Synthesizing Realistic Facial Expressions from Photographs

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The Artist's Complete Guide to Facial Expression: Gary Faigin

- There is no landscape that we know as well as the human face. The twenty-five-odd square inches containing the features is the most intimately scrutinized piece of territory in existence, examined constantly, and carefully, with far more than an intellectual interest. Every detail of the nose, eyes, and mouth, every regularity in proportion, every variation from one individual to the next, are matters about which we are all authorities.

Main Points

- One view is not enough.
- Fitting of wire frame model to the image is a complex problem (pose estimation)
- Texture mapping is an important problem

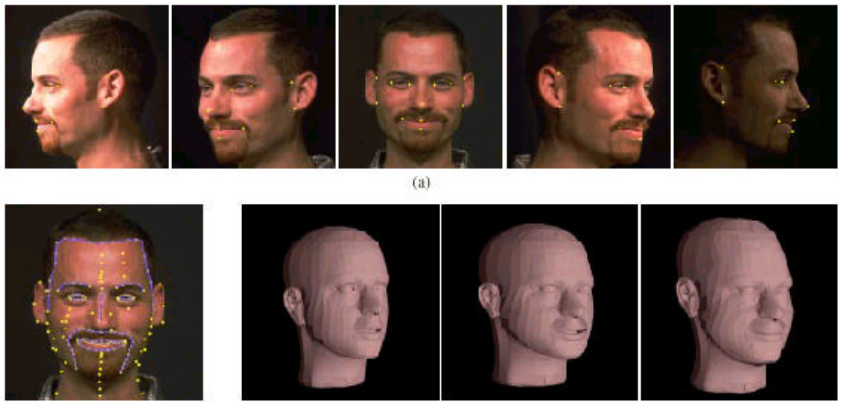
Synthesizing Realistic Facial Expressions

- Select 13 feature points manually in face image corresponding to points in face model created with Alias.
- Estimate camera poses and deformed 3d model points.
- Use these deformed values to deform the remaining points on the mesh using interpolation.

Synthesizing Realistic Facial Expressions

- Introduce more feature points (99) manually, and compute deformations as before by keeping the camera poses fixed.
- Use these deformed values to deform the remaining points on the mesh using interpolation as before.
- Extract texture.
- Create new expressions using morphing.

Synthesizing Realistic Facial Expressions



3D Rigid Transformation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + T = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

Camera coordinates

Wireframe coordinates

$$x_i'^k = f^k \frac{X_i'^k}{Z_i'^k}, y_i'^k = f^k \frac{Y_i'^k}{Z_i'^k}$$ perspective

3D Rigid Transformation

$$x_i'^k = f^k \frac{X_i'^k}{Z_i'^k}, y_i'^k = f^k \frac{Y_i'^k}{Z_i'^k}$$

$$x_i'^k = f_k \frac{r_{11}^k X_i + r_{12}^k Y_i + r_{13}^k Z_i + T_X^k}{r_{31}^k X_i + r_{32}^k Y_i + r_{33}^k Z_i + T_Z^k}$$

$$y_i'^k = f_k \frac{r_{21}^k X_i + r_{22}^k Y_i + r_{23}^k Z_i + T_Y^k}{r_{31}^k X_i + r_{32}^k Y_i + r_{33}^k Z_i + T_Z^k}$$

Model Fitting

$$x'^{k}_{i} = f_k \frac{\mathbf{r}^{\mathbf{k}}_{x}\mathbf{p}_{\mathbf{i}} + T^{k}_{X}}{\mathbf{r}^{\mathbf{k}}_{z}\mathbf{p}_{\mathbf{i}} + T^{k}_{Z}}$$

$$y'^{k}_{i} = f_k \frac{\mathbf{r}^{\mathbf{k}}_{y}\mathbf{p}_{\mathbf{i}} + T^{k}_{Y}}{\mathbf{r}^{\mathbf{k}}_{z}\mathbf{p}_{\mathbf{i}} + T^{k}_{Z}}$$

Model Fitting

$$x'^{k}_{i} = f_k \frac{\mathbf{r}^{\mathbf{k}}_{x}\mathbf{p}_{\mathbf{i}} + T^{k}_{X}}{\mathbf{r}^{\mathbf{k}}_{z}\mathbf{p}_{\mathbf{i}} + T^{k}_{Z}}$$

$$y'^{k}_{i} = f_k \frac{\mathbf{r}^{\mathbf{k}}_{y}\mathbf{p}_{\mathbf{i}} + T^{k}_{Y}}{\mathbf{r}^{\mathbf{k}}_{z}\mathbf{p}_{\mathbf{i}} + T^{k}_{Z}} \qquad h^k = \frac{1}{T^{k}_{Z}}, s^k = f^k h^k$$

$$x'^{k}_{i} = s_k \frac{\mathbf{r}^{\mathbf{k}}_{x}\mathbf{p}_{\mathbf{i}} + T^{k}_{X}}{1 + h^k \mathbf{r}^{\mathbf{k}}_{z}\mathbf{p}_{\mathbf{i}}}$$

$$y'^{k}_{i} = s_k \frac{\mathbf{r}^{\mathbf{k}}_{y}\mathbf{p}_{\mathbf{i}} + T^{k}_{Y}}{1 + h^k \mathbf{r}^{\mathbf{k}}_{z}\mathbf{p}_{\mathbf{i}}}$$

