Lecture-15

Discussion of Program-2

Lucas & Kanade (Least Squares)

- Optical flow eq

$$f_x u + f_y v = -f_t$$

- Consider 3 by 3 window

$$f_{x1} u + f_{y1} v = -f_{t1}$$

$$\vdots$$

$$f_{x9} u + f_{y9} v = -f_{t9}$$

$$\begin{bmatrix} f_{x1} & f_{y1} \\ \vdots & \vdots \\ f_{x9} & f_{y9} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -f_{t1} \\ \vdots \\ -f_{t9} \end{bmatrix}$$

$$\mathbf{Au} = \mathbf{f_t}$$

Lucas & Kanade

$$\mathbf{Au} = \mathbf{f_t}$$

$$\mathbf{A^T A u} = \mathbf{A^T f_t}$$

$$\mathbf{u} = (\mathbf{A^T A})^{-1} \mathbf{A^T f_t}$$

⇧

$$\min \sum_{i=-2}^{2} \sum_{j=-2}^{2} (f_{xi} u + f_{yi} v + f_{ti})^2$$

Lucas & Kanade

$$\min \sum_{i=-2}^{2} \sum_{j=-2}^{2} (f_{xi} u + f_{yi} v + f_{ti})^2$$

min ⇩

$$\sum (f_{xi} u + f_{yi} v + f_{ti}) f_{xi} = 0$$

$$\sum (f_{xi} u + f_{yi} v + f_{ti}) f_{yi} = 0$$

⇩

$$\sum (f_{xi} f_{xi} u + f_{xi} f_{yi} v + f_{xi} f_{ti}) = 0$$

$$\sum (f_{yi} f_{xi} u + f_{yi} f_{yi} v + f_{yi} f_{ti}) = 0$$

⇩

$$\sum f_{xi}^2 u + \sum f_{xi} f_{yi} v + \sum f_{xi} f_{ti} = 0$$

$$\sum f_{yi} f_{yi} u + \sum f_{yi}^2 v + \sum f_{yi} f_{ti} = 0$$

⇩

$$\begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

Lucas & Kanade

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi} f_{yi} \\ \sum f_{xi} f_{yi} & \sum f_{yi}^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum f_{xi} f_{ti} \\ -\sum f_{yi} f_{ti} \end{bmatrix}$$

Least Squares Fit

$$a_j . x - b_j = 0$$

$$\sum_j (a_j . x - b_j)^2$$

$$\sum_j (a_j a_j^T) x = \sum_j b_j a_j$$

$$a_j^T x - b_j = 0$$

$$\min \sum_j (a_j^T x - b_j)^2$$

differentiate

$$\sum_j 2(a_j^T x - b_j) a_j = 0$$

$$\sum_j a_j (a_j^T x - b_j) = 0$$

$$\sum_j (a_j a_j^T) x = \sum_j b_j a_j$$

Anandan

$$\mathbf{u}(\mathbf{x}) = \mathbf{X}(\mathbf{x})\mathbf{a}$$

$$f_{\mathbf{x}} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

$$E(da) = \sum_x (f_t + f_{\mathbf{x}}^T du)^2$$

$$E(da) = \sum_x (f_t + f_{\mathbf{x}}^T \mathbf{X} da\ )^2$$

min

$$\left[\sum \mathbf{X}^{\mathbf{T}}(\mathbf{f_X})(\mathbf{f_x})^T \mathbf{X}\right] d\mathbf{a} = -\sum \mathbf{X}^T \mathbf{f_X} f_t$$

$$Ax = b$$

Optical flow constraint eq

$$f_x u + f_y v = -f_t$$

Homework 2: Show this due 9/21

$$a_j^T x - b_j = 0$$

$$\min \sum_j (a_j^T x - b_j)^2$$

$$\sum_j (a_j a_j^T) x = \sum_j b_j a_j$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

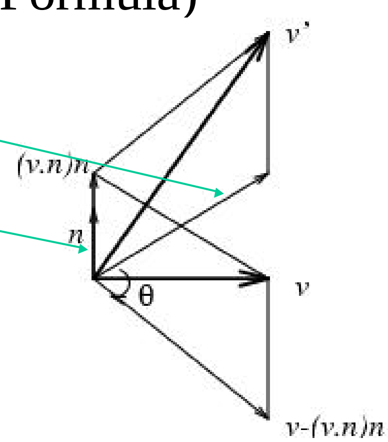
$$V = (V.n)n + (V - (V.n)n)$$

HW 4.1

$$V'_{\perp} = \cos q (V - (V.n)n) + \sin q (n \times (V - (V.n)n)$$

$$V'_{II} = (V.n)n$$

$$V' = V'_{\perp} + V'_{II}$$



$$V' = \cos q\ V + \sin q\ n \times V + (1 - \cos q)(V.n)n$$

$$V' = V + \sin q\ n \times V + (1 - \cos q\ ) n \times (n \times V)$$

$$n \times (n \times V) = (V.n)n - V$$

$$n \times (n \times V) + V = (V.n)n$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$V' = V + \sin q \; n \times V + (1 - \cos q)\; n \times (n \times V)$$

