Binary Trees

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

Outline

- Tree Stuff
  - Trees
  - Binary Trees
  - Implementation of a Binary Tree
- Tree Traversals – Depth First
  - Preorder
  - Inorder
  - Postorder
- Breadth First Tree Traversal
- Binary Search Trees

Tree Stuff

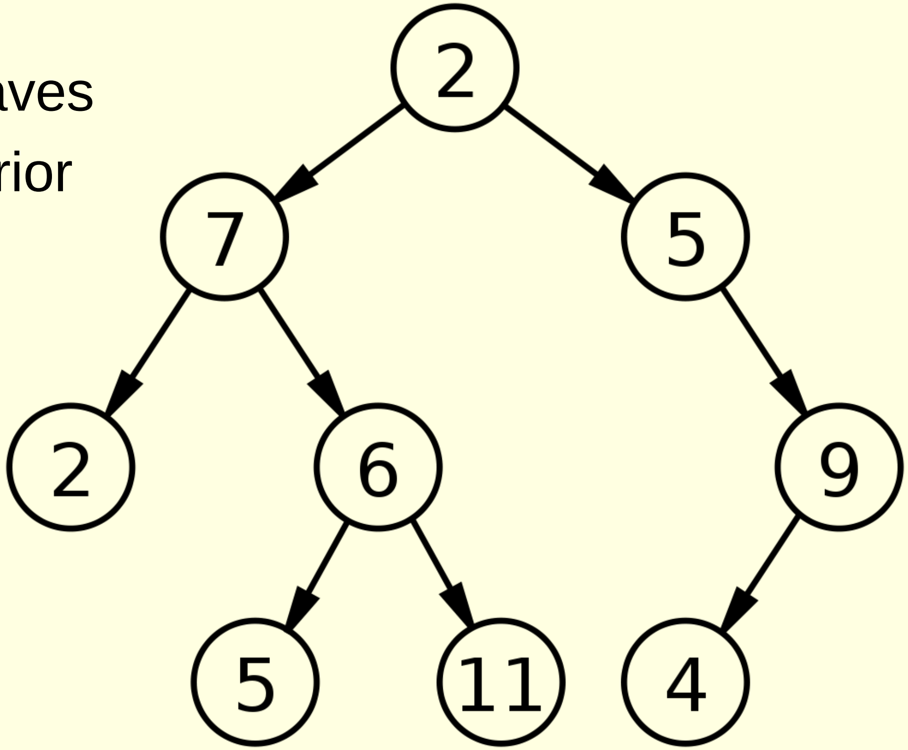
- Trees:
  - Another Abstract Data Type
  - Data structure made of nodes and pointers
    - Much like a linked list
      - The difference between the two is how they are organized.
    - A linked list represents a linear structure
      - A predecessor/successor relationship between the nodes of the list
    - A tree represents a hierarchical or ancestral relationship between the nodes
      - A node in a tree can have several successors, which we refer to as children

Tree Stuff

- Trees:
  - General Tree Information:
    - Top node in a tree is called the **root**
      - the root node has no parent above it
    - Every node in the tree can have “children” nodes
      - Each child node can, in turn, be a parent to its children and so on
    - Nodes having no children are called **leaves**
    - Any node that is not a root or a leaf is an **interior node**
    - The **height** of a tree is defined to be the length of the longest path from the root to a leaf in that tree.
      - A tree with only one node (the root) has a height of zero.

Tree Stuff

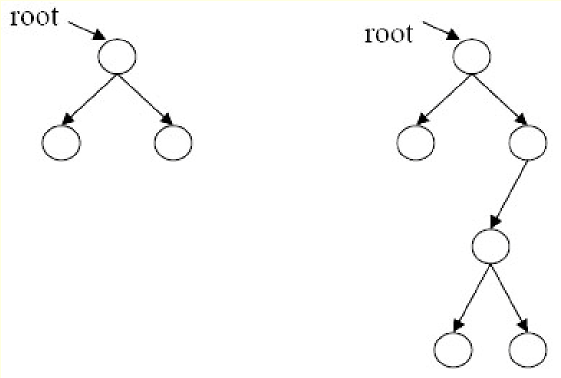
- Trees:
  - Here's a purty picture of a tree:
    - 2 is the root
    - 2, 5, 11, and 4 are leaves
    - 7, 5, 6, and 9 are interior nodes



Tree Stuff

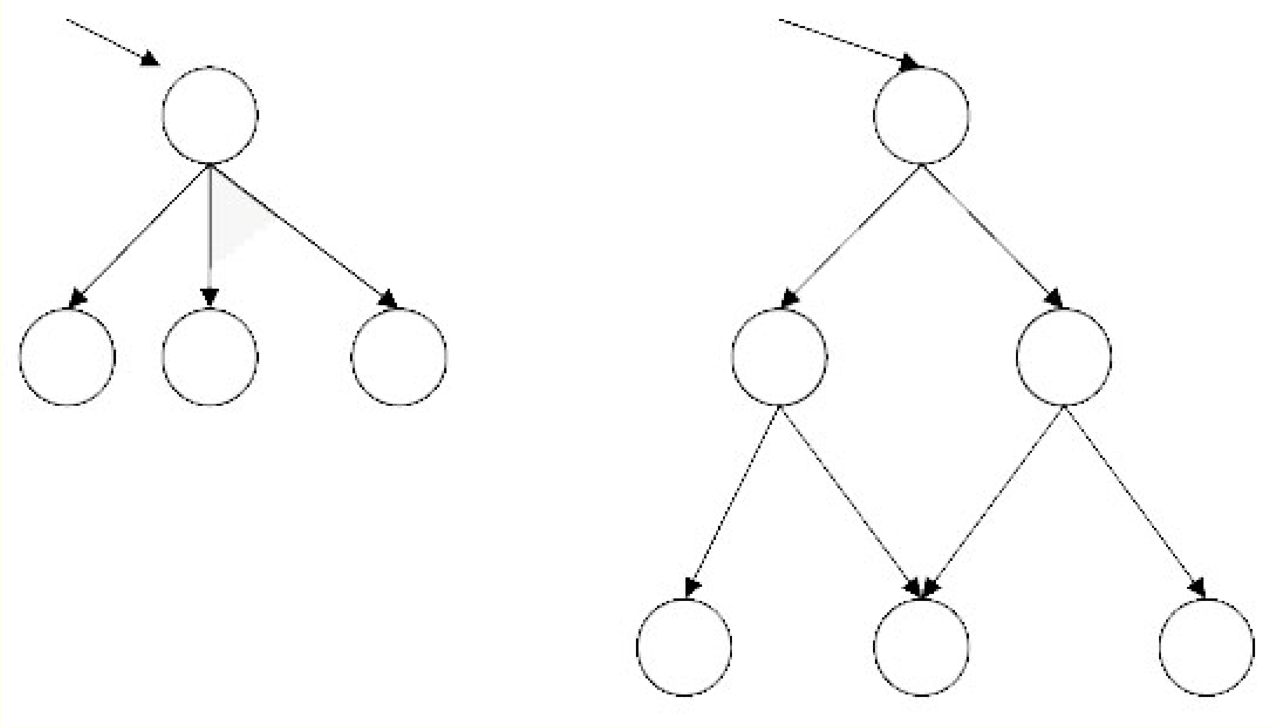
- Binary Trees:
  - A tree in which each node can have a maximum of two children
    - Each node can have no child, one child, or two children
    - And a child can only have one parent
    - Pointers help us to identify if it is a right child or a left one

