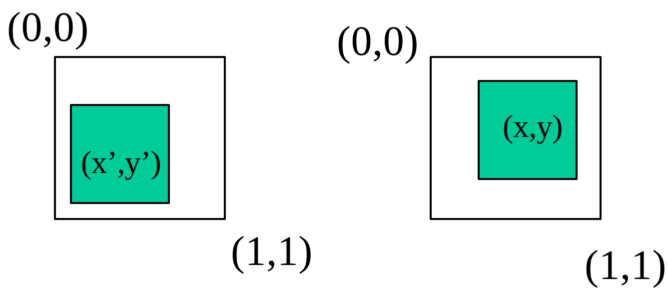
Anandan

Affine

Affine



$$u(x, y) = a_1 x + a_2 y + b_1$$
$$v(x, y) = a_3 x + a_4 y + b_2$$

Anandan

$$u(x,y) = a_1 x + a_2 y + b_1$$
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•Affine

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ a_3 \\ a_4 \\ b_2 \end{bmatrix}$$

$$\mathbf{u}(\mathbf{x}) = \mathbf{X}(\mathbf{x})\mathbf{a}$$

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$$\mathbf{u}(\mathbf{x}) = \mathbf{X}(\mathbf{x})\mathbf{a}$$

$$f_{\mathbf{x}} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

$$E(da) = \sum_x (f_t + f_{\mathbf{x}}^T du)^2$$

$$E(da) = \sum_x (f_t + f_{\mathbf{x}}^T \mathbf{X} da)^2$$

min

Optical flow constraint eq

$$f_x u + f_y v = -f_t$$

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$$\left[\sum \mathbf{X}^{\mathbf{T}}(\mathbf{f}_{\mathbf{X}})(\mathbf{f}_{\mathbf{x}})^{T}\mathbf{X}\right]d\mathbf{a} = -\sum \mathbf{X}^{T}\mathbf{f}_{\mathbf{X}} f_t$$

Homework 1.1

Basic Components

- Pyramid construction
- Motion estimation
- Image warping
- Coarse-to-fine refinement

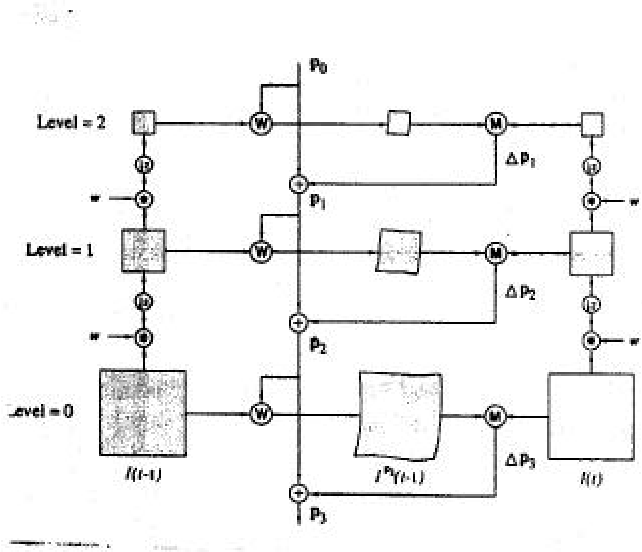
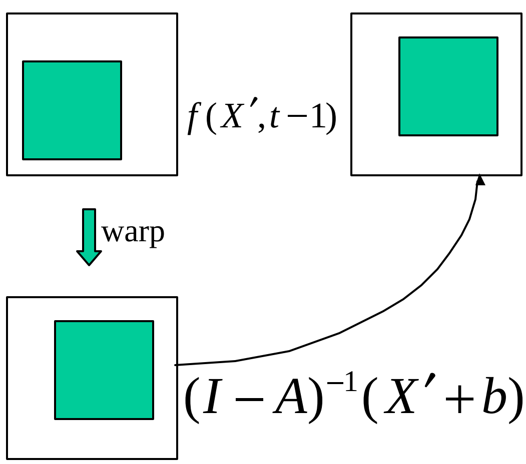


Image Warping

$$X' = X - U$$
$$= X - (AX + b)$$



$f(X', t-1)$

$f(X, t)$

$$(I - A)^{-1}(X' + b) = X''$$

Image Warping

$X' = X - U = X - (AX + b)$
$X' = (I - A)X - b$
$X' = A'X - b$
$X' + b = A'X$
$(A')^{-1}(X' + b) = X$

Image at time t: X
Image at time t-1: X'

$$(A')^{-1}(X' + b) = X''$$

Image Warping

- How about values in $X'' = (x'', y'')$ are not integer.
- But image is sampled only at integer rows and columns
  - Instead of converting X' to X'' and copying $f(X', t-1)$ at $f(X'', t-1)$ we can convert integer values X'' to X' and copy $f(X', t-1)$ at $f(X'', t-1)$

Image Warping

• But how about the values in X' are not integer.

• Perform bilinear interpolation to compute at non-integer values.

Image Warping

$$(A')^{-1}(X'+b) = X''$$

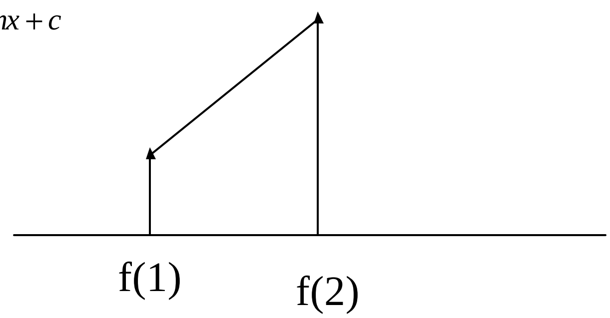
$$(X'+b) = (A')X''$$

$$X' = (A')X'' - b$$

1-D Interpolation

$y = mx + c$

$f(x) = mx + c$



2-D Interpolation

$f(x, y) = a_1 + a_2 x + a_3 y + a_4 xy$

Bilinear

X X

O X

Bi-linear Interpolation

Four nearest points of (x,y) are:

$$(\underline{x},\underline{y}),(\overline{x},\underline{y}),(\underline{x},\overline{y}),(\overline{x},\overline{y})$$

$$(3,5),(4,5),(3,6),(4,6)$$

$\underline{x}=\text{int}(x)$ 3 (3.2,5.6)

$\underline{y}=\text{int}(y)$ 5

$\overline{x}=\underline{x}+1$ 4

$\overline{y}=\underline{y}+1$ 6

X X
O X

$$f'(x,y)=\overline{e_x}\,\overline{e_y}\,f(\underline{x},\underline{y})+\underline{e_x}\,\overline{e_y}\,f(\overline{x},\underline{y})+$$
$$\overline{e_x}\,\underline{e_y}\,f(\underline{x},\overline{y})+\underline{e_x}\,\underline{e_y}\,f(\overline{x},\overline{y})$$

$$\overline{e_x}=\overline{x}-x$$

$$\overline{e_y}=\overline{y}-y$$

$$\underline{e_x}=x-\underline{x}$$

$$\underline{e_y}=y-\underline{y}$$

Program-1 Due Sept 23

- Implement Anandan’s algorithm