Fall 2022 CIS 3362 Homework #5: Number Theory
Check WebCourses for the due date

1) (6 pts) Without the aid of a computer program, determine the prime factorization of 1,561,032,000. Show your work. You may do division on a calculator.

2) (8 pts) What is $\varphi(1{,}561{,}032{,}000)$?

3) (8 pts) Use Fermat's Theorem to calculate the remainder when 6^{5094} is divided by 1019?

4) (8 pts) Use Euler's Theorem to calculate the remainder when 18^{57026} is divided by 24455?

5) (10 pts) Show the steps of running the Miller-Rabin algorithm, testing n = 561 for primality with the randomly chosen value of a = 2. Please use a calculator or computer program to calculate the modular exponents and just show the result of each squaring/mod operations

6) (20 pts) Trace through the Fermat Factoring algorithm to factor 161,423 as the product of two prime numbers. You may use a calculator or computer program to execute each calculation, but print out the result of each number being tested as a perfect square.

7) (40 pts) We can create a sequence from a starting integer n by repeatedly taking the previous value in the sequence and generating the next number in the sequence as $\varphi(n)$ until we generate the number 1. For example, if we start with n = 11, the sequence generated would be 11, 10, 4, 2, 1. Define a function f(n) to equal the number of terms in the sequence generated above, with the first term equal to n. Write a computer program to calculate the smallest integer n such that f(n) = 10. **(Note: This question can be solved by writing a very inefficient phi function. For a few extra credit points, write your code with an efficient phi function and find the minimal integer n such that f(n) = 20. In Python, my efficient version took 17 seconds on my laptop.)** Please attach your program separately and in your write up state the answer your program produced. In the write up, describe the algorithm you used to solve the problem.