Examples of two Binary Trees:



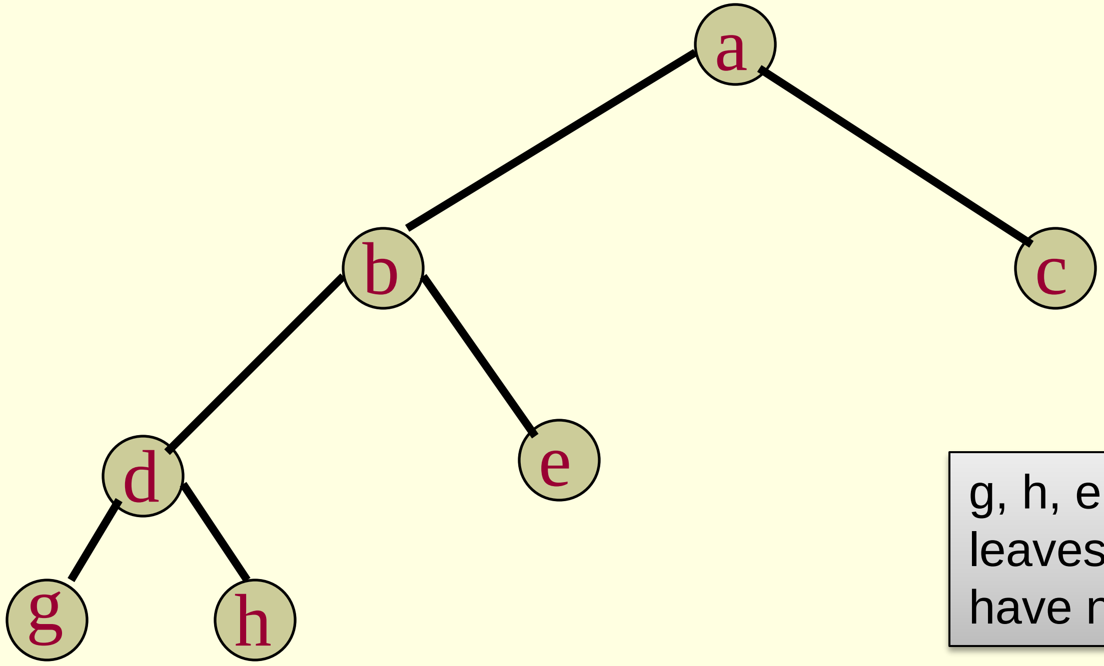
Tree Stuff

- Examples of trees that are NOT Binary Trees:



Tree Stuff

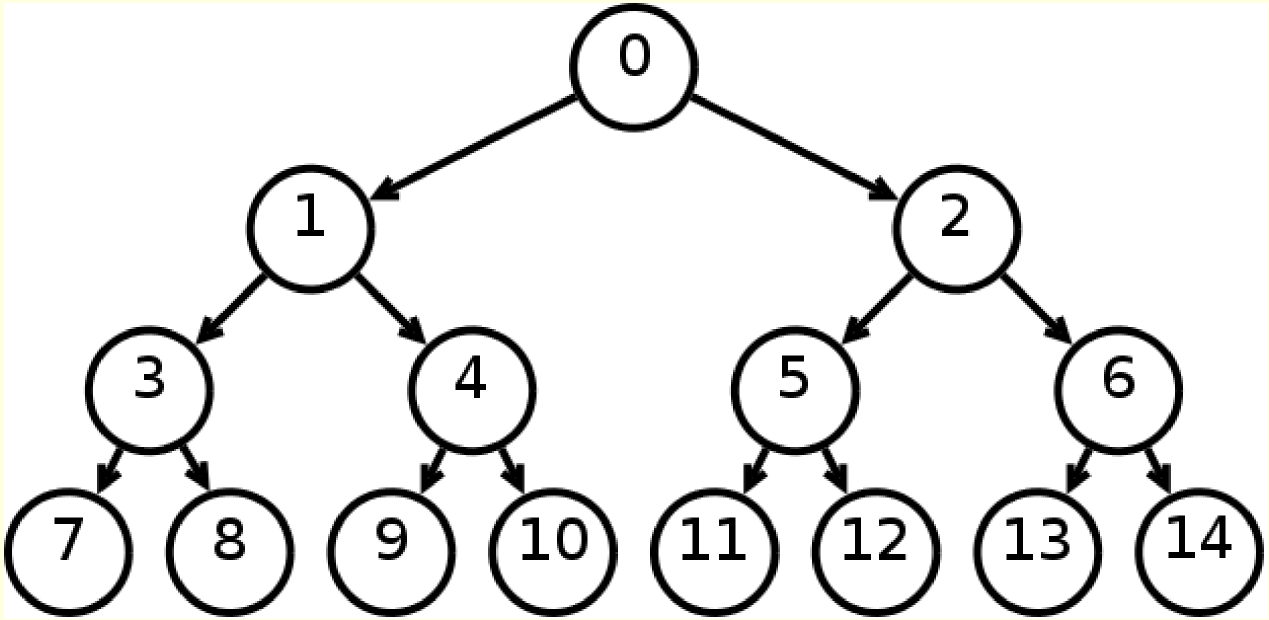
- More Binary Tree Goodies:
  - A **full** binary tree:
    - Every node, other than the leaves, has two children



g, h, e, and c are leaves: so they have no children.

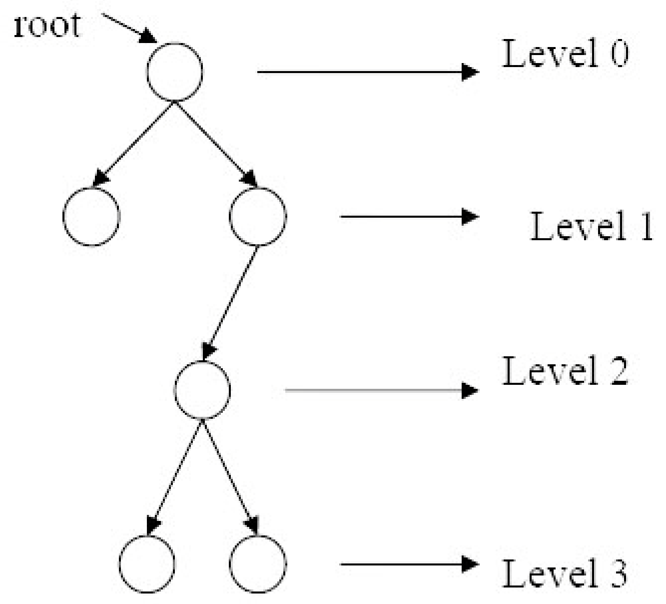
Tree Stuff

- More Binary Tree Goodies:
  - A **complete** binary tree:
    - Every level, except possibly the last, is completely filled, and all nodes are as far left as possible.



