudo Perspective
$$x' = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy$$
$$y' = a_6 + a_7 x + a_8 y + a_4 xy + a_5 y^2$$

Displacement Models (contd)

- Translation
  - simple
  - used in block matching
  - no zoom, no rotation, no pan and tilt
- Rigid
  - rotation and translation
  - no zoom, no pan and tilt

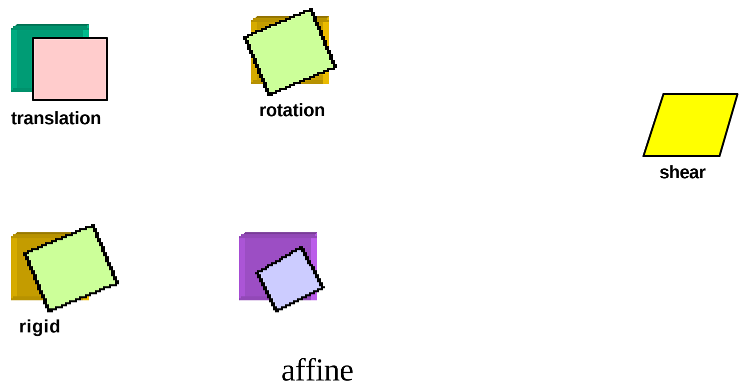
Displacement Models (contd)

- Affine
  - rotation about optical axis only
  - can not capture pan and tilt
  - orthographic projection
- Projective
  - exact eight parameters (3 rotations, 3 translations and 2 scalings)
  - difficult to estimate

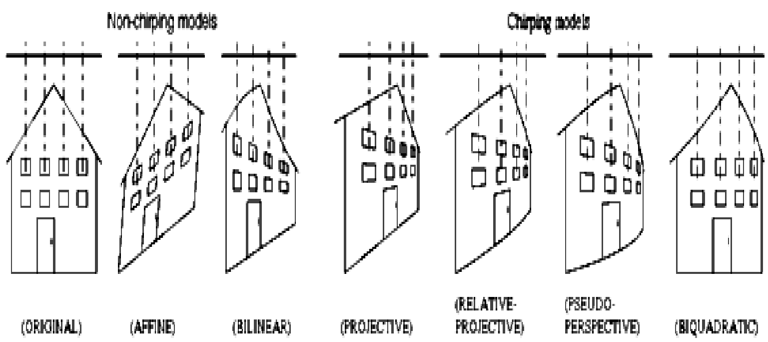
Displacement Models (contd)

- Biquadratic
  - obtained by second order Taylor series
  - 12 parameters
- Bilinear
  - obtained from biquadratic model by removing square terms
  - most widely used
  - not related to any physical 3D motion
- Pseudo-perspective
  - obtained by removing two square terms and constraining four remaining to 2 degrees of freedom

Spatial Transformations



Displacement Models (contd)



Affine Mosaic

Projective Mosaic

Instantaneous Velocity Model

3-D Rigid Motion

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 1 & -a & b \\ a & 1 & -g \\ -b & g & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\begin{bmatrix} X'-X \\ Y'-Y \\ Z'-Z \end{bmatrix} = \begin{bmatrix} 0 & -a & b \\ a & 0 & -g \\ -b & g & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\dot{\mathbf{X}} = \Omega \times \mathbf{X} + \mathbf{V}$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \left(\begin{bmatrix} 0 & -a & b \\ a & 0 & -g \\ -b & g & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

Orthographic Projection

$$u = \dot{x} = \Omega_2 Z - \Omega_3 y + V_1 \qquad \text{(u,v) is optical flow}$$
$$v = \dot{y} = \Omega_3 x - \Omega_1 Z + V_2$$

$$\dot{\mathbf{X}} = \Omega \times \mathbf{X} + \mathbf{V}$$
$$\dot{X} = \Omega_2 Z - \Omega_3 Y + V_1$$
$$\dot{Y} = \Omega_3 X - \Omega_1 Z + V_2$$
$$\dot{Z} = \Omega_1 Y - \Omega_2 X + V_3$$

Perspective Projection (arbitrary flow)

$$x = \frac{fX}{Z} \qquad u = \dot{x} = \frac{fZ\dot{X} - fX\dot{Z}}{Z^2} = f\frac{\dot{X}}{Z} - x\frac{\dot{Z}}{Z}$$
$$y = \frac{fY}{Z} \qquad v = \dot{y} = \frac{fZ\dot{Y} - fY\dot{Z}}{Z^2} = f\frac{\dot{Y}}{Z} - y\frac{\dot{Z}}{Z}$$

$$u = f(\frac{V_1}{Z} + \Omega_2) - \frac{V_3}{Z}x - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2$$
$$v = f(\frac{V_2}{Z} - \Omega_1) + \Omega_3 x - \frac{V_3}{Z}y + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

