COP 3502 - 10/8/21

① Quiz 2 (Avg 14 out 35)

② Sums

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + \dots + f(b)$$

```
int sum = 0;
for (int i = a; i <= b; i++)
    sum += f(i);
```

$$\underbrace{\sum_{i=a}^{b} c = (b-a+1) \cdot c}_{\text{repeated addition is multiplication}}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=a}^{b} f(i) = \sum_{i=1}^{b} f(i) - \sum_{i=1}^{a-1} f(i)$$

$\boxed{1 < a < b}$ $\quad f(1) + f(2) + f(3) + \dots + f(a-1) + \underline{f(a) + f(a+1) + \dots + f(b)}$

Subtract out what we don't want.

$$\sum_{i=a}^{b} \left(f(i) + g(i)\right) = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$

$$f(a) + g(a)$$
$$f(a+1) + g(a+1)$$
$$\vdots$$
$$f(b) + g(b)$$

$$\sum_{i=a}^{b} c \cdot f(i) = c \sum_{i=a}^{b} f(i)$$

$$cf(a) + cf(a+1) + cf(a+2) + \dots + cf(b) = c\left[f(a) + f(a+1) + \dots + f(b)\right]$$

$$\sum_{i=2n}^{3n} (4i+3) = \sum_{i=1}^{3n} (4i+3) - \sum_{i=1}^{2n-1} (4i+3)$$

$$= \sum_{i=1}^{3n} 4i + \sum_{i=1}^{3n} 3 - \left[\sum_{i=1}^{2n-1} 4i + \sum_{i=1}^{2n-1} 3\right]$$

$$= \frac{4 \cdot 3n(3n+1)}{2} + 3(3n) - \left[\frac{4(2n-1)2n}{2} + 3(2n-1)\right]$$

$$= 6n(3n+1) + 9n - 4n(2n-1) - 6n + 3$$

$$= 18n^2 + 6n + 3n + 3 - 8n^2 + 4$$

$$= \boxed{10n^2 + 9n + 7} = O(n^2)$$

$$S = 1 + \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{4}} + \cancel{\tfrac{1}{8}} + \cdots = \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i$$

$$-\ \frac{S}{2} = \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{4}} + \cancel{\tfrac{1}{8}} + \cdots$$

$$S - \frac{S}{2} = 1$$

$$\frac{S}{2} = 1 \implies \boxed{S = 2}$$

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}, \text{ for all } x,\ |x| < 1.$$

Hint: derive on your own.

What about $\sum_{i=0}^{n-1} x^i$? $(x \neq 0)$

$$S = 1 + \cancel{x} + \cancel{x^2} + \cdots + \cancel{x^{n-1}}$$

$$-\ xS = \cancel{x} + \cancel{x^2} + \cancel{x^2} \cdots + \cancel{x^{n-1}} + x^n$$

$$S - xS = 1 - x^n$$

$$S(1-x) = 1 - x^n$$

$$S = \frac{1-x^n}{1-x} = \frac{x^n - 1}{x - 1}$$

$|x|<1$

$$\left(\sum_{i=1}^{\infty} i x^{i-1}\right)$$

$$S = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

$$-\,xS = \quad x + 2x^2 + 3x^3 + \cdots$$

$$S - xS = \underline{1 + x + x^2 + x^3 + \cdots}$$

$$S(1-x) = \frac{1}{1-x}$$

$$S = \frac{1}{(1-x)^2}$$

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1. Double Summations
2. Connect Code to Sums
3. Expenential Run time?

$$\sum_{i=0}^{n} \underbrace{\sum_{j=1}^{i} j} = \sum_{i=0}^{n} \frac{i(i+1)}{2} = \frac{1}{2}\sum_{i=0}^{n}\left(i^2 + i\right)$$

$$= \frac{1}{2}\left(\sum_{i=0}^{n} i^2 + \sum_{i=0}^{n} i\right)$$

Note:
$$\sum_{i=0}^{n} 1 = (n+1)$$

$$= \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right)$$

$$= \frac{1}{2}\cdot\frac{n(n+1)}{2}\left[\frac{2n+1}{3} + \frac{3}{3}\right] = \frac{n(n+1)}{4}\cdot\frac{(2n+4)}{3} = \boxed{\frac{n(n+1)(n+2)}{6}}$$

$$\sum_{i=0}^{n} \sum_{j=0}^{i} 2^j = \sum_{i=0}^{n} (2^{i+1} - 1)$$

$$= \sum_{i=0}^{n} 2^{i+1} - \sum_{i=0}^{n} 1$$

$$= 2 \sum_{i=0}^{n} 2^i - (n+1)$$

$$= 2(2^{n+1} - 1) - n - 1$$

$$= 2^{n+2} - 2 - n - 1$$

$$= \boxed{2^{n+2} - n - 3}$$

CODE

```
int sum = 0;
for (int i = 1; i <= n; i++) {
    for (int j = 0; j <= i; j += 2) {
        sum++;
    }
}
```

only 1 stmt here

$$\sim \sum_{i=1}^{n} \left(\sum_{j=1}^{i/2} 1 \right)$$

$$= O(n^2)$$

$$= \sum_{i=1}^{n} \frac{i}{2} = \frac{n(n+1)}{4}$$

```
for (i=1; i<=n; i++) {
    for (j=0; j<n/i; j++) {
        sum++;
    }
}
```

loop runs this many times
each iteration const time.

$$\sum_{i=1}^{n} \frac{n}{i} = n \underbrace{\sum_{i=1}^{n} \frac{1}{i}}_{} \sim \boxed{n \ln n}$$

$O(n \lg n)$