Tree Stuff

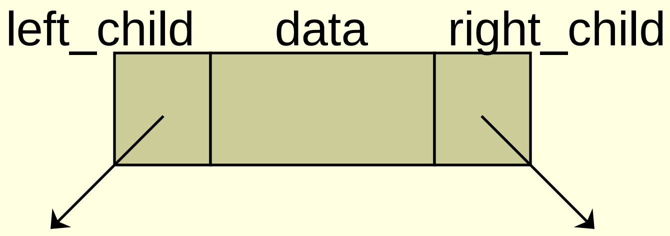
- More Binary Tree Goodies:
  - The root of the tree is at level 0
  - The level of any other node in the tree is one more than the level of its parent
  - Total # of nodes (n) in a complete binary tree:
    - $n = 2^{h+1} - 1$ (maximum)
  - Height (h) of the tree:
    - $h = \log((n + 1)/2)$
    - If we have 15 nodes
    - $h = \log(16/2) = \log(8) = 3$



Tree Stuff

- Implementation of a Binary Tree:
  - A binary tree has a natural implementation using linked storage
  - Each node of a binary tree has both left and right subtrees that can be reached with pointers:

```
struct tree_node {
    int data;
    struct tree_node *left_child;
    struct tree_node *right_child;
}
```



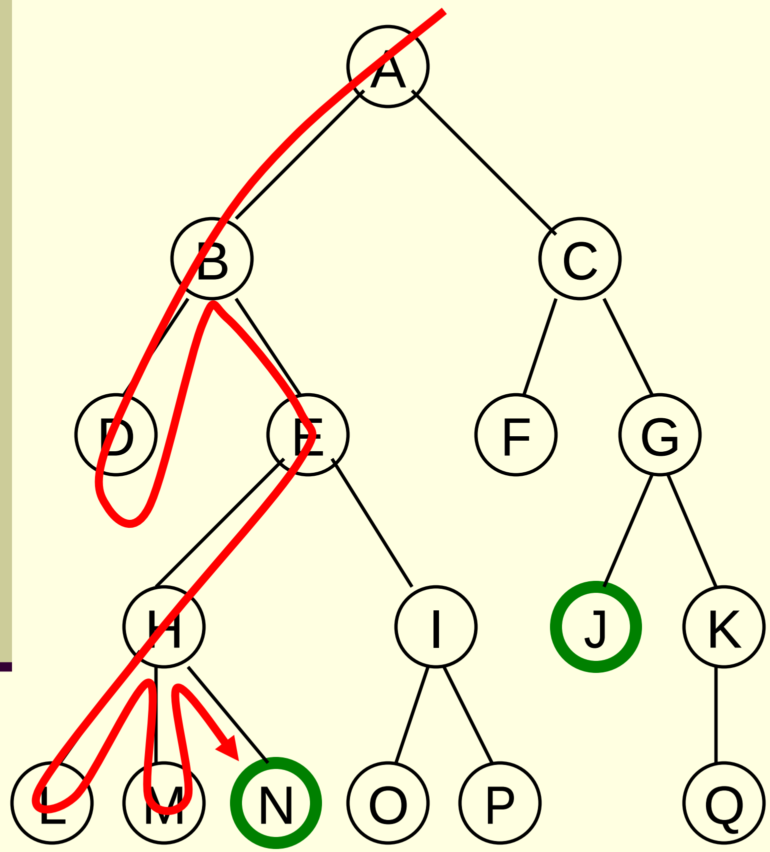
Tree Traversals – Depth First

- Traversal of Binary Trees:
  - We need a way of zipping through a tree for searching, inserting, etc.
    - But how can we do this?
    - If you remember…
      - Linked lists are traversed from the head to the last node, sequentially
      - Can't we just "do that" for binary trees.?.
        - NO! There is no such natural linear ordering for nodes of a tree.
  - Turns out, there are THREE ways/orderings of traversing a binary tree:
    - Preorder, Inorder, and Postorder

Tree Traversals – Depth First

But before we get into the nitty gritty of those three, let's describe..

Tree Traversals – Depth First



- A depth-first search (DFS) explores a path all the way to a leaf before backtracking and exploring another path
- For example, after searching A, then B, then D, the search backtracks and tries another path from B
- Node are explored in the order A B D E H L M N I O P C F G J K Q
- N will be found before J

Tree Traversals – Depth First

- Traversal of Binary Trees:
  - There are 3 ways/orderings of traversing a binary tree (all 3 are depth first search methods):
    - Preorder, Inorder, and Postorder
    - These names are chosen according to the step at which the root node is visited:
      - With **preorder** traversal, the root is visited before its left and right subtrees.
      - With **inorder** traversal, the root is visited between the subtrees.
      - With **postorder** traversal, the root is visited after both subtrees.

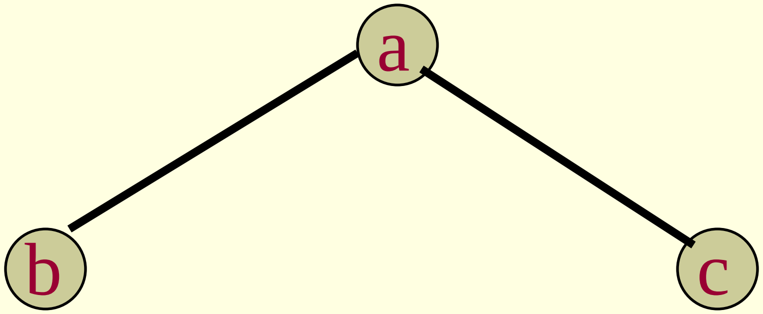
Tree Traversals - Preorder

- Preorder Traversal
  - the root is visited before its left and right subtrees
    - For the following example, we assume we are printing the nodes out
  - Code for Preorder Traversal:

```
void preorder (struct tree_node *p) {
    if (p != NULL) {
        printf("%d ", p->data);
        preorder(p->left_child);
        preorder(p->right_child);
    }
}
```

Tree Traversals - Preorder

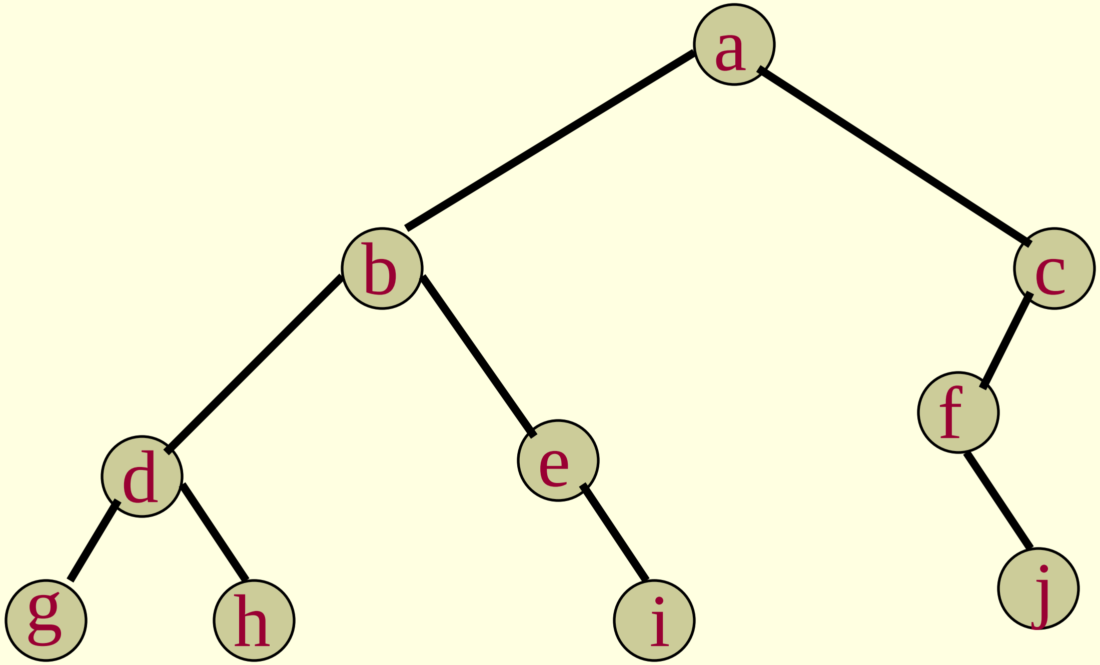
- Preorder Traversal – Example 1
  - the root is visited before its left and right subtrees



