

Estimation Using Flexible Wireframe Model

Main Points

- Model photometric effects
- Simultaneously compute 3D motion and adapt the wireframe model.

Generalized Optical Flow Constraint

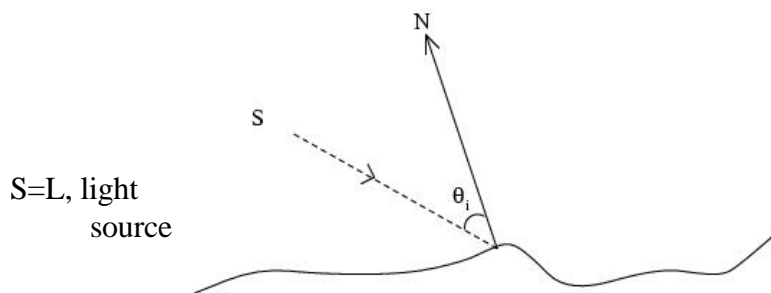
$$f(x, y, t) = \mathbf{r}N(t) \cdot \mathbf{L} \quad \text{Lambertian Model}$$

$$\frac{df(x, y, t)}{dt} = \mathbf{rL} \cdot \frac{dN}{dt}$$

Albedo
Surface Normal
(-p, -q, 1)

$$f_x u + f_y v + f_t = \mathbf{rL} \cdot \frac{dN}{dt}$$

Lambertian Model



$$f(x, y) = n \cdot L = (n_x, n_y, n_z) \cdot (l_x, l_y, l_z)$$

$$f(x, y) = n \cdot L = \left(\frac{1}{\sqrt{p^2 + q^2 + 1}} (-p, -q, 1) \right) \cdot (l_x, l_y, l_z)$$

Sphere

$$z = \sqrt{(R^2 - x^2 - y^2)}$$

$$p = \frac{\partial z}{\partial x} = -\frac{x}{R}$$

$$p = \frac{\partial z}{\partial y} = -\frac{y}{R}$$

$$(n_x, n_y, n_z) = \frac{1}{R}(x, y, z)$$

Orthographic Projection

$$u = \dot{x} = \Omega_2 Z - \Omega_3 y + V_1 \quad (u, v) \text{ is optical flow}$$

$$v = \dot{y} = \Omega_3 x - \Omega_1 Z + V_2$$

$$\dot{\mathbf{X}} = \boldsymbol{\Omega} \times \mathbf{X} + \mathbf{V}$$

$$\dot{X} = \Omega_2 Z - \Omega_3 Y + V_1$$

$$\dot{Y} = \Omega_3 X - \Omega_1 Z + V_2$$

$$\dot{Z} = \Omega_1 Y - \Omega_2 X + V_3$$

$$f_x(\Omega_2 Z - \Omega_3 y + V_1) + f_y(\Omega_3 x - \Omega_1 Z + V_2) + f_t = \mathbf{rL} \cdot \frac{dN}{dt}$$

$$f_x(\Omega_2 Z - \Omega_3 y + V_1) + f_y(\Omega_3 x - \Omega_1 Z + V_2) + f_t =$$

$$\mathbf{rL} \cdot \left[\frac{(-p', -q', 1)^T}{\sqrt{p'^2 + q'^2 + 1}} - \frac{(-p, -q, 1)^T}{\sqrt{p^2 + q^2 + 1}} \right]$$

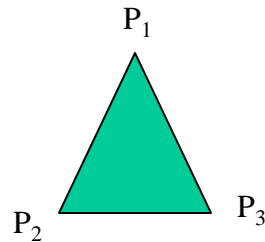
Equation of a Planar Patch

$$P_1^{(i)} = (X_1^{(i)}, Y_1^{(i)}, Z_1^{(i)})$$

$$P_2^{(i)} = (X_2^{(i)}, Y_2^{(i)}, Z_2^{(i)})$$

$$P_3^{(i)} = (X_3^{(i)}, Y_3^{(i)}, Z_3^{(i)})$$

$$P^{(i)} = (X^{(i)}, Y^{(i)}, Z^{(i)})$$



$$\overline{P^{(i)} P_1^{(i)}} \cdot (\overline{P_2^{(i)} P_1^{(i)}} \times \overline{P_3^{(i)} P_1^{(i)}}) = 0$$

