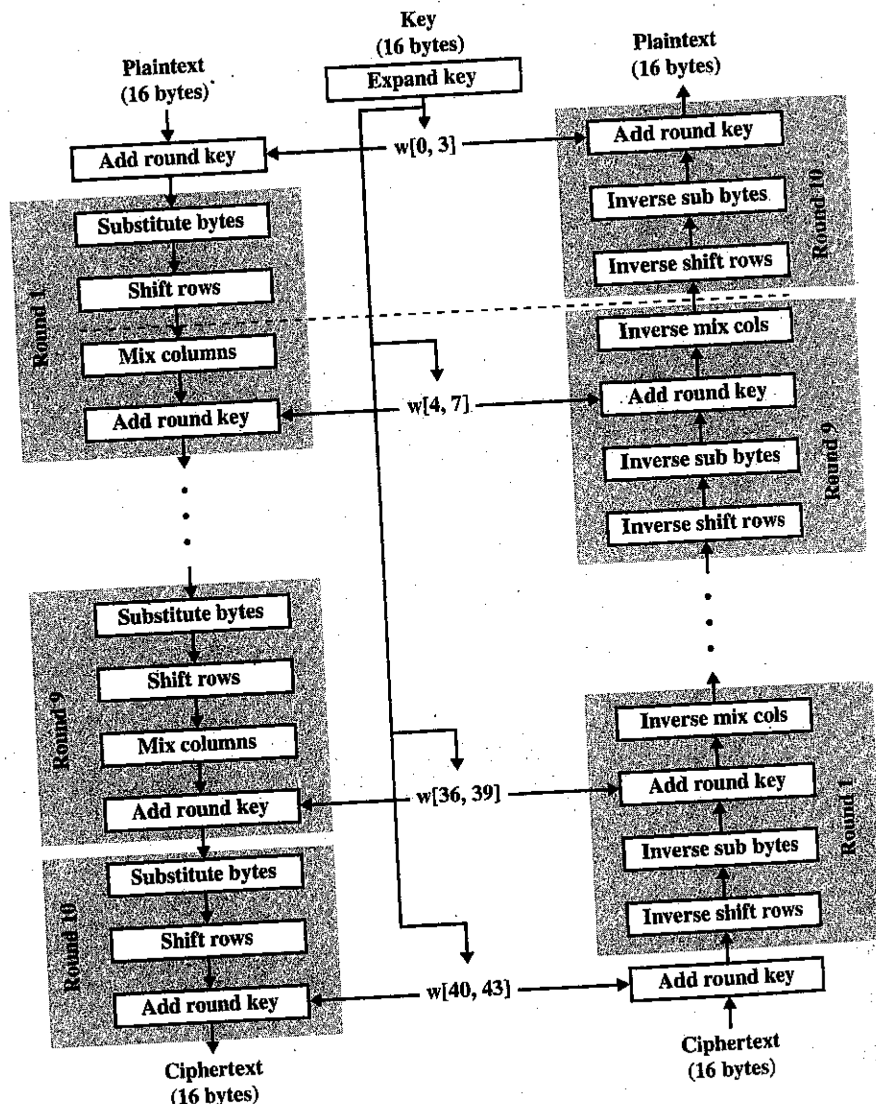


(a) Encryption

(b) Decryption

Figure 5.3 AES Encryption and Decryption

of the block (the other three stages), followed by XOR encryption, and s
This scheme is both efficient and highly secure.