a b c

Tree Traversals - Preorder

- Preorder Traversal – Example 2



Order of Visiting Nodes: a b d g h e i c f j

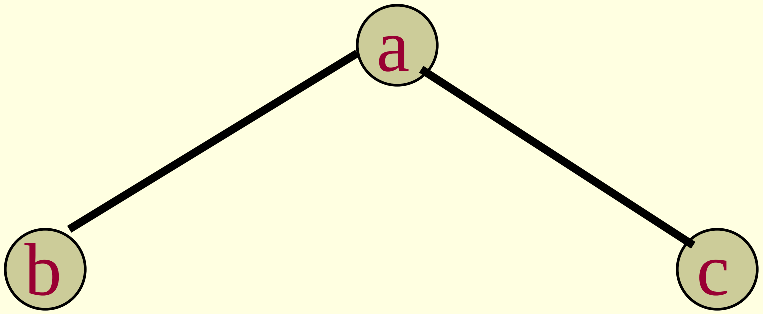
Tree Traversals - Inorder

- Inorder Traversal
  - the root is visited between the subtrees
    - For the following example, we assume we are printing the nodes out
  - Code for Inorder Traversal:

```
void inorder(struct tree_node *p) {
    if (p !=NULL) {
        inorder(p->left_child);
        printf("%d ", p->data);
        inorder(p->right_child);
    }
}
```

Tree Traversals - Inorder

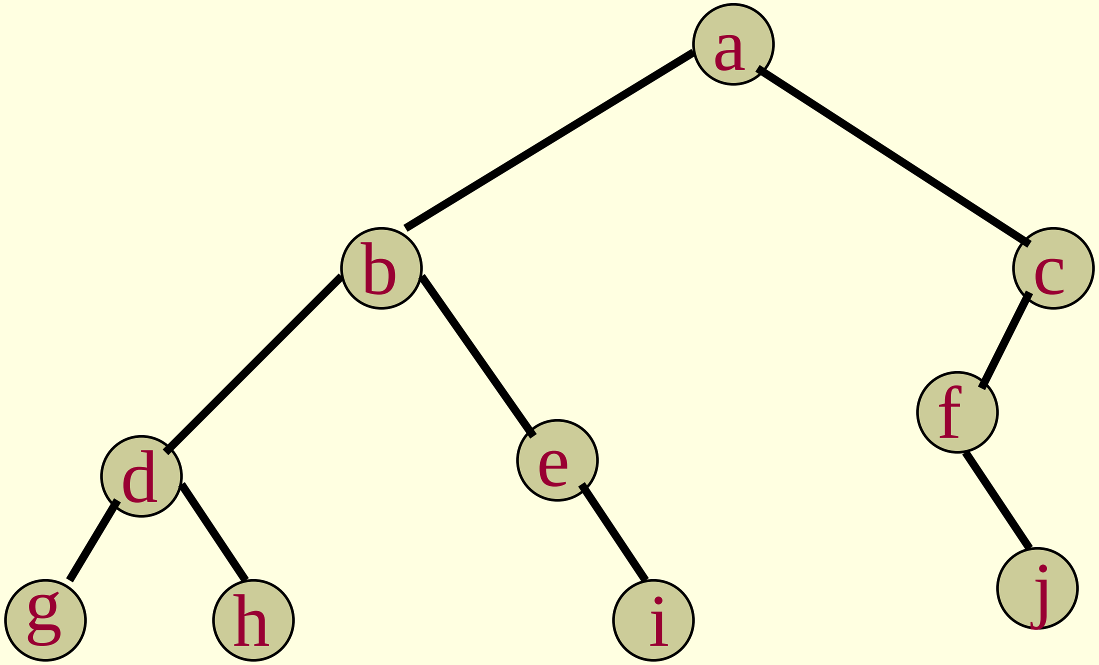
- Inorder Traversal – Example 1
  - the root is visited between the subtrees



b a c

Tree Traversals - Inorder

- Inorder Traversal – Example 2



Order of Visiting Nodes: g d h b e i a f j c

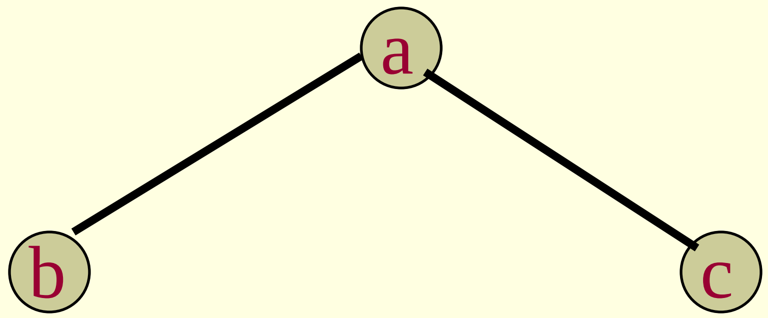
Tree Traversals – Postorder

- Postorder Traversal
  - the root is visited after both subtrees
    - For the following example, we assume we are printing the nodes out
  - Code for Postorder Traversal:

```
void postorder (struct tree_node *p) {
    if (p !=NULL) {
        postorder(p->left_child);
        postorder(p->right_child);
        printf("%d\n", p->data);
    }
}
```

Tree Traversals – Postorder

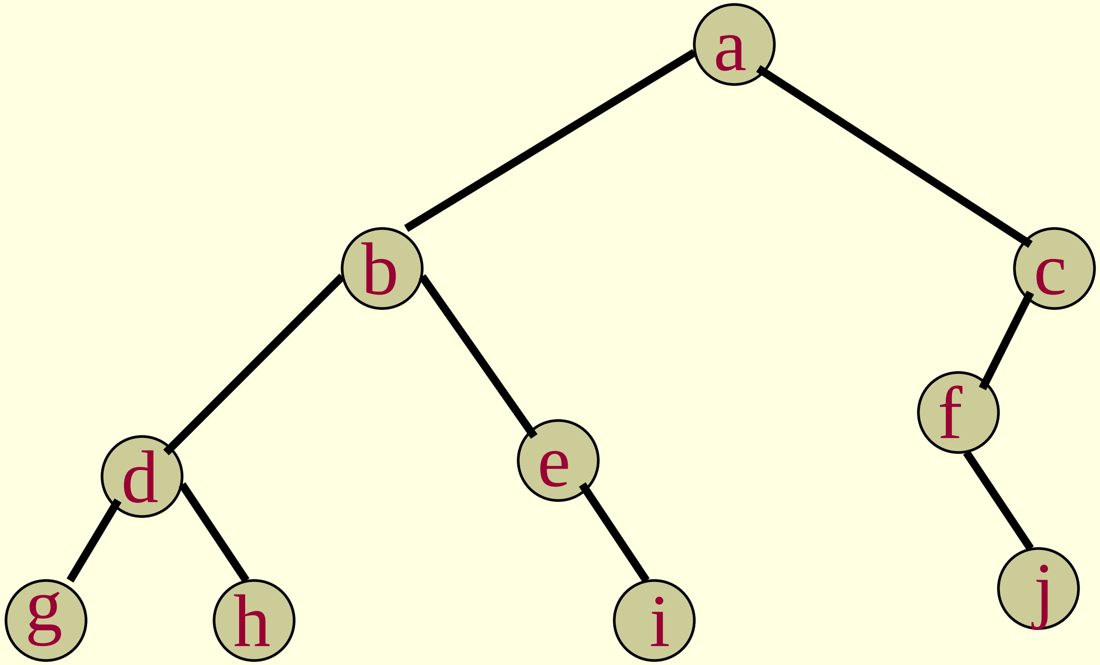
- Postorder Traversal – Example 1
  - the root is visited after both subtrees