Plane+orthographic(Affine)

$$u = V_1 + \Omega_2 Z - \Omega_3 y$$
$$v = V_2 + \Omega_3 x - \Omega_1 Z$$
$$u = b_1 + a_1 x + a_2 y$$
$$v = b_2 + a_3 x + a_4 y$$

$$\mathbf{u} = \mathbf{A}\mathbf{x} + \mathbf{b}$$

$$b_1 = V_1 + a\Omega_2$$
$$a_1 = b\Omega_2$$
$$a_2 = c\Omega_2 - \Omega_3$$
$$b_2 = V_2 - a\Omega_1$$
$$a_3 = \Omega_3 - b\Omega_1$$
$$a_4 = -c\Omega_1$$

Plane+Perspective (pseudo perspective)

$$u = f\left(\frac{V_1}{Z} + \Omega_2\right) - \frac{V_3}{Z}x - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2$$
$$v = f\left(\frac{V_2}{Z} - \Omega_1\right) + \Omega_3 x - \frac{V_3}{Z}y + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

$$Z = a + bX + cY$$
$$\frac{1}{Z} = \frac{1}{a} - \frac{b}{a}x - \frac{c}{a}y$$

$$u = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy$$
$$v = a_{64} + a_7 x + a_8 y + a_4 xy + a_5 y^2$$

Measurement of Image Motion

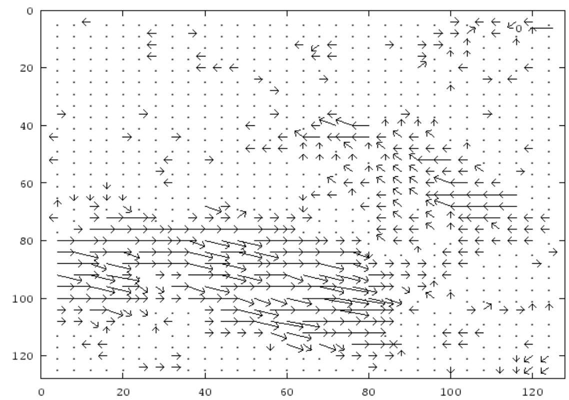
- Local Motion (Optical Flow)
- Global Motion (Frame Alignment)

Computing Optical Flow

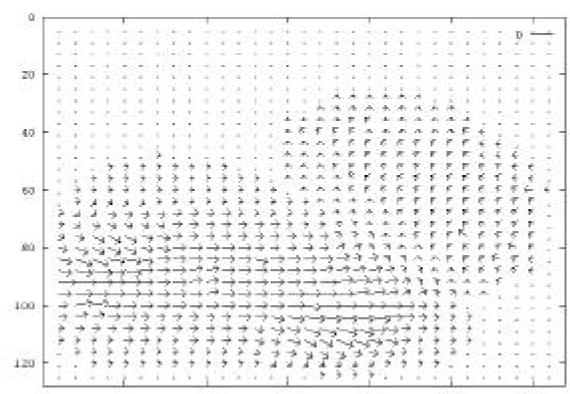
Image from Hamburg Taxi seq

Image from Hamburg Taxi seq

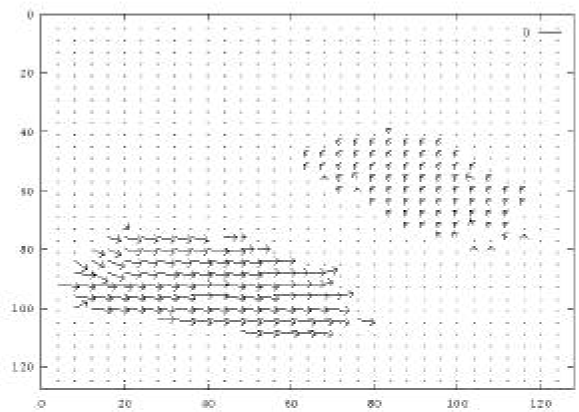
Fleet & Jepson optical flow



Horn & Schunck optical flow



Tian & Shah optical flow



Horn&Schunck Optical Flow

$$f(x,y,t) = f(x+dx, y+dy, t+dt)$$

Taylor Series

$$f(x,y,t) = f(x,y,t) + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt$$

$$f_x dx + f_y dy + f_t dt = 0$$

$$f_x u + f_y v + f_t = 0$$

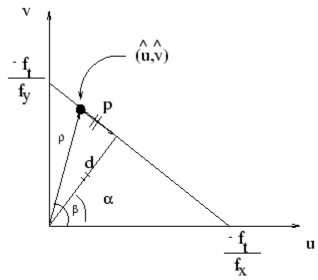
brightness constancy eq

Interpretation of optical flow eq

$$f_x u + f_y v + f_t = 0$$

$$v = -\frac{f_x}{f_y}u - \frac{f_t}{f_y}$$

Equation of st.line



d=normal flow

p=parallel flow

$$d = \frac{f_t}{\sqrt{f_x^2 + f_y^2}}$$

Horn&Schunck (contd)