7. Each stage is easily reversible. For the Substitute Byte, ShiftRows MixColumns stages, an inverse function is used in the decryption algo For the AddRoundKey stage, the inverse is achieved by XORing the round key to the block, using the result that A $\oplus$ B $\oplus$ B = A.
8. As with most block ciphers, the decryption algorithm makes use of expanded key in reverse order. However, the decryption algorithm

Table 5.4 AES S-Boxes

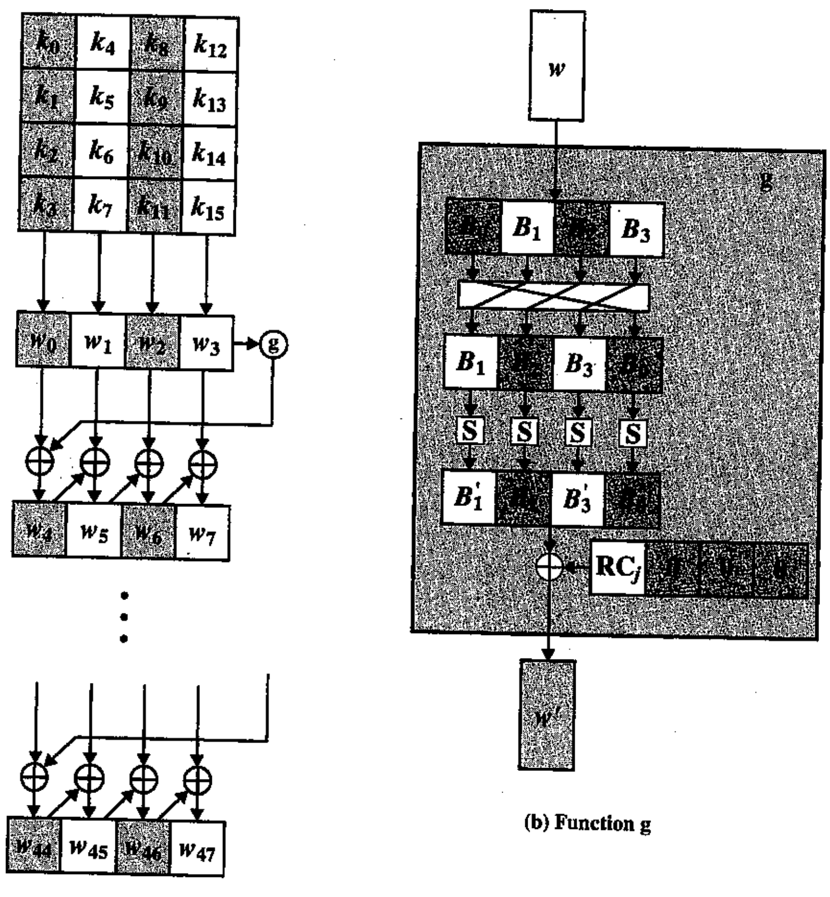
(a) S-box

x \ y	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(b) Inverse S-box

x \ y	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++)    w[i] = (key[4*i], key[4*i+1],
                                       key[4*i+2],
                                       key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
      temp = w[i - 1];
      if (i mod 4 = 0)    temp = SubWord (RotWord (temp))
                                  ⊕ Rcon[i/4];
      w[i] = w[i-4] ⊕ temp
    }
}
```



(b) Function g

(a) Overall algorithm

Figure 5.9 AES Key Expansion

cation defined over the field GF(2^8). The values of RC[j] in hexadecimal are

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

ShiftRows Transformation

Forward and Inverse Transformations The **forward shift row transformation,** called ShiftRows, is depicted in Figure 5.5a. The first row of **State** is not altered. For the second row, a 1-byte circular left shift is performed. For the third row, a 2-byte circular left shift is performed. For the fourth row, a 3-byte circular left shift is performed. The following is an example of ShiftRows:

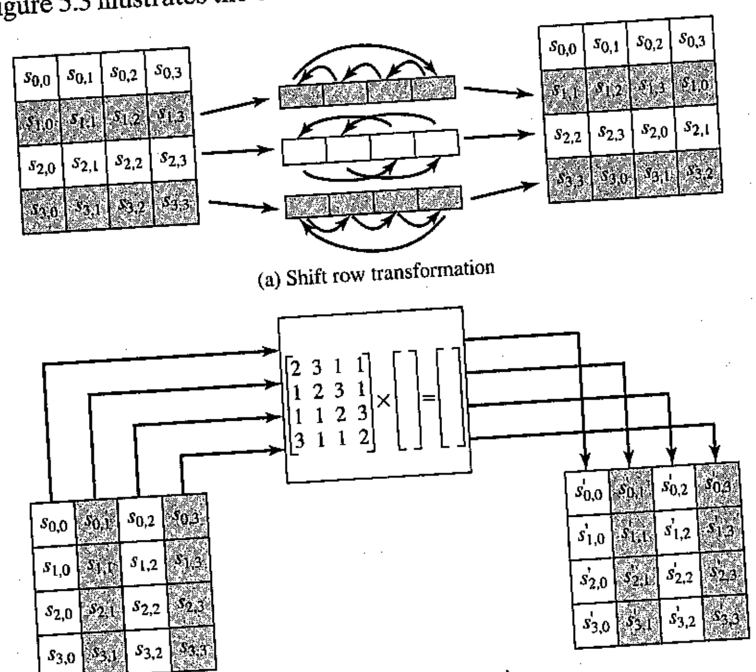
87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

→

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

The **inverse shift row transformation**, called InvShiftRows, performs the circular shifts in the opposite direction for each of the last three rows, with a one-byte circular right shift for the second row, and so on.

Rationale The shift row transformation is more substantial than it may first appear. This is because the **State**, as well as the cipher input and output, is treated as an array of four 4-byte columns. Thus, on encryption, the first 4 bytes of the plaintext are copied to the first column of **State**, and so on. Further, as will be seen, the round key is applied to **State** column by column. Thus, a row shift moves an individual byte from one column to another, which is a linear distance of a multiple of 4 bytes. Also note that the transformation ensures that the 4 bytes of one column are spread out to four different columns. Figure 5.3 illustrates the effect.