b c a

Tree Traversals – Postorder

- Postorder Traversal – Example 2



Order of Visiting Nodes: g h d i e b j f c a

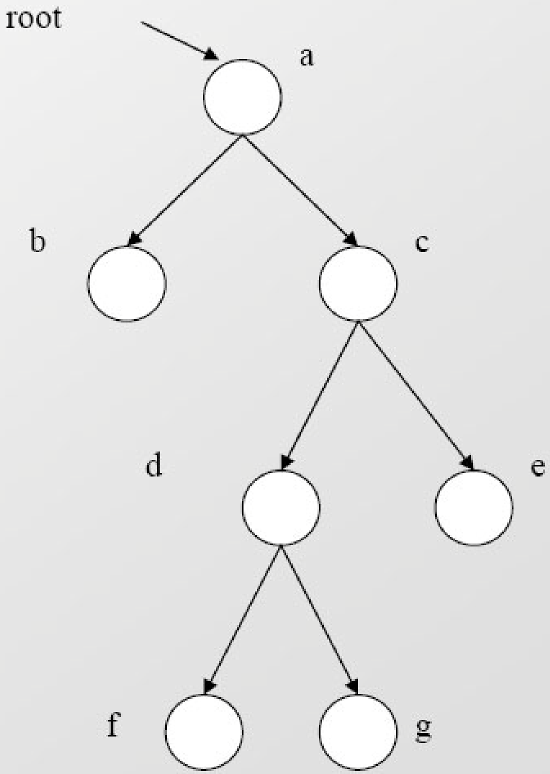
Tree Traversals

- Final Traversal Example

• <u>Preorder</u>: a b c d f g e

• <u>Inorder:</u> b a f d g c e

• <u>Postorder</u>: b f g d e c a

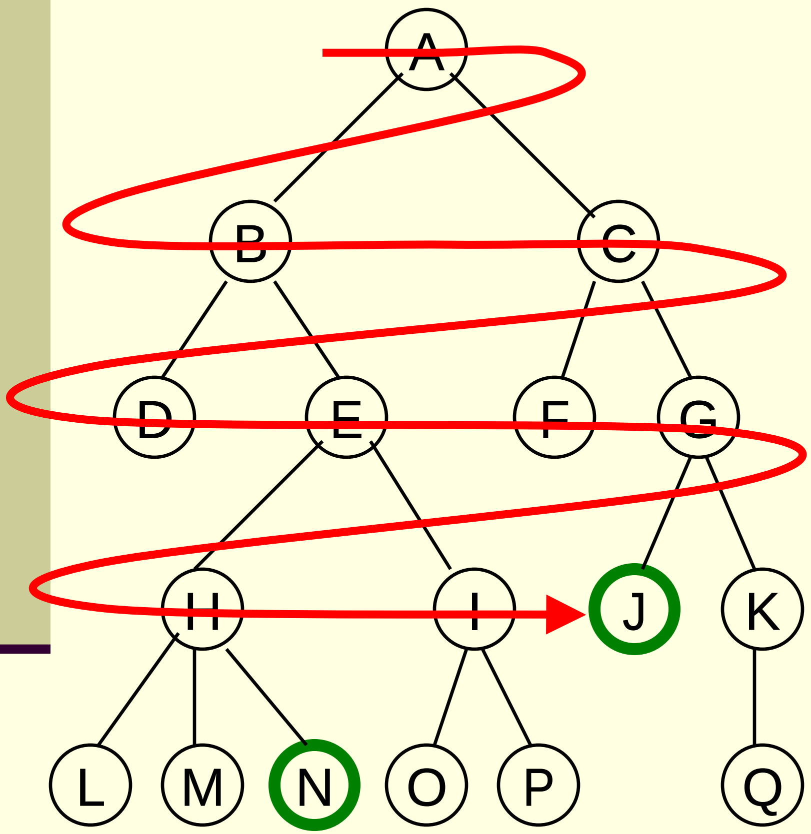


Brief Interlude: Human Stupidity

Unfortunately, this was here at UCF near the Student Union.

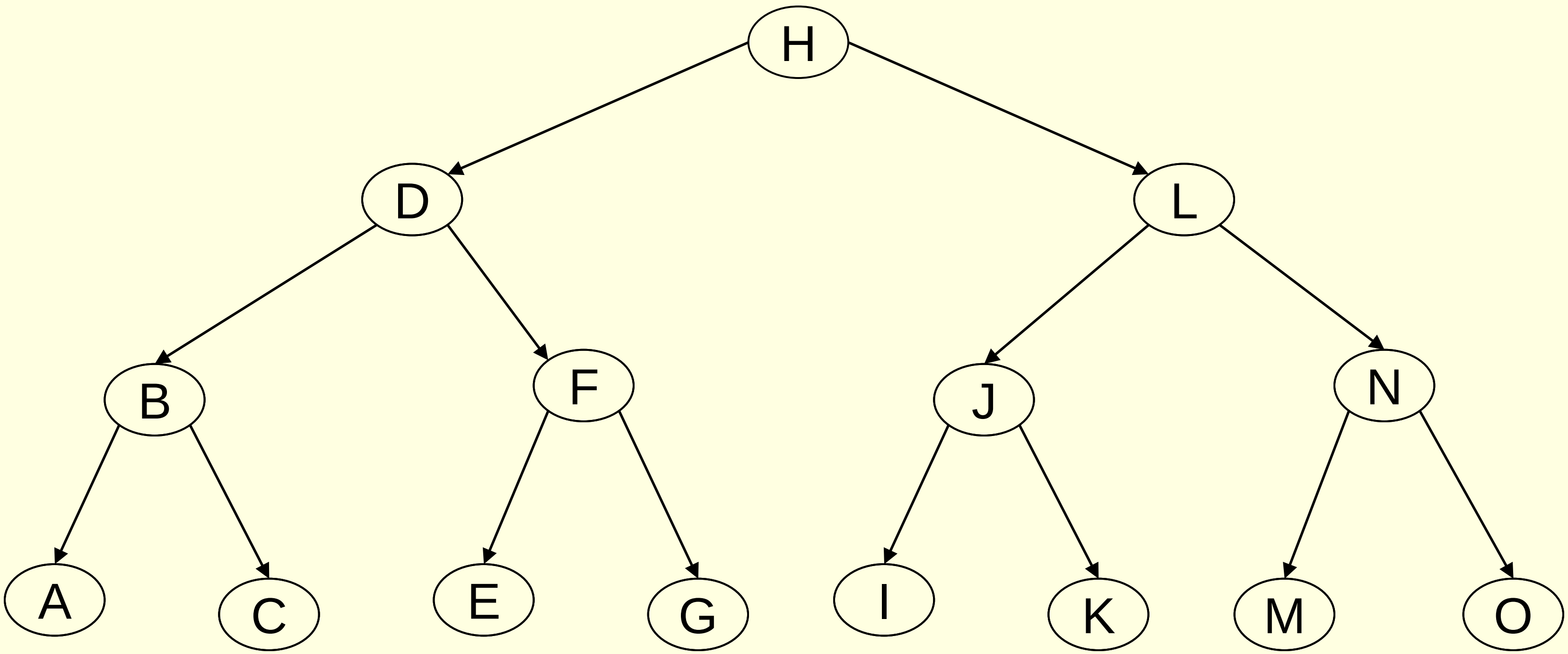
Picture courtesy of Joe Gravelle.