Model Fitting

$$x_i'^k = s_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_X^k}{1 + h^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}}$$

$$y_i'^k = s_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Y^k}{1 + h^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}} \quad w_i^k = (1 + h^k (\mathbf{r}_z^k . \mathbf{p}_i))^{-1}$$

$$w_i^k (x_i'^k + x_i'^k h^k (\mathbf{r}_{\mathbf{z}}^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}}) - s^k (\mathbf{r}_X^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}} + T_X^k)) = 0$$

$$w_i^k (y_i'^k + y_i'^k h^k (\mathbf{r}_{\mathbf{z}}^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}}) - s^k (\mathbf{r}_Y^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}} + T_Y^k)) = 0$$

Model Fitting

- Solve for unknowns in five steps:

  $s^k; \mathbf{p}_i; \mathbf{R}^k; T_X^k, T_Y^k; h^k$

  - Use linear least squares fit.

  - When solving for an unknown, assume other parameters are known.

Least Squares Fit

$a_j.x - b_j = 0$

$\sum_j (a_j.x - b_j)^2$

$w_i^k(x_i'^k + x_i'^k h^k(\mathbf{r_z^k}.\mathbf{p_i}) - s^k(\mathbf{r}_X^k.\mathbf{p_i} + T_X^k)) = 0$

$w_i^k(y_i'^k + y_i'^k h^k(\mathbf{r_z^k}.\mathbf{p_i}) - s^k(\mathbf{r}_Y^k.\mathbf{p_i} + T_Y^k)) = 0$

$\sum_j (a_j a_j^T)x = \sum_j b_j a_j$

Update for p

$a_{2k+0} = w_i^k(x_i^k \boldsymbol{h}^k r_z^k - s^k r_x^k)$ $\quad$ $\mathrm{b}_{2k+0} = w_i^k(s^k T_x^k - x_i^k)$

$a_{2k+1} = w_i^k(y_i^k \boldsymbol{h}^k r_z^k - s^k r_y^k)$ $\quad$ $\mathrm{b}_{2k+1} = w_i^k(s^k T_y^k - y_i^k)$

$a_j.x - b_j = 0$

$\sum_j (a_j.x - b_j)^2$

$w_i^k(x_i'^k + x_i'^k h^k(\mathbf{r_z^k}.\mathbf{p_i}) - s^k(\mathbf{r}_X^k.\mathbf{p_i} + T_X^k)) = 0$

$w_i^k(y_i'^k + y_i'^k h^k(\mathbf{r_z^k}.\mathbf{p_i}) - s^k(\mathbf{r}_Y^k.\mathbf{p_i} + T_Y^k)) = 0$

$\sum_j (a_j a_j^T)x = \sum_j b_j a_j$

Update for s^k

$a_{2k+0} = w_i^k(r_x^k.p_i + t_x^k)$ $\quad$ $\mathrm{b}_{2k+0} = w_i^k(x_i^k + x_i^k \boldsymbol{h}^k(r_z^k.p_i))$

$a_{2k+1} = w_i^k(r_y^k.p_i + t_y^k)$ $\quad$ $\mathrm{b}_{2k+1} = w_i^k(y_i^k + y_i^k \boldsymbol{h}^k(r_z^k.p_i))$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

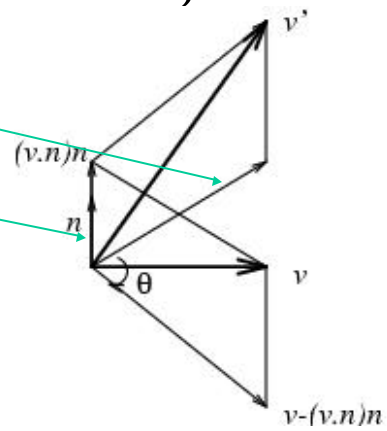
$$V = (V.n)n + (V - (V.n)n)$$

HW 4.1

$$V'_{\perp} = \cos q(V - (V.n)n) + \sin q(n \times (V - (V.n)n)$$

$$V'_{II} = (V.n)n$$

$$V' = V'_{\perp} + V'_{II}$$



$$V' = \cos q\, V + \sin q\; n \times V + (1 - \cos q)(V.n)n$$

$$V' = V + \sin q\; n \times V + (1 - \cos q)n \times (n \times V)$$

$$n \times (n \times V) = (V.n)n - V$$

$$n \times (n \times V) + V = (V.n)n$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$V' = V + \sin q\; n \times V + (1 - \cos q)\, n \times (n \times V)$$

$$V' = R(n,q)V$$

$$R(n,\boldsymbol{q}) = I + \sin \boldsymbol{q}\; X(n) + (1 - \cos \boldsymbol{q}) X^2(n)$$