$$V' = R(n, q) V$$

$$R(n, \mathbf{q}) = I + \sin \mathbf{q} \; X(n) + (1 - \cos \mathbf{q}) X^2(n)$$ HW 4.2

$$X(n) = \begin{bmatrix} 0 & -n_z & n_x \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$r = \| r \| \frac{r}{\| r |} = qn$$

$$R(r, \mathbf{q}) = I + \sin \mathbf{q} \frac{X(r)}{\| r \|} + (1 - \cos \mathbf{q}) \frac{X^2(r)}{\| r \|^2}$$

$$X(r) = \begin{bmatrix} 0 & -r_z & r_x \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$$

$$R(n,\theta) = I + \sin\theta X(n) + (1-\cos\theta) X^2(n)$$

$$R(m) = I + \sin\theta \frac{X(m)}{\theta} + \frac{X^2(m)}{\theta^2}(1-\cos\theta)$$

$$\tilde{R} \approx I + X(m)$$

$$m = \theta n = (m_x, m_y, m_z)$$

$$R^{it+1} \leftarrow \tilde{R} R^{it}$$

3D Rigid Transformation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + T = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

Camera coordinates

Wireframe coordinates

$$x_i'^k = f^k \frac{X_i'^k}{Z_i'^k}, \quad y_i'^k = f^k \frac{Y_i'^k}{Z_i'^k}$$ perspective

K=camera no.

3D Rigid Transformation

$$x_i'^k = f^k \frac{X_i'^k}{Z_i'^k}, y_i'^k = f^k \frac{Y_i'^k}{Z_i'^k}$$

$$x_i'^k = f_k \frac{r_{11}^k X_i + r_{12}^k Y_i + r_{13}^k Z_i + T_X^k}{r_{31}^k X_i + r_{32}^k Y_i + r_{33}^k Z_i + T_Z^k}$$

$$y_i'^k = f_k \frac{r_{21}^k X_i + r_{22}^k Y_i + r_{23}^k Z_i + T_Y^k}{r_{31}^k X_i + r_{32}^k Y_i + r_{33}^k Z_i + T_Z^k}$$

Model Fitting

$$x_i'^k = f_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_X^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Z^k}$$

$$y_i'^k = f_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Y^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Z^k}$$

Model Fitting

$$x_i'^k = f_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_X^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Z^k}$$

$$y_i'^k = f_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Y^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Z^k} \qquad h^k = \frac{1}{T_Z^k}, s^k = f^k h^k$$

$$x_i'^k = s_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_X^k}{1 + h^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}}$$

$$y_i'^k = s_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Y^k}{1 + h^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}}$$

Model Fitting

$$x_i'^k = s_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_X^k}{1 + h^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}}$$

$$y_i'^k = s_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_{\mathbf{i}} + T_Y^k}{1 + h^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_{\mathbf{i}}} \qquad w_i^k = (1 + h^k (\mathbf{r}_z^k . \mathbf{p}_i))^{-1}$$

$$w_i^k (x_i'^k + x_i'^k h^k (\mathbf{r}_{\mathbf{z}}^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}}) - s^k (\mathbf{r}_X^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}} + T_X^k)) = 0$$

$$w_i^k (y_i'^k + y_i'^k h^k (\mathbf{r}_{\mathbf{z}}^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}}) - s^k (\mathbf{r}_Y^{\mathbf{k}} . \mathbf{p}_{\mathbf{i}} + T_Y^k)) = 0$$

Model Fitting

- Solve for unknowns in five steps:

$$s^k; \mathbf{p}_i; \mathbf{R}^k; T_X^k, T_Y^k; h^k$$

- Use linear least squares fit.
- When solving for an unknown, assume other parameters are known.

$$a_j.x - b_j = 0$$

$$\sum_j (a_j.x - b_j)^2$$

$$\sum_j (a_j a_j^T) x = \sum_j b_j a_j$$

$$w_i^k (x_i'^k + x_i'^k h^k (\mathbf{r_z^k}.\mathbf{p_i}) - s^k (\mathbf{r}_X^\mathbf{k}.\mathbf{p_i} + T_X^k)) = 0$$

$$w_i^k (y_i'^k + y_i'^k h^k (\mathbf{r_z^k}.\mathbf{p_i}) - s^k (\mathbf{r}_Y^\mathbf{k}.\mathbf{p_i} + T_Y^k)) = 0$$