Breadth-First Traversal



- A breadth-first search (BFS) explores nodes nearest the root before exploring nodes further away
- For example, after searching A, then B, then C, the search proceeds with D, E, F, G
- Node are explored in the order A B C D E F G H I J K L M N O P Q
- J will be found before N

Breadth-First Traversal



H	**D**	**L**	**B**	**F**	**J**	**N**	**A**	**C**	**E**	**G**	**I**	**K**	**M**	**O**

Breadth-First Traversal

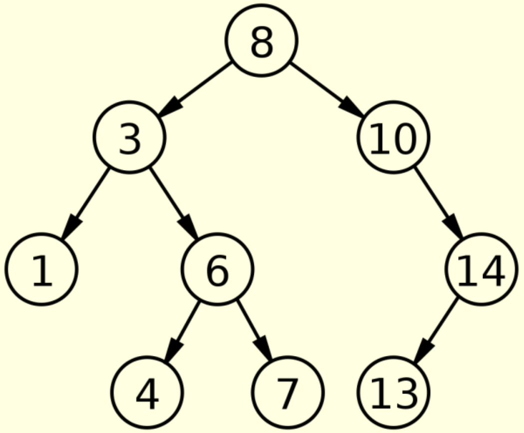
- Coding the Breadth-First Traversal
  - How would you do this?
    - Think about it, how would you make this happen?
    - SOLUTION:

1) **Enqueue** the root node.
2) **Dequeue** a node and examine it.
   - If the element sought is found in this node, quit the search and return a result.
   - Otherwise enqueue any successors (the direct child nodes) that have not yet been discovered.

- If the queue is empty, every node on the graph has been examined – quit the search and return "not found".
- Repeat from Step 2.

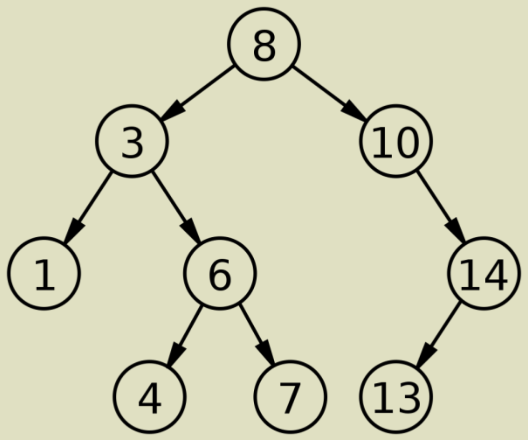
Binary Search Tree

- Binary Search Trees
  - We've seen how to traverse binary trees
  - But it is not quite clear how this data structure helps us
    - What is the purpose of binary trees?
  - What if we added a restriction…
  - Consider the following binary tree:



  - What pattern can you see?

Binary Search Tree

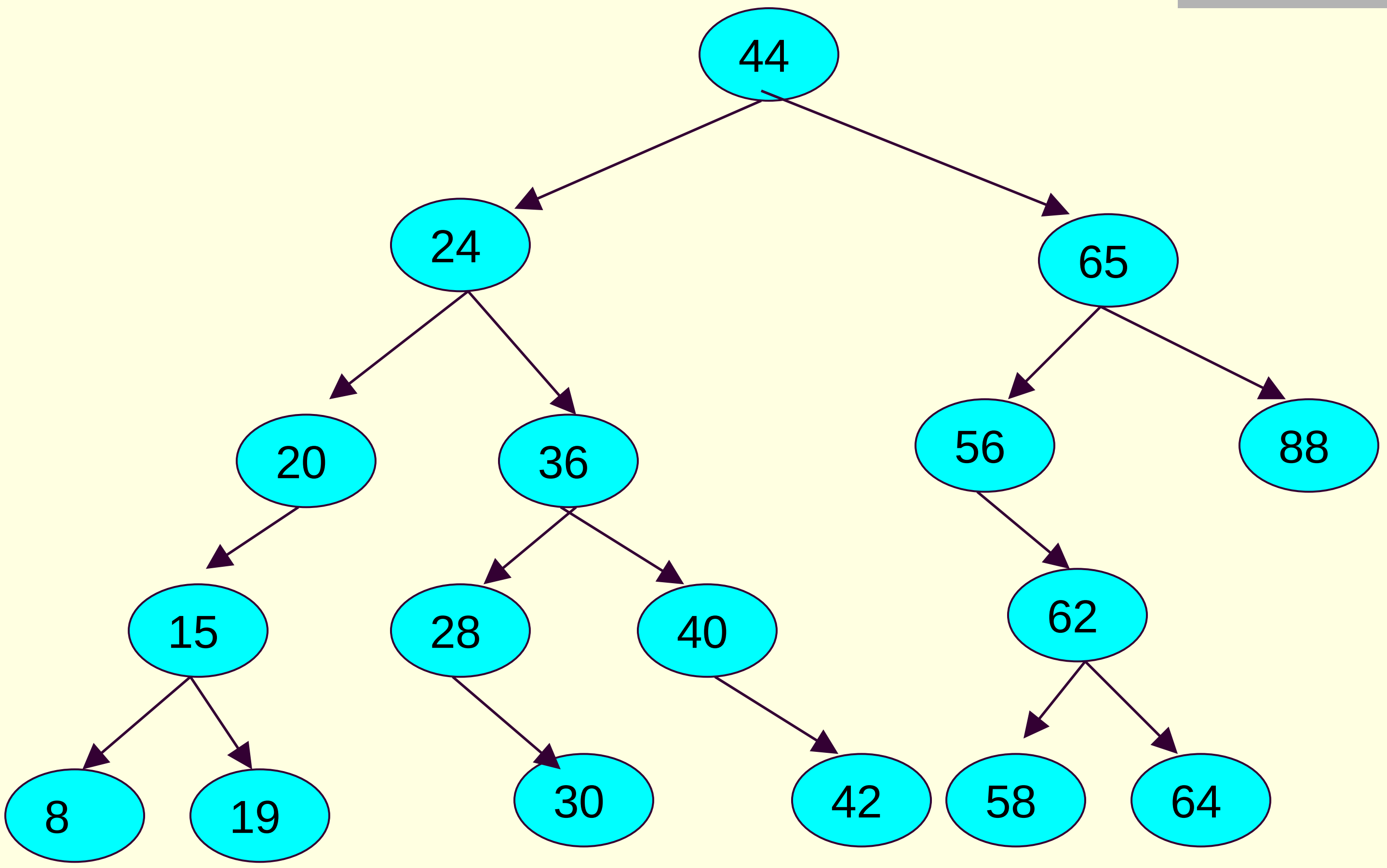


- Binary Search Trees
  - What pattern can you see?
    - For each node N, all the values stored in the left subtree of N are LESS than the value stored in N.
    - Also, all the values stored in the right subtree of N are GREATER than the value stored in N.
    - Why might this property be a desireable one?
      - **Searching for a node is super fast!**
    - Normally, if we search through n nodes, it takes O(n) time
    - But notice what is going on here:
      - This ordering property of the tree tells us where to search
      - We choose to look to the left or look to the right of a node
      - We are **HALVING** the search space …**O(log n)** time

Binary Search Tree

- Binary Search Trees
  - Details:
    - All of the data values in the left subtree of each node are smaller than the data value in the node (root of the subtree) itself.
      - Stated another way, the value of the node itself is larger than the value of every node in its left subtree.
    - All of the data values in the right subtree of each node are larger than the data value in the node (root of the subtree) itself.
      - Stated another way, the value of the node itself is smaller than the value of every node in its right subtree.
    - Both the left and right subtrees of the node are themselves binary search trees.

