9/28/23

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(1) LAs - NOT to help you debug 2nd Programming Assignment

MATH!!!

Summations - adding a bunch of numbers

$$1+2+3+4+5+6 = \sum_{i=1}^{6} i$$

$$13+17+21+25 = \sum_{i=0}^{3} 13+4i$$

$$6+12+24+48 = \sum_{i=0}^{3} 6\cdot 2^{i}$$

$a \le b$
int

$$\sum_{i=a}^{b} f(i) = f(a)+f(a+1)+f(a+2)+\dots+f(b)$$

```
int sum = 0;
for (int i = a; i <= b; i++)
    sum += f(i);
```

$$\sum_{i=a}^{b} c = c(b-a+1)$$

$$\underbrace{c + c + c + c \cdots + c}_{a \quad a+1 \quad a+2 \quad \cdots \quad b}$$

there are $b-a+1$ copies of c.

$$S = \sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n$$

$$S = \sum_{i=1}^{n} (n+1-i) = n + (n-1) + (n-2) + \cdots + 1$$

$$2S = \sum_{i=1}^{n} \left(i + (n+1-i)\right) = \underbrace{(n+1) + (n+1) + (n+1)}$$

$$2S = \sum_{i=1}^{n} (n+1) = (n+1)n$$

$$\boxed{S = \frac{n(n+1)}{2}}$$

$$\sum_{i=1}^{n^2} i = \frac{n^2(n^2+1)}{2}$$

$$\sum_{i=a}^{b} c\cdot f(i) = c\cdot f(a) + c\cdot f(a+1) + c\cdot f(a+2) + \dots + c f(b)$$

$$= c\sum_{i=a}^{b} f(i) \quad , \quad \sum_{i=1}^{3n+7} 5i = 5\sum_{i=1}^{3n+7} i = \frac{5(3n+7)(3n+8)}{2}$$

$$\sum_{i=a}^{b}(f(i)+g(i)) = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$

Commutative Property of Addition

$$\sum_{i=1}^{2n+3}(2i+7) = 2\sum_{i=1}^{2n+3} i + \sum_{i=1}^{2n+3} 7 = \frac{\cancel{2}(2n+3)(2n+4)}{\cancel{2}} + 7(2n+3)$$

$$= (2n+3)(2n+4+7)$$

$$= \boxed{(2n+3)(2n+11)}$$

$$\sum_{i=a}^{b} f(i)*g(i) \neq \left(\sum_{i=a}^{b} f(i)\right) * \left(\sum_{i=a}^{b} g(i)\right)$$

$a=1, b=2$

$$f(1)g(1)+f(2)g(2) \neq (f(1)+f(2))(g(1)+g(2))$$

$$f(1)g(1)+f(1)g(2)+f(2)g(1)+f(2)g(2)$$

DO NOT DO THIS!

$$\sum_{i=a}^{b} f(i) = \sum_{i=1}^{b} f(i) - \sum_{i=1}^{a-1} f(i)$$

$1 < a < b$

$$\cancel{f(1)} + \cancel{f(2)} + \cancel{f(3)} + \cdots + \cancel{f(a-1)} + f(a) + f(a+1) \cdots f(b)$$

$$-\left[\cancel{f(1)} + \cancel{f(2)} + \cancel{f(3)} \cdots + \cancel{f(a-1)}\right]$$

$$f(a) + f(a+1) + \cdots + f(b)$$

$$\sum_{i=n}^{2n} (3i+5) = \sum_{i=1}^{2n} (3i+5) - \sum_{i=1}^{n-1} (3i+5)$$

$$= 3\sum_{i=1}^{2n} i + 5\sum_{i=1}^{2n} 1 - \left[\sum_{i=1}^{n-1} 3i + 5\sum_{i=1}^{n-1} 1\right]$$

$$= \frac{3(2n)(2n+1)}{2} + 5(2n) - \frac{3(n-1)n}{2} - 5(n-1)$$

$$= \frac{3n\left[2(2n+1) - (n-1)\right]}{2} + 10n - 5n + 5$$

$$= \frac{3n\left[4n + 2 - n + 1\right]}{2} + 5n + 5$$

$$= \frac{3n(3n+3)}{2} + 5n + 5$$

$$= \frac{9n^2}{2} + \frac{9n}{2} + 5n + 5$$

$$= \frac{9n^2}{2} + \frac{19n}{2} + \frac{10}{2} = \frac{9n^2+19n+10}{2}$$

Alternate Method $\sum_{i=n}^{2n} (3i+5)$ this is an arithmetic sequence

$$S = \left(\frac{a_1+a_n}{2}\right) \times n$$

$n+1$ terms

$a_1 = 3n+5$

$a_{n+1} = 6n+5$

$$S = \frac{((3n+5)+(6n+5))}{2} \times (n+1)$$

$$= \frac{(9n+10)}{2} \times (n+1)$$

$$= \frac{9n^2+19n+10}{2}$$ ✓

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$S = 1 + 2 + 4 + 8 + 16 + \cdots + 2^{n-1}$$

$$\begin{aligned} S &= a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \cdots \cancel{a_1 r^{n-1}} \\ -rS &= \quad\;\; \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \quad \cancel{a_1 r^{n-1}} + a_1 r^n \end{aligned}$$

$$S - rS = a_1 \qquad\qquad - a_1 r^n$$

$$S(1-r) = a_1(1-r^n)$$

$$S = \frac{a_1(1-r^n)}{1-r}$$

Consider when $|r| < 1$ and the sequence never ends.