Equation of a Planar Patch

$$Z^{(i)} = p_i X^{(i)} + q_i Y^{(i)} + c_i$$

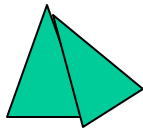
$$p_i = \frac{(Y_2^{(i)} - Y_1^{(i)})(Z_3^{(i)} - Z_1^{(i)}) - (Z_2^{(i)} - Z_1^{(i)})(Y_3^{(i)} - Y_1^{(i)})}{(X_2^{(i)} - X_1^{(i)})(Y_3^{(i)} - Y_1^{(i)}) - (Y_2^{(i)} - Y_1^{(i)})(X_3^{(i)} - X_1^{(i)})}$$

$$q_i = \frac{(Z_2^{(i)} - Z_1^{(i)})(X_3^{(i)} - X_1^{(i)}) - (X_2^{(i)} - X_1^{(i)})(Z_3^{(i)} - Z_1^{(i)})}{(X_2^{(i)} - X_1^{(i)})(Y_3^{(i)} - Y_1^{(i)}) - (Y_2^{(i)} - Y_1^{(i)})(X_3^{(i)} - X_1^{(i)})}$$

$$c_i = Z_1^{(i)} + X_1^{(i)} \frac{(Y_2^{(i)} - Y_1^{(i)})(Z_3^{(i)} - Z_1^{(i)}) - (Z_2^{(i)} - Z_1^{(i)})(Y_3^{(i)} - Y_1^{(i)})}{(X_2^{(i)} - X_1^{(i)})(Y_3^{(i)} - Y_1^{(i)}) - (Y_2^{(i)} - Y_1^{(i)})(X_3^{(i)} - X_1^{(i)})} +$$

$$Y_1^{(i)} \frac{(Z_2^{(i)} - Z_1^{(i)})(X_3^{(i)} - X_1^{(i)}) - (X_2^{(i)} - X_1^{(i)})(Z_3^{(i)} - Z_1^{(i)})}{(X_2^{(i)} - X_1^{(i)})(Y_3^{(i)} - Y_1^{(i)}) - (Y_2^{(i)} - Y_1^{(i)})(X_3^{(i)} - X_1^{(i)})}$$

(x^{ij}, y^{ij})



$$p_i x^{(ij)} + q_i y^{(ij)} + c_i = p_j x^{(ij)} + q_j y^{(ij)} + c_j$$

Neighboring patches must intersect at a straight line.

$$E = \sum_i \sum_{(x,y) \in \text{patch } i} e_i^2$$

$$p_i x_1^{(ij)} + q_i x_2^{(ij)} + c_i = p_j x_1^{(ij)} + q_j x_2^{(ij)} + c_j$$

$$e_i(x, y) = f_x(\Omega_3 y - \Omega_2(p_i x + q_i y + c_i) + V_1)$$

$$+ f_y(-\Omega_3 x + \Omega_1(p_i x + q_i y + c_i) + V_2) + f_t$$

$$- \mathbf{r}(L_1, L_2, L_3) \cdot \left(\frac{-\Omega_2 + p_i}{1 + \Omega_2 p_i}, \frac{-\Omega_1 + q_i}{1 - \Omega_1 q_i} \right) -$$

$$\frac{(-p_i, -q_i, 1)}{(p_i^2 + q_i^2 + 1)^{1/2}}$$

Updating of (X,Y):

$$p_i X^{(n)} + q_i Y^{(n)} + c_i = p_j X^{(n)} + q_j Y^{(n)} + c_j$$

$$p_i X^{(n)} + q_i Y^{(n)} + c_i = p_k X^{(n)} + q_k Y^{(n)} + c_k$$

$$\begin{bmatrix} X^{(n)} \\ Y^{(n)} \end{bmatrix} = \begin{bmatrix} p_i - p_j & p_j - p_k \\ q_i - q_j & q_j - q_k \end{bmatrix}^{-1} \begin{bmatrix} c_j - c_i \\ -c_j + c_k \end{bmatrix}$$

Patches i, j, k intersect at node n.

Algorithm

- Estimate light source direction
- Initialize coordinates of all nodes using approximately scaled wireframe model.
Determine initial values of surface normals for each triangle.
- Determine initial motion parameters based on selected feature correspondences and their depth values from wireframe model.
- Compute the adjustments to motion parameters and depth values by minimizing the error function E .