(a) Shift row transformation

(b) Mix column transformation

Figure 5.5 AES Row and Column Operations

MixColumns Transformation

Forward and Inverse Transformations The **forward mix column transformation**, called MixColumns, operates on each column individually. Each byte of a column is mapped into a new value that is a function of all four bytes in that column. The transformation can be defined by the following matrix multiplication on **State** (Figure 5.5b):

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix} \quad \textbf{(5.3)}$$

Each element in the product matrix is the sum of products of elements of one row and one column. In this case, the individual additions and multiplications[6] are performed in GF(2^8). The MixColumns transformation on a single column $j (0 \le j \le 3)$ of **State** can be expressed as

$$\begin{aligned} s'_{0,j} &= (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j} \\ s'_{1,j} &= s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j} \\ s'_{2,j} &= s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j}) \\ s'_{3,j} &= (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j}) \end{aligned} \quad \textbf{(5.4)}$$

The following is an example of MixColumns:

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

→

47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

Let us verify the first column of this example. Recall from Section 4.6 that, in GF(2^8), addition is the bitwise XOR operation and that multiplication can be performed according to the rule established in Equation (4.10). In particular, multiplication of a value by x (i.e., by $\{02\}$) can be implemented as a 1-bit left shift followed by a conditional bitwise XOR with (0001 1011) if the leftmost bit of the original value (prior to the shift) is 1. Thus, to verify the MixColumns transformation on the first column, we need to show that

$$\begin{array}{lllll} (\{02\} \cdot \{87\}) & \oplus (\{03\} \cdot \{6E\}) & \oplus \{46\} & \oplus \{A6\} & = \{47\} \\ \{87\} & \oplus (\{02\} \cdot \{6E\}) & \oplus (\{03\} \cdot \{46\}) & \oplus \{A6\} & = \{37\} \\ \{87\} & \oplus \{6E\} & \oplus (\{02\} \cdot \{46\}) & \oplus (\{03\} \cdot \{A6\}) & = \{94\} \\ (\{03\} \cdot \{87\}) & \oplus \{6E\} & \oplus \{46\} & \oplus (\{02\} \cdot \{A6\}) & = \{ED\} \end{array}$$

[6]We follow the convention of FIPS PUB 197 and use the symbol • to indicate multiplication over the finite field GF(2^8) and $\oplus$ to indicate bitwise XOR, which corresponds to addition in GF(2^8).