HW 4.2

$$X(n) = \begin{bmatrix} 0 & -n_z & n_x \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$r = \| r \| \frac{r}{\| r |} = qn$$

$$R(r,\boldsymbol{q}) = I + \sin \boldsymbol{q} \frac{X(r)}{\| r \|} + (1 - \cos \boldsymbol{q}) \frac{X^2(r)}{\| r \|^2}$$

$$X(r) = \begin{bmatrix} 0 & -r_z & r_x \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$$

$$R^k = \tilde{R} R^k$$

$$\tilde{R}(n,\boldsymbol{q}) = I + \sin \boldsymbol{q} X(n) + (1 - \cos \boldsymbol{q}) X^2(n)$$

$$\tilde{R} \approx I + \boldsymbol{q} X(m) \qquad m = \boldsymbol{q} n = (m_x, m_y, m_z)$$

$$w_i^k (x_i'^k + x_i'^k h^k (\mathbf{r}_\mathbf{z}^\mathbf{k}.\mathbf{p_i}) - s^k (\mathbf{r}_X^\mathbf{k}.\mathbf{p_i} + T_X^k)) = 0$$
$$w_i^k (y_i'^k + y_i'^k h^k (\mathbf{r}_\mathbf{z}^\mathbf{k}.\mathbf{p_i}) - s^k (\mathbf{r}_Y^\mathbf{k}.\mathbf{p_i} + T_Y^k)) = 0$$

$$R^k \leftarrow \tilde{R}(n^k, q^k) R^k$$

$$\tilde{r}_x^k = (1, -m_z, m_y)$$
$$\tilde{r}_y^k = (m_z, 1, -m_x)$$
$$\tilde{r}_z^k = (-m_y, m_x, 1)$$
$$q_i = R^k p_i$$

$$w_i^k (x_i^k + x_i^k h r_x^k (\tilde{r}_z^k . q_i) - s^k (\tilde{r}_x^k . q_i + t_x^k)) = 0$$
$$w_i^k (y_i^k + y_i^k h r_x^k (\tilde{r}_z^k . q_i) - s^k (\tilde{r}_y^k . q_i + t_y^k)) = 0$$

Interpolation

- Use initial set of coordinates for the feature points (13 points), to deform the remaining vertices using interpolation.

Interpolation

$$f(\mathbf{p}) = \sum_i c_i f(\| \mathbf{p} - \mathbf{p}_i \|) + \mathbf{M}\mathbf{p} + \mathbf{t}$$

Original point position: $\mathbf{p_i}$
Updated point position: $\mathbf{p_i^0}$

$$u_i = \mathbf{p}_i - \mathbf{p_i^0}, u_i = f(\mathbf{p}_i)$$

$$\sum_i c_i = 0, \sum_i c_i \mathbf{p}_i = 0$$

1.First estimate c_i's , M, and t using the known 13 points
2.Compute the displacement of Other points using *f(p)*

$$f(r) = e^{\frac{-r}{64}}$$

Texture Extraction

- Given a collection of photographs, the recovered viewing parameters, and the fitted face model, compute for each point p on the face model its texture color T(p).

Texture Extraction

$$T(\mathbf{p}) = \frac{\sum_k m^k(\mathbf{p}) I^k(x^k, y^k)}{\sum_k m^k(\mathbf{p})}$$

I^k is k-th image

m^k is weight

Weights

- Self-occlusion
  - m^k should be zero unless P is front facing with k-th image and is visible in it.
- Smoothness
  - The weight map should vary smoothly, in order to ensure a seamless blend between different images.
- Positional certainty
  - It is the dot product of surface normal at P and k-th direction of projection.
- View similarity
  - This depends on the angle between direction of projection of P onto j-th image and its direction of projection in the new image.

Texture Extraction

•Visibility map $F^k(u,v)$ is set to 1 if the corresponding point p is visible in k-th image, and zero otherwise.

• Positional certainty, $P^k(\mathbf{p})$ is define as a dot product of surface normal at p and the k-th direction of projection.

