

COP 3502 10/19/23

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① Tom Post Calc Grade, post curr A, B, C lines (approx)

① Exam 1 Solutions posted tomorrow

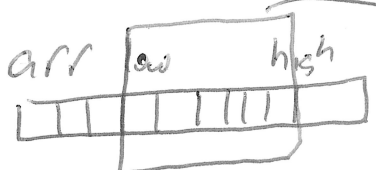
② Leave early?

a) Merge Sort

b) Quick Sort

c) Comparison

Last time - finished Merge function algorithm

void MergeSort ( arr, low, high ) {  
 Sort this subarray  
}

if (low >= high) return;

int mid = (low + high) / 2;

$T(\frac{n}{2}) \rightarrow$  MergeSort (arr, low, mid);

$T(\frac{n}{2}) \rightarrow$  MergeSort (arr, mid+1, high);

$O(n) \rightarrow$  Merge (arr, low, mid, high);

~

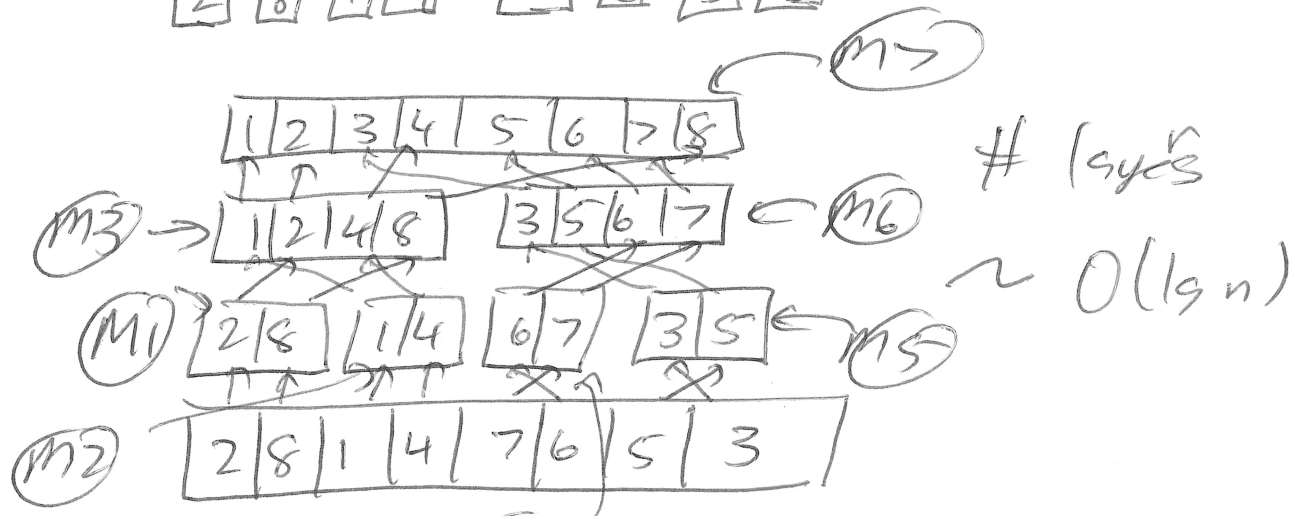
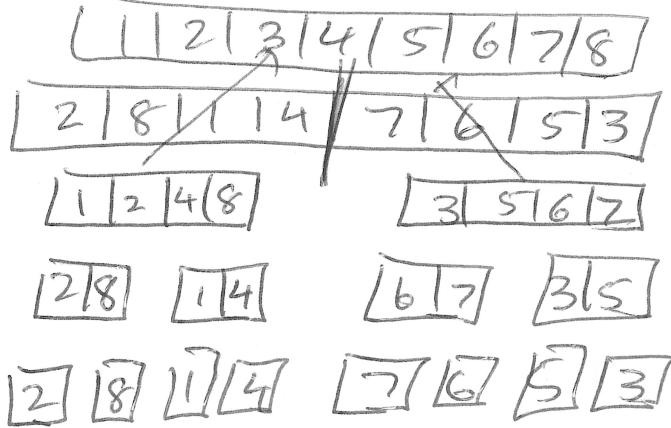
~~MS(8,8)~~ MS(2,2)  
~~MS(7,7)~~ MS(1,1)  
~~MS(6,6)~~ MS(0,0)  
~~MS(5,5)~~ MS(0,1)  
~~MS(4,4)~~ MS(0,2) MS(3,3)  
~~MS(3,3)~~ MS(0,3)  
~~MS(2,2)~~ MS(0,4)  
~~MS(1,1)~~ MS(0,5)  
MS(0,10)