Binary Search Tree



A Binary Search Tree

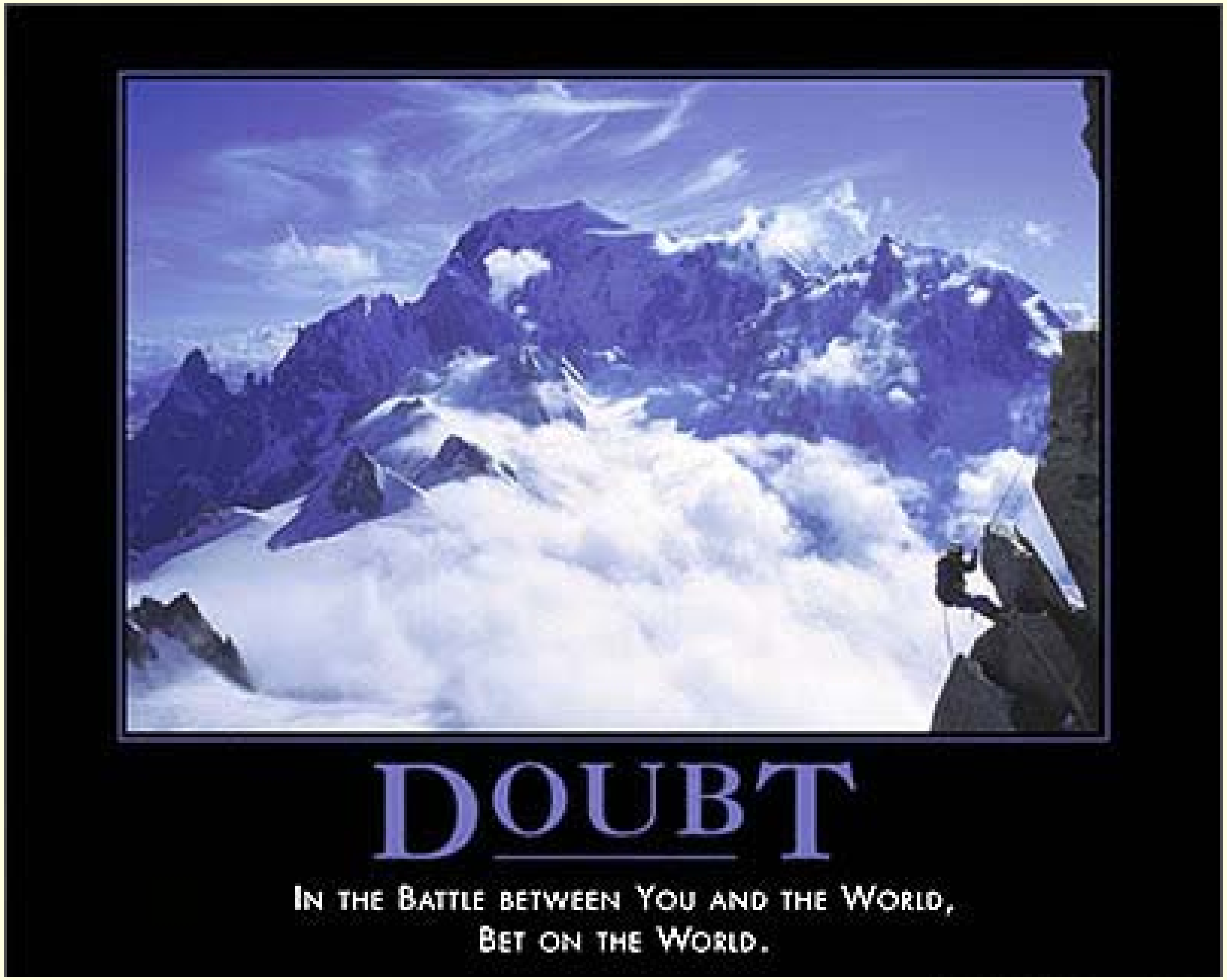
Binary Search Tree

- Binary Search Trees
  - Details:
    - A binary search tree, commonly referred to as a BST, is extremely useful for efficient searching
    - Basically, a BST amounts to embedding the binary search into the data structure itself.
    - Notice how the root of every subtree in the BST on the previous page is the root of a BST.
    - This ordering of nodes in the tree means that insertions into a BST are not placed arbitrarily
    - Rather, there is a specific way to insert
    - …and that is for next time

Binary Trees

WASN'T THAT HISTORIC!