$$\iint \{(f_x u + f_y v + f_t)^2 + l(u_x^2 + u_y^2 + v_x^2 + v_y^2)\} dx dy$$

min

$$(f_x u + f_y v + f_t) f_x + l(\Delta^2 u) = 0$$

$$(f_x u + f_y v + f_t) f_y + l((\Delta^2 v) = 0$$

discrete version

$$(f_x u + f_y v + f_t) f_x + l(u - u_{av}) = 0$$

$$(f_x u + f_y v + f_t) f_y + l((v - v_{av}) = 0$$

variational calculus

$$u = u_{av} - f_x \frac{P}{D}$$

$$v = v_{av} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = l + f_x^2 + f_y^2$$

Algorithm-1

- k=0
- Initialize $u^K \quad v^K$
- Repeat until some error measure is satisfied

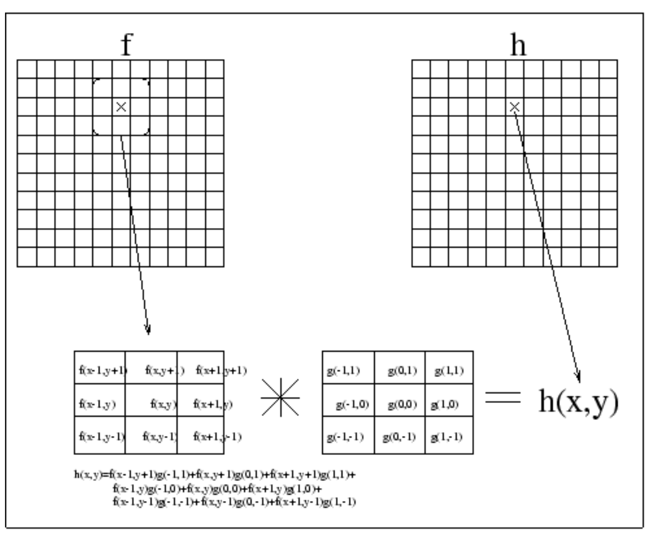
$$u^K = u_{av}^{k-1} - f_x \frac{P}{D}$$

$$v = v_{av}^{K-1} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = l + f_x^2 + f_y^2$$

Convolution



Convolution (contd)

$$h(x, y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} f(x+i, y+j) g(i, j)$$

$$h(x, y) = f(x, y) * g(x, y)$$

Derivative Masks

(a)

frame-1

-1	1
-1	1

frame-2

-1	1
-1	1

(b)

frame-1

1	1
-1	-1

frame-2

1	1
-1	-1

(c)

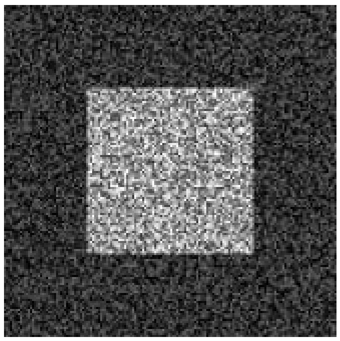
frame-1

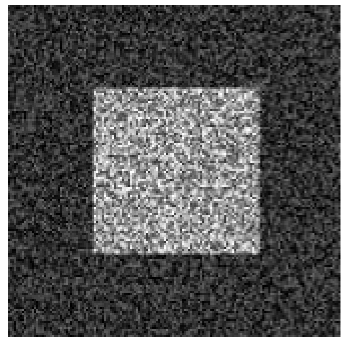
1	1
1	1

frame-2

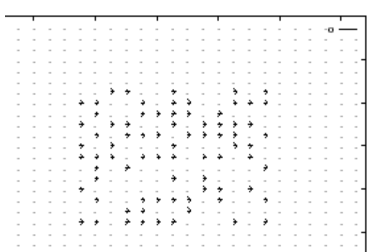
-1	-1
-1	-1

Synthetic Images

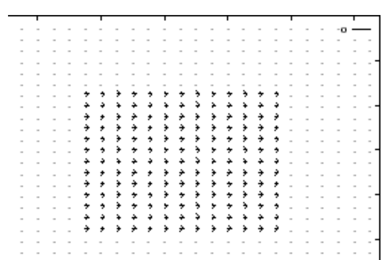




Results



One iteration



10 iterations