COP3502 - 11/15/21

1. Rubber Duck Strategy
2. BASE CONVERSION

$135 = 1\times10^2 + 3\times10^1 \times 5\times10^0$

10 is the base (decimal)

Using 10 symbols to write numbers

$b \leq 10$, our symbols are $0, 1, 2, \ldots, b-1$

$10110_2 = 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$

$= 22_{10}$

In code, bases 2 to 10.

```c
int   baseBtobase10(char * num, int base){

    int res = 0;
    int len = strlen(num);
    for (int i=0; i<len; i++)
        res = base*res + (num[i]-'0');
    return res;
}
```

num | 1 | 0 | 1 | 1 | 0 | \0 |

res $\boxed{\cancel{0}}$ $\cancel{1}$ $\cancel{2}$ $\cancel{5}$ $\cancel{11}$ 22

1	1
2	10
5	101
11	1011
22	10110

$\underbrace{2(d_0\times2^k + d_1\times2^{k-1} \ldots)}$

$d_0\times2^{k+1} + d_1\times2^k + \ldots$

all shifted left by 1 spot.

if $b > 10$ we say symbols are
$0, 1, 2, \ldots 9, a, b, c, \ldots, z$. (Upto base 36)

base 16 = hexadecimal $0, .. 9, a .. f$
$10, 11, .. 15$

a	10	1010
b	11	1011
c	12	1100
d	13	1101
e	14	1110
f	15	1111

Other common bases: 2, 4, 8

BASE B ⟹ BASE 10

Def $d_{k-1}d_{k-2}d_{k-3} \cdots d_0 = d_{k-1} \times b^{k-1} + d_{k-2} \times b^{k-2} + \ldots + d_0 \times b^0$
$$= \sum_{i=0}^{k-1} d_i \times b^i$$

Other way is code I showed you which "builds" up the value from left to right.

CONVERT FROM BASE 10 ⟹ BASE B

$\underline{22_{10}}$ ⟹ base 2 (binary)
10110_2

$2 \,|\, 22$
$2 \,|\, 11$ R 0 (last digit)
$2 \,|\, 5$ R 1 (2^{nd} to last digit)
$2 \,|\, 2$ R 1 (3^{rd} to last digit)
$2 \,|\, 1$ R 0 (4^{th} to last digit)
0 R 1 (1^{st} digit)

base	
2	binary
3	ternary
8	octal
10	decimal
16	hexadecimal

Convert from base 10 to base b:

Repeatedly mod and divide by the base b to "peel off" digits from the end to the front.

```
void printInBaseB(int n, int b) {
    if (n == 0) return;
    printInBaseB(n/b, b);
    printf("%d", n%b);
}
```

base b_1 to b_2 and neither are base 10

2 strategies

① Slow: $b_1 \to 10 \to b_2$

② If b_1 and b_2 have a common base b_3 such that $b_1 = b_3^x$ and $b_2 = b_3^y$ for positive integers x and y, then do

$$b_1 \to b_3 \to b_2$$

Examples of #2

$32130323_4 \Longrightarrow$ base 8

$4 = 2^2,\ 8 = 2^3$

base 4	base 2
0	00
1	01
2	10
3	11

$$3\times 4^7 + 2\times 4^6 \cdots + 3\times 4^2 + 2\times 4^1 + 3\times 4^0$$

$$(2^1+2^0)\times 2^{14} + (2^1)\times 2^{12} \cdots + (2^1+2^0)\times 2^4 + (2^1)\times 2^2 + (2^1+2^0)\times 2^0$$

$$32130323_4 = 1110011100111011_2$$

$$= 163473_8$$

base 8	base 2
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Note: if either $b_1 = b_2^k$ or $b_2 = b_1^k$ for positive int $k > 1$, then one of the conversions isn't necessary!

① Convert 435_7 to base 10

$$= 4\times 7^2 + 3\times 7^1 + 5\times 7^0$$

$$= 4\times 49 + 21 + 5$$

$$= 196 + 26$$

$$= \boxed{222_{10}}$$

(2) 222_{10} to base 7

$7\,\overline{)222}$

$7\,\overline{)31}$ R5

$7\,\overline{)4}$ R3

0 R4

$= \boxed{435_7}$

(3) $3CE_{16}$ to base 5

$$3CE_{16} = 3\times16^2 + 12\times16^1 + 14\times16^0$$
$$= 3\times256 + 192 + 14$$
$$= 768 + 206 = \boxed{974_{10}}$$

$5\,\overline{)974}$

$5\,\overline{)194}$ R4

$5\,\overline{)38}$ R4

$5\,\overline{)7}$ R3

$5\,\overline{)1}$ R2

0 R1

$\boxed{12344_5}$

Problem: Odometer ~~a~~ 5 digit display $\boxed{0|0|0|0|0}$ but digit 6 isn't working:

00005
00007
...
00059
00070
etc.

Start at 00000

Advance Odometer 2021 times

What will the odometer read?

Odometer is counting in base 9 w/symbols 0, 1, 2, 3, 4, 5, 7, 8, 9. Convert 2021 to base 9:

$9 \,|\, 2021$
$9 \,|\, 224$ R5
$9 \,|\, 24$ R8
$9 \,|\, 2$ R6
0 R2

$2021_{10} = 2685_9$

$= \boxed{02795}$
ODOMETER

Odom	base 9
0	0
1	1
2	2
3	3
4	4
5	5
7	6
8	7
9	8

---

base 8 to base 16

$7320415_8 \Rightarrow$ Base 16

$8 \Rightarrow 2 \Rightarrow 16$

base 8	base 2
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

$7320415_8 \rightarrow 111011010000100001101_2$

$\rightarrow \boxed{1da10d_{16}}$

base16	base 2
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
a	1010
b	1011
c	1100
d	1101
e	1110
f	1111

$320142_5 \rightarrow$ base 7

$320142_5 = 3\times5^5 + 2\times5^4 + 0\times5^3 + 1\times5^2 + 4\times5^1 + 2\times5^0$
$= 3\times3125 + 1250 + 25 + 20 + 2$
$= 9375 +$
$+1297$
10672 base 10

$7 \,\underline{)\, 10672}$

$7 \,\underline{)\, 1524}$ R4

$7 \,\underline{)\, 217}$ R5

$7 \,\underline{)\, 31}$ R0

$7 \,\underline{)\, 4}$ R3

0 R4

$$\boxed{\begin{array}{l} 320142_5 \\ = 43054_7 \end{array}}$$