Daily Demotivator



Binary Trees

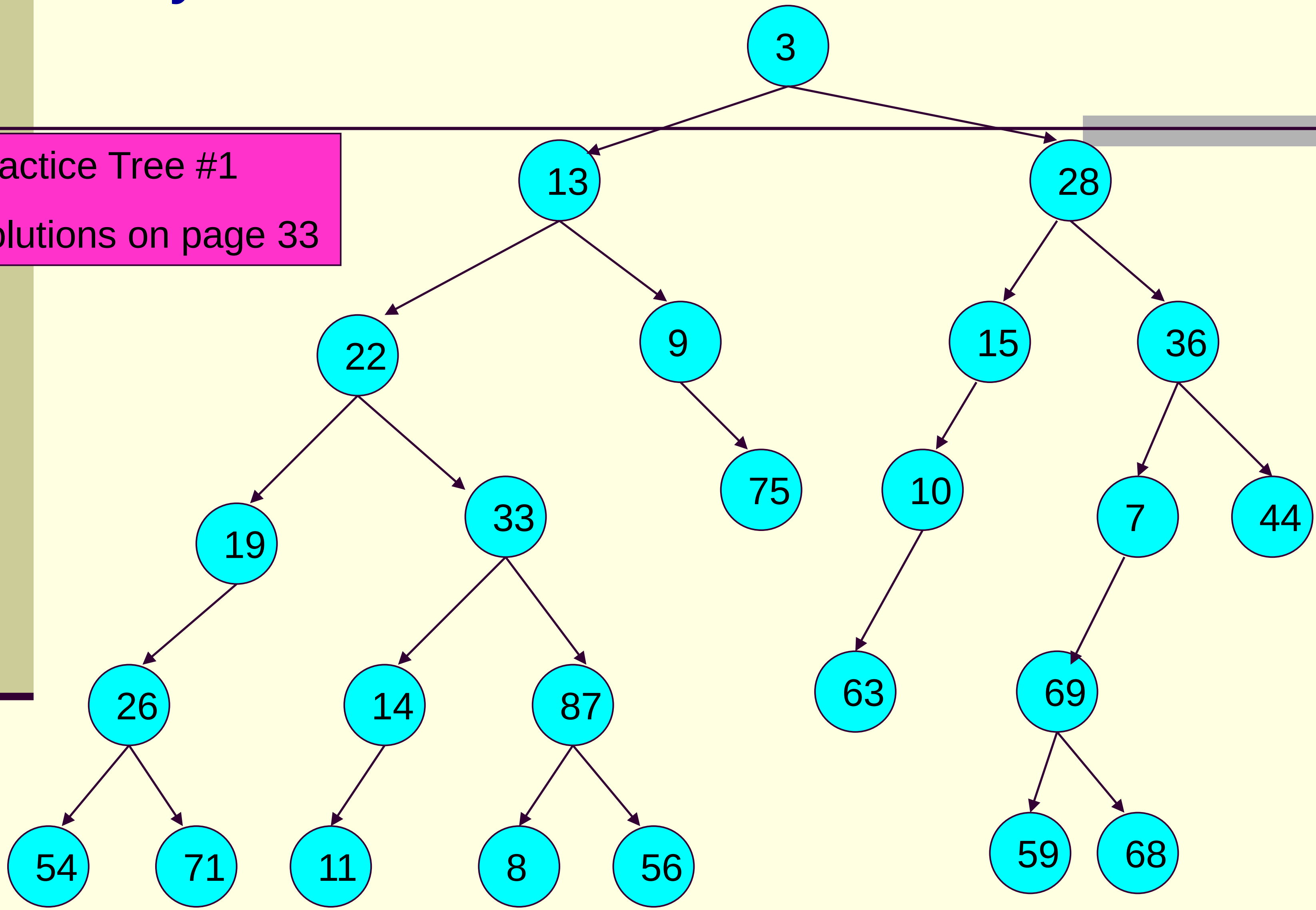
Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

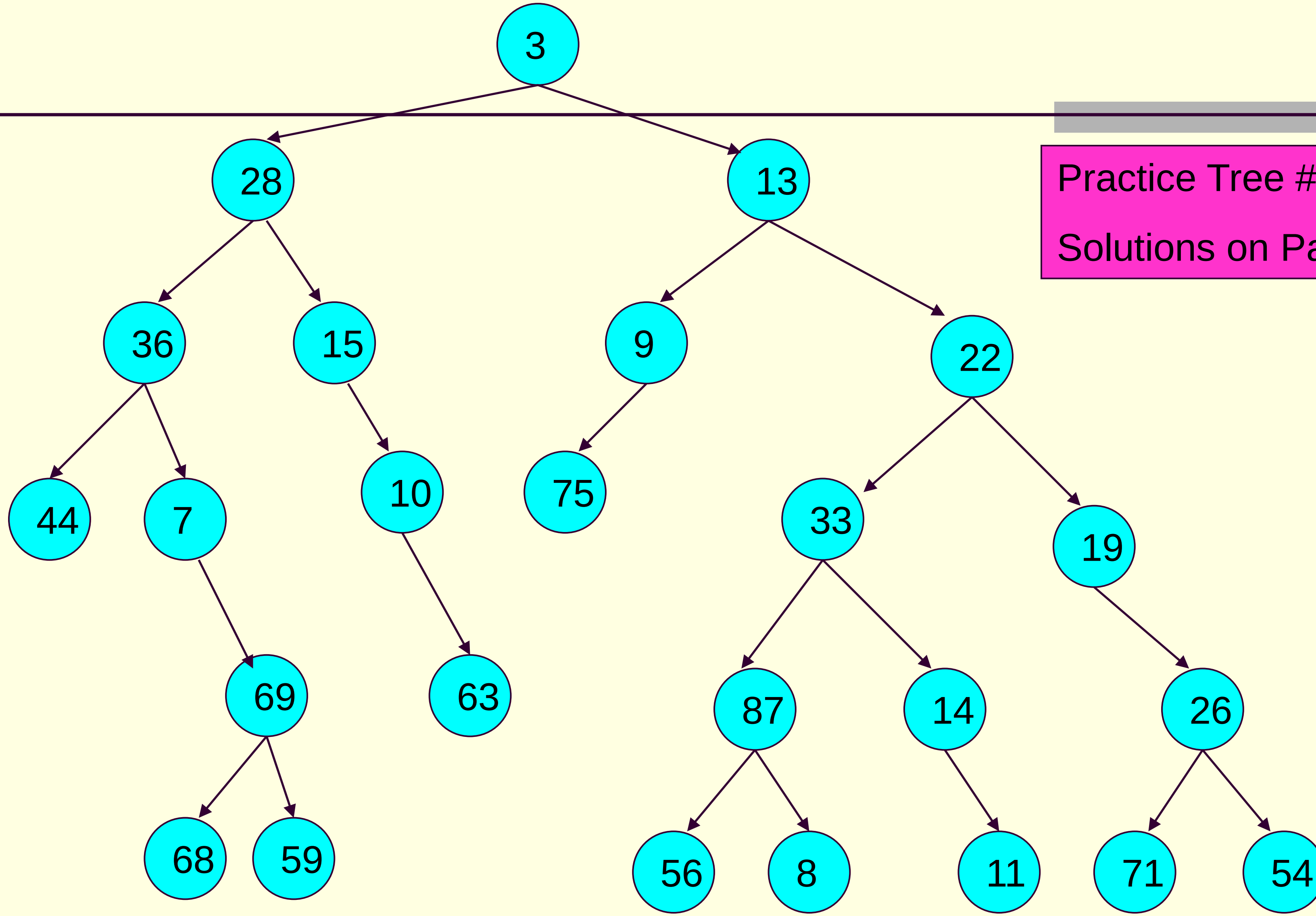
Binary Tree Traversals – Practice Problems

Practice Tree #1

Solutions on page 33



Binary Tree Traversals – Practice Problems



Practice Tree #2

Solutions on Page 34

Practice Problem Solutions – Tree #1

- Preorder Traversal:

  3, 13, 22, 19, 26, 54, 71, 33, 14, 11, 87, 8, 56, 9, 75, 28, 15, 10, 63, 36, 7, 69, 59, 68, 44

- Inorder Traversal:

  54, 26, 71, 19, 22, 11, 14, 33, 8, 87, 56, 13, 9, 75, 3, 63, 10, 15, 28, 59, 69, 68, 7, 36, 44

- Postorder Traversal:

  54, 71, 26, 19, 11, 14, 8, 56, 87, 33, 22, 75, 9, 13, 63, 10, 15, 59, 68, 69, 7, 44, 36, 28, 3

Practice Problem Solutions – Tree #2

- Preorder Traversal:

  3, 28, 36, 44, 7, 69, 68, 59, 15, 10, 63, 13, 9, 75, 22, 33, 87, 56, 8, 14, 11, 19, 26, 71, 54

- Inorder Traversal:

  44, 36, 7, 68, 69, 59, 28, 15, 10, 63, 3, 75, 9, 13, 56, 87, 8, 33, 14, 11, 22, 19, 71, 26, 54

- Postorder Traversal:

  44, 68, 59, 69, 7, 36, 63, 10, 15, 28, 75, 9, 56, 8, 87, 11, 14, 33, 71, 54, 26, 19, 22, 13, 3