Texture Extraction

• View-independent texture mapping:

$$m^k(u,v) = F^k(u,v)P^k(\mathbf{p})$$

• View-dependent texture mapping:

$$m^k(u,v) = F^k(x^k, y^k)P^k(\mathbf{p})V^k(d)$$

$$V^k(\mathbf{d}) = \mathbf{d}.\mathbf{d}^k - \mathbf{d}^l.\mathbf{d}^{l+1}$$

Show Video Clip.

Face Recognition

Simple Algorithm

- Recognize faces (mug shots) using gray levels (appearance)
- Each image is mapped to a long vector of gray levels
- Several views of each person are collected in the model-base during training
- During recognition a vector corresponding to an unknown face is compared with all vectors in the model-base
- The face from model-base, which is closest to the unknown face is declared as a recognized face.

Problems

- Problems :
  - Dimensionality of each face vector will be very large (250,000 for a 512X512 image!)
  - Raw gray levels are sensitive to noise, and lighting conditions.
- Solution:
  - Reduce dimensionality of face space by finding principal components (eigen vectors) to span the face space
  - Only a few most significant eigen vectors can be used to represent a face, thus reducing the dimensionality

Eigen Vectors and Eigen Values

The eigen vector, x, of a matrix B is a special vector, with the following property

$Bx = \lambda x$ Where ë is called eigen value

To find eigen values of a matrix A first find the roots of:

$$\det(A - \lambda I) = 0$$

Then solve the following linear system for each eigen value to find corresponding eigen vector

$$(A - \lambda I)x = 0$$

Example

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix} \qquad \lambda_1 = 7,\ \lambda_2 = 3,\ \lambda_3 = -1$$

$$x_1 = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix},\ x_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix},\ x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Collect all gray levels in a long vector *u:*

$$u = (I(1,1),\ldots,I(1,N),I(2,1),\ldots,I(2,N),\ldots,I(M,1),\ldots,I(M,N))^T$$

Collect *n* samples (views) of each of *p* persons in matrix A (MN X pn):

$$A = \left[u_1^1,\ldots u_n^1,u_1^2\ldots,u_n^2,\ldots,u_1^p\ldots,u_n^p\right]$$

Form a correlation matrix L (MN X MN):

$$L = AA^T$$

Compute eigen vectors, $f_1, f_2, f_3, \ldots fn_1$, of L, which form a bases for whole face space

Each face, *u,* can now be represented as a linear combination of eigen vectors

$$u = \sum_{i=1}^{n_1} a_i f_i$$

Where

$$a_i = u_x^T . f_i$$

L is a large matrix, computing eigen vectors of a large matrix is time consuming. Therefore compute eigen vectors of a smaller matrix, C:

$$C = A^T A$$

Let l_i be eigen vectors of C, then Al_i are the eigen vectors of A:

$$C\alpha_i = \lambda_i \alpha_i$$

$$A^T A\alpha_i = \lambda_i \alpha_i$$

$$AA^T(A\alpha_i) = \lambda_i(A\alpha_i)$$
$$L(A\alpha_i) = L(A\alpha_i)$$

Training

- Create A matrix from training images
- Compute C matrix from A.
- Compute eigenvectors of C.
- Compute eigenvectors of L from eigenvectors of C.
- Select few most significant eigenvectors of L for face recognition.
- Compute coefficient vectors corresponding to each training image.
- For each person, coefficients will form a cluster, compute the mean of cluster.

Recognition

- Create a vector *u* for the image to be recognized.
- Compute coefficient vector for this *u*.
- Decide which person this image belongs to, based on the distance from the cluster mean for each person.

MATLAB Progam

Face Recognition

```
load faces.mat
C=A'*A;
[vectorC,valueC]=eig(C);
ss=diag(valueC);
[ss,iii]=sort(-ss);
vectorC=vectorC(:,iii);
vectorL=A*vectorC(:,1:5);
Coeff=A'*vectorL;
for I=1:30
        model(i, :)=mean(coeff((5*(i-1)+1):5*I,:));
end
while (1)
        imagename=input('Enter the filename of the image to
        Recognize(0 stop):');
        if (imagename <1)
        break;
        end;
        imageco=A(:,imagename)'*vectorL;
        disp ('');
        disp ('The coefficients for this image are:');
```

```
        mess1=sprintf('%.2f  %.2f  %.2f  %.2f  %.2f',
        imageco(1),imageco(2),imageco(3),imageco(4),
        imageco(5));
        disp(mess1);
T       top=1;
        for I=2:30
                if (norm(model(i,:)-imageco,1)<norm(model
                (top, : )-imageco,1))
                top=i
                end
        end
        mess1=sprintf('The image input was a image of person
        number %d',top);
        disp(mess1);
        end
b=A(:,81);
b=reshape(b,34,51);
imshow(b,gray(255)):
```

Webpage

http://vismod.www.media.mit.edu/vismod/demos/