COP 3502 9/29/23

⓪ LAs - they can't help w/assignments

① Bug/Typo P3 - fixed (10)

② Sums + Recurrences

$3+4+5+6+7 = 25 = \sum_{i=3}^{7} i$

$10+13+16+19 = 58 = \sum_{i=0}^{3} 10+3i$

$16+8+4+2+1+\frac{1}{2} = 31\frac{1}{2} = \sum_{i=0}^{5} 16(\frac{1}{2})^i$

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + \dots + f(b)$$

$a \le b$

$a \in \mathbb{Z}$

$b \in \mathbb{Z}$

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int sum=0;
for(int i=a; i<=b; i++)
    sum+= f(i);
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$$\sum_{i=a}^{b} c = \underset{a}{c} + \underset{a+1}{c} + \underset{a+2}{c} + \dots \underset{b}{c} = c(b-a+1)$$

$$\sum_{i=n}^{2n} (3n-2) = (3n-2)(2n-n+1)$$

$$= (3n-2)(n+1)$$

$$S = \sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n$$

$$S = \sum_{i=1}^{n} (n+1-i) = n + (n-1) + (n-2) \cdots + 1$$

$+$

$$2S = \sum_{i=1}^{n} (i+n+1-i) = (n+1) + (n+1) + (n+1) \quad + (n+1)$$

$$2S = \sum_{i=1}^{n} (n+1) = (n+1)n$$

$$\boxed{S = \frac{n(n+1)}{2}}$$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$$\sum_{i=1}^{2n^2} i = \frac{\cancel{2}n^2(2n^2+1)}{\cancel{2}} = n^2(2n^2+1) = 2n^4 + n^2$$

$$\sum_{i=a}^{b} c \cdot f(i) = c \sum_{i=a}^{b} f(i)$$

$$c \cdot f(a) + c \cdot f(a+1) + c \cdot f(a+2) \cdots + c \cdot f(b)$$

$$c\left[f(a) + f(a+1) + f(a+2) \cdots + f(b) \right]$$

$$\sum_{i=a}^{b} f(i) = \overbrace{\sum_{i=1}^{b} f(i)}^{S} - \overbrace{\sum_{i=1}^{a-1} f(i)}^{T}$$

$1 \le a < b$

$$\begin{array}{rl} S & f(1) + f(2) + \cdots + f(a-1) + f(a) + \cdots f(b) \\ -T & f(1) + f(2) + \cdots + f(a-1) \\ \hline & f(a) + f(a+1) \cdots f(b) \end{array}$$

$$\sum_{i=\square}^{\triangle} \bigcirc = \sum_{i=1}^{\triangle} \bigcirc - \sum_{i=1}^{\square - 1} \bigcirc$$

$$\sum_{i=a}^{b}(f(i)+g(i)) = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$

$$f(a)+g(a)+f(a+1)+g(a+1)+\cdots+f(b)+g(b)$$

$$[f(a)+f(a+1)+\cdots+f(b)]+[g(a)+g(a+1)+\cdots g(b)]$$

BUT $\displaystyle\sum_{i=a}^{b} f(i) * g(i) \neq \left(\sum_{i=a}^{b} f(i)\right)\left(\sum_{i=a}^{b} g(i)\right)$

$a=1, b=2$ $\quad f(1)g(1)+f(2)g(2)$

$$(f(1)+f(2))(g(1)+g(2))$$

$$\underline{f(1)g(1)}+f(1)g(2)+f(2)g(1)+\underline{f(2)g(2)}$$

$$\sum_{i=n+1}^{2n}(3i+7) = \sum_{i=1}^{2n}(3i+7) - \sum_{i=1}^{n}(3i+7)$$

$$= \sum_{i=1}^{2n} 3i + \sum_{i=1}^{2n} 7 - \left[\sum_{i=1}^{n} 3i + \sum_{i=1}^{n} 7\right]$$

$$= \boxed{\frac{3\cdot 2n(2n+1)}{2}} + 7(2n) \boxed{-\frac{3n(n+1)}{2}} - 7n$$

$$= \frac{3n(4n+2-(n+1))}{2} + \frac{14n}{2}$$

$$= \frac{3n(3n+1)+14n}{2} = \boxed{\frac{9n^2+17n}{2}}$$

Arithmetic Sequence

$3, 5, 7, 9, 11$

$a_1 = 3$, $d = 2$

$$S = \frac{(a_1 + a_n)}{2} \times n$$

$$\sum_{i=n+1}^{2n} (3i+7)$$

a.s.

$a_1 = 3(n+1)+7 = 3n+10$

$a_n = 3(2n)+7 = 6n+7$

$$\cancel{S = \frac{((3n+10)(6n+7))}{2} \times n}$$

$$S = \frac{((3n+10)+(6n+7))}{2} \times n$$

$$= \frac{(9n+17)}{2} \times n = \boxed{\frac{9n^2+17n}{2}}$$

$$S = \boxed{a_1} + \boxed{a_1 r} + a_1 r^2 + \cdots \boxed{a_1 r^{n-1}}$$

$$- rS = a_1 r + a_1 r^2 + \cdots a_1 r^{n-1} + a_1 r^n$$

$$S - rS = a_1 - a_1 r^n$$

$S(1-r) = a_1(1-r^n)$, if $r \neq 1$

$$S = \frac{a_1(1-r^n)}{1-r} = \frac{a_1(r^n-1)}{r-1}$$

$$\sum_{i=0}^{n} 3\cdot 2^i = \qquad a_1 = 3 \quad r = 2$$

terms = $\underline{\underline{n+1}}$

$$S = \frac{3(2^{n+1}-1)}{2-1} = \boxed{3(2^{n+1}-1)}$$

$$\sum_{i=1}^{\infty} i\cdot\left(\tfrac{1}{2}\right)^i$$

$$S = \boxed{1\times\tfrac{1}{2}} + \boxed{2\times\tfrac{1}{4}} + \boxed{3\times\tfrac{1}{8}} + 4\times\tfrac{1}{16} + \cdots$$

$$-\tfrac{1}{2}S = \quad 1\times\tfrac{1}{4} + 2\times\tfrac{1}{8} + 3\times\tfrac{1}{16} + \cdots$$

$$S - \tfrac{1}{2}S = \underbrace{\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \tfrac{1}{16} + \cdots}_{T}$$

$$\tfrac{1}{2}S = 1 \implies \boxed{S = 2}$$

$$T = \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \cdots$$

$$-\tfrac{1}{2}T = \tfrac{1}{4} + \tfrac{1}{8} + \cdots$$

$$T - \tfrac{1}{2}T = \tfrac{1}{2}$$

$$\tfrac{1}{2}T = \tfrac{1}{2}$$

$$T = 1$$

$$|r| < 1$$

$$T = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots$$

$$-rT = \quad a_1 r + a_1 r^2 + a_1 r^3 + \cdots$$

$$T - rT = a_1$$

$$\boxed{T = \frac{a_1}{1-r}}$$ Sum Inf Geo Seq