

① Quiz 2 (Avg 14 out 35)

② Sums

$$\sum_{i=a}^b f(i) = f(a) + f(a+1) + \dots + f(b)$$

int sum = 0;

for (int i = a; i <= b; i++)

sum += f(i);

$$\sum_{i=a}^b c = (b-a+1) \cdot c$$

repeated addition is multiplication

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=a}^b f(i) = \sum_{i=1}^b f(i) - \sum_{i=1}^{a-1} f(i)$$

$$1 < a < b$$

$$f(1) + f(2) + f(3) + \dots + f(a-1) + f(a) + f(a+1) + \dots + f(b)$$

Subtract out what we don't want.

$$\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$$

Diagram illustrating the distributive property of summation. The left side shows a single sum of $f(i) + g(i)$ from $i=a$ to b . The right side shows the sum of $f(i)$ plus the sum of $g(i)$ from $i=a$ to b . A list of terms $f(a)+g(a)$, $f(a+1)+g(a+1)$, ..., $f(b)+g(b)$ is shown, with arrows indicating how these terms are grouped into the two separate sums on the right.

$$\sum_{i=a}^b c \cdot f(i) = c \sum_{i=a}^b f(i)$$

$$cf(a) + cf(a+1) + cf(a+2) + \dots + cf(b) = c[f(a) + f(a+1) + \dots + f(b)]$$

$$\begin{aligned} \sum_{i=2n}^{3n} (4i+3) &= \sum_{i=1}^{3n} (4i+3) - \sum_{i=1}^{2n-1} (4i+3) \\ &= \sum_{i=1}^{3n} 4i + \sum_{i=1}^{3n} 3 - \left[\sum_{i=1}^{2n-1} 4i + \sum_{i=1}^{2n-1} 3 \right] \\ &= \frac{4 \cdot 3n(3n+1)}{2} + 3(3n) - \left[\frac{4(2n-1)2n}{2} + 3(2n-1) \right] \\ &= 6n(3n+1) + 9n - 4n(2n-1) - 6n + 3 \\ &= 18n^2 + 6n + 3n + 3 - 8n^2 + 4 \\ &= \boxed{10n^2 + 9n + 7} = O(n^2) \end{aligned}$$

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i$$

$$- \frac{S}{2} = \cancel{\frac{1}{2}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{8}} + \dots$$

$$S - \frac{S}{2} = 1$$

$$\frac{S}{2} = 1 \Rightarrow \boxed{S=2}$$

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}, \text{ for all } x, |x| < 1.$$

Hint: derive on your own.

What about $\sum_{i=0}^{n-1} x^i$? ($x \neq 0$)

$$S = 1 + \cancel{x} + \cancel{x^2} + \dots + \cancel{x^{n-1}}$$

$$- xS = \cancel{x} + \cancel{x^2} + \cancel{x^3} + \dots + \cancel{x^{n-1}} + x^n$$

$$S - xS = 1$$

$$-x^n$$

$$S(1-x) = 1 - x^n$$

$$S = \frac{1 - x^n}{1 - x} = \frac{x^n - 1}{x - 1}$$

$$|x| < 1$$

$$S = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$-xS = x + 2x^2 + 3x^3 + \dots$$

$$S - xS = 1 + x + x^2 + x^3 + \dots$$

$$S(1-x) = \frac{1}{1-x}$$

$$S = \frac{1}{(1-x)^2}$$

① Double Summations

② Connect Code to Sums

③ Exponential Runtime?

$$\sum_{i=0}^n \sum_{j=1}^i j = \sum_{i=0}^n \frac{i(i+1)}{2} = \frac{1}{2} \sum_{i=0}^n (i^2 + i)$$

$$= \frac{1}{2} \left(\sum_{i=0}^n i^2 + \sum_{i=0}^n i \right)$$

Note:

$$\sum_{i=0}^n 1 = (n+1)$$

$$= \frac{1}{2} \left(\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right)$$

$$= \frac{1}{2} \cdot \frac{n(n+1)}{2} \left[\frac{2n+1}{3} + \frac{3}{3} \right] = \frac{n(n+1)}{4} \cdot \frac{(2n+4)}{3} = \boxed{\frac{n(n+1)(n+2)}{6}}$$

$$\sum_{i=0}^n \sum_{j=0}^i 2^j = \sum_{i=0}^n (2^{i+1} - 1)$$

$$= \sum_{i=0}^n 2^{i+1} - \sum_{i=0}^n 1$$

$$= 2 \sum_{i=0}^n 2^i - (n+1)$$

$$= 2(2^{n+1} - 1) - n - 1$$

$$= 2^{n+2} - 2 - n - 1$$

$$= \boxed{2^{n+2} - n - 3}$$

CODE

```
int sum = 0;
```

```
for (int i = 1; i <= n; i++) {
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```
    for (int j = 0; j <= i; j += 2) {
```

```
        sum++;
```

```
    }
```

```
}
```

$= O(n^2)$

$$\sim \sum_{i=1}^n \left(\sum_{j=1}^{i/2} 1 \right)$$

only 1 start here

$$= \sum_{i=1}^n \frac{i}{2} = \frac{n(n+1)}{4}$$

for ($i = 1; i \leq n; i++$) {

for ($j = 0; j < n/i; j++$) {

sum++;

}

}

loop runs this many times
each iteration const time.

$$\sum_{i=1}^n \frac{n}{i} = n \sum_{i=1}^n \frac{1}{i} \sim \boxed{n \ln n}$$

$O(n \lg n)$