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$$\begin{aligned} S &= a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{\cdots \to} \\ -rS &= \quad\;\; \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{\cdots \to} \end{aligned}$$

$$S - rS = a_1$$

$$S = \frac{a_1}{1-r}$$, sum of infinite geo seq

$$S = n + \frac{3}{4}n + \frac{9}{16}n + \frac{3^3}{4^3}n + \cdots$$

$$S = \frac{a_1}{1-r} = \frac{n}{1-\frac{3}{4}} = \frac{n}{\frac{1}{4}} = 4n$$

$$\sum_{i=0}^{\infty} n \cdot \left(\frac{3}{4}\right)^i = 4n \qquad \sum_{i=1}^{n} i \cdot \left(\frac{1}{2}\right)^i$$

$$S = \boxed{1 \times \frac{1}{2}} + \boxed{2 \times \frac{1}{4}} + \boxed{3 \times \frac{1}{8}} + 4 \times \frac{1}{16} + \cdots + \boxed{n \cdot \frac{1}{2^n}}$$

$$-\frac{1}{2}S = 1 \times \frac{1}{4} + 2 \times \frac{1}{8} + 3 \times \frac{1}{16} + \cdots (n-1)\frac{1}{2^n} + n \cdot \frac{1}{2^{n+1}}$$

$$S - \frac{1}{2}S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots + \frac{1}{2^n} - \frac{n}{2^{n+1}}$$

$$S = \boxed{1 + \frac{1}{2} + \frac{1}{4} + \cdots \frac{1}{2^{n-1}}} - \frac{n}{2^n}$$

$$S = \left(2 - \frac{1}{2^{n-1}}\right) - \frac{n}{2^n}$$

$$= \boxed{2 - \frac{(n+2)}{2^n}}$$

Recurrence Relations

hanoi (int n) {

$\boxed{1. \text{hanoi}(n-1)}$

$\boxed{2. \text{move bottom}}$

$\boxed{3. \text{hanoi}(n-1)}$

}

let $T(n)$ = run time
\# moves

towers of hanoi on
n disks

$$T(n) = \overbrace{T(n-1)}^{①} + \overbrace{1}^{②} + \overbrace{T(n-1)}^{③}$$

Recurrence Relation $\left\{ T(n) = 2T(n-1) + 1 \right.$, $\left\{ \begin{array}{l} T(1) = 1 \\ T(0) = 0 \end{array} \right\}$

Iteration Technique

$$T(n) = \boxed{2T(n-1)+1}$$
$$= 2\left[2T(n-2)+1\right]+1$$
$$= 4T(n-2)+2+1$$
$$= \boxed{4T(n-2)+3}$$
$$= 4\left[2T(n-3)+1\right]+3$$
$$= 8T(n-3)+4+3$$
$$3^{rd} \text{ iter} = \boxed{8T(n-3)+7}$$

After k^{th} iterations we guess

$$T(n) = 2^k T(n-k) + 2^k - 1$$

$$T(\boxed{n-1}) = 2T(\boxed{n-2}) + 1$$

n-1-1

$$T(n-2) = 2T(n-3)+1$$

$T(1) = 1$
let $n-k = 1$
so $k = n-1$

$$T(n) = 2^{n-1}T(1) + 2^{n-1} - 1$$
$$= \underbrace{2^{n-1} + 2^{n-1}}-1$$
$$= 2^1 \times 2^{n-1} - 1$$
$$= \boxed{2^n - 1}$$

Binary Search

$$T(n) = T\left(\frac{n}{2}\right) + \overset{\text{if stmt}}{1}$$ rec call $\quad T(1) = 1$

$$= T\left(\frac{n}{4}\right) + 1 + 1$$

$$= T\left(\frac{n}{4}\right) + 2$$

$$= T\left(\frac{n}{8}\right) + 1 + 2$$

$$= T\left(\frac{n}{8}\right) + 3$$

After k iterations, we have

$$= T\left(\frac{n}{2^k}\right) + k$$

let $\frac{n}{2^k} = 1 \Rightarrow 2^k = n$ and $k = \log_2 n$

$$= T(1) + \log_2 n$$

$$= 1 + \log_2 n$$

$$= O(\lg n)$$

expected run time of binary search on an array of size $n=2^k-1$

3	2	3	1	3	2	3
2	5	6	8	12	13	19

assume that we're searching for a value in the array, and each value is equally likely.

Prob	# steps
$\frac{1}{n}$	1
$\frac{2}{n}$	2
$\frac{4}{n}$	3
$\frac{8}{n}$	4
$\vdots$	
$\frac{2^{k-1}}{n}$	k

$$S = \frac{1}{n}\times 1 + 2\times\frac{2}{n} + 3\times\frac{4}{n} + 4\times\frac{8}{n} +$$

$$\text{new}S = 1\times\underline{1} + 2\times 2 + 3\times 4 + 4\times 8 + \cdots + k\times 2^{k-1}$$

$$-2(\text{new}S) = 1\times 2 + 2\times 4 + 3\times 8 + \cdots + (k-1)\times 2^{k-1} + k\cdot 2^k$$

$$-\text{new}S = 1 + 2 + 4 + 8 + \cdots + 2^{k-1} - k\cdot 2^k$$

$$\text{new}S = k\cdot 2^k - (1+2+4+\cdots+2^{k-1})$$

$$= k\cdot 2^k - (2^k-1) = k\cdot 2^k - 2^k + 1 = \boxed{(k-1)2^k+1}$$

Exp Value $= \dfrac{(k-1)2^k + 1}{2^k - 1} \sim (k-1)$

$\sim \boxed{\log_2 n - 1}$