Update for s^k

$$a_{2k+0} = w_i^k (r_x^k.p_i + t_x^k) \qquad \mathrm{b}_{2k+0} = w_i^k (x_i^k + x_i^k \boldsymbol{h}^k (r_z^k.p_i))$$

$$a_{2k+1} = w_i^k (r_y^k.p_i + t_y^k) \qquad \mathrm{b}_{2k+1} = w_i^k (y_i^k + y_i^k \boldsymbol{h}^k (r_z^k.p_i))$$

$$x_i' + x_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_x.\mathbf{p}_i + T_x) = 0$$
$$y_i' + y_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_y.\mathbf{p}_i + T_y) = 0$$

Solving for T_x and T_y

$$sT_x = x_i' + x_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_x.\mathbf{p}_i)$$
$$a_0 = s, b_0 = x_i' + x_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_x.\mathbf{p}_i)$$

$$sT_y = y_i' + y_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_y.\mathbf{p}_i)$$
$$a_0 = s, b_0 = y_i' + y_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_y.\mathbf{p}_i)$$

$$x_i' + x_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_x.\mathbf{p}_i + T_x) = 0$$
$$y_i' + y_i' h(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_y.\mathbf{p}_i + T_y) = 0$$

Solving for h

$$a_0 = x_i'(\mathbf{r}_z.\mathbf{p}_i), b_0 = s(\mathbf{r}_x.\mathbf{p}_i + T_x) - x_i'$$
$$a_1 = y_i'(\mathbf{r}_z.\mathbf{p}_i), b_1 = s(\mathbf{r}_y.\mathbf{p}_i + T_y) - y'$$

Solving for rotation:

$$w_i^k(x_i'^k + x_i'^k h^k(\mathbf{r_z^k}.\mathbf{p_i}) - s^k(\mathbf{r_X^k}.\mathbf{p_i} + T_X^k)) = 0$$
$$w_i^k(y_i'^k + y_i'^k h^k(\mathbf{r_z^k}.\mathbf{p_i}) - s^k(\mathbf{r_Y^k}.\mathbf{p_i} + T_Y^k)) = 0$$

⇩

$$w_i^k(x_i'^k + x_i'^k \boldsymbol{h}(\tilde{r}_z^k.q_i) - s^k(\tilde{r}_x^k.q_i + T_x^k)) = 0$$
$$w_i^k(y_i'^k + y_i'^k(\tilde{r}_z^k.q_i) - s^k(\tilde{r}_y^k.q_i + T_y^k)) = 0$$

⇩

$$x_i' + x_i' h(-m_y q_x + m_x q_y + q_z) - s(q_x - m_z q_y + m_y q_z + T_x) = 0$$
$$y_i' + y_i' h(-m_y q_x + m_x q_y + q_z) - s(m_z q_x + q_y - m_x q_z + T_y) = 0$$

$$\tilde{r}_z^k = (-m_y, m_x, 1)$$
$$\tilde{r}_x^k = (1, -m_z, m_y)$$
$$\tilde{r}_y^k = (m_z, 1, -m_x)$$

$$q_i = R^{it} p_i$$

$$R^{it+1} \leftarrow \tilde{R} R^{it}$$

$$x_i' + x_i' h(-m_y q_x + m_x q_y + q_z) - s(q_x - m_z q_y + m_y q_z + T_x) = 0$$
$$y_i' + y_i' h(-m_y q_x + m_x q_y + q_z) - s(m_z q_x + q_y - m_x q_z + T_y) = 0$$

$$\begin{bmatrix} x' h q_y & -x' h q_x + s q_z & s q_y \\ & & \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} sT_x - x' - x' h q_z + s q_x \\ \end{bmatrix}$$

$$\begin{bmatrix} & & \\ y' h q_y - s q_z & -y' h q_x & s q_x \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} \\ sT_y - y' - y' h q_z + s q_y \end{bmatrix}$$

$$a_0 = \begin{bmatrix} x' \boldsymbol{h} q_y & -x' \boldsymbol{h} q_x + s q_z & s q_y \\ & & \end{bmatrix}, b_0 = \begin{bmatrix} sT_x - x' - x' \boldsymbol{h} q_z + s q_x \\ \end{bmatrix}$$

$$a_1 = \begin{bmatrix} & & \\ y' \boldsymbol{h} q_y - s q_z & -y' \boldsymbol{h} q_x & s q_x \end{bmatrix}, b_1 = \begin{bmatrix} \\ sT_y - y' - y' \boldsymbol{h} q_z + s q_y \end{bmatrix}$$

$$\sum_j (a_j a_j^T) x = \sum_j b_j a_j$$