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + O(n)$$

MS on  $T(n) = 2T\left(\frac{n}{2}\right) + O(n)$

n elements  $A=2, B=2, k=1 \rightarrow O(n \lg n)$

best, avg, worst



# cycles

$\sim O(\lg n)$

$M_4$  IS STABLE (keeps ties in original order)  
~~IS IN PLACE~~ (can merge in place)

benefit: worst-case  $O(n \lg n)$

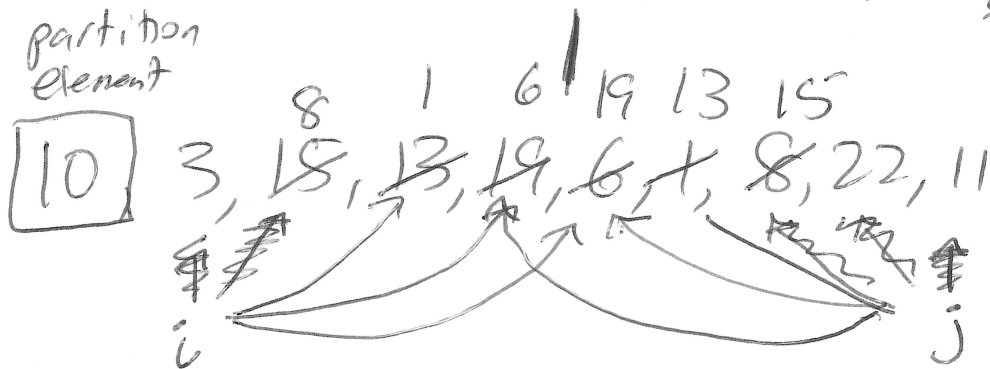
drawbacks: extra memory (can't do in place)  
~~NOT IN PLACE?~~

# Partition function

partition - split up into groups

in math split a set into multiple sets where each element in the original set belongs to exactly 1 of the new sets

$$\{A, B, C, D, E\} \rightarrow \{A, D\}, \{E, B, C, E\}$$



while  $(i < j)$  {

move  $i$  (left ptr) to the right until we find value  $> a[\text{part}]$

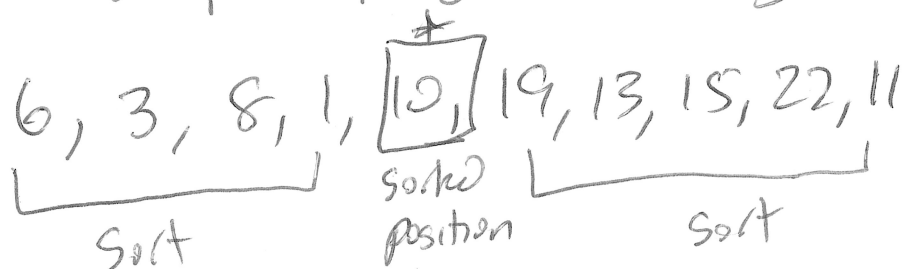
move  $j$  (right ptr) to the left

until we find value  $\leq a[\text{part}]$

if  $(i < j)$  swap  $a[i]$  and  $a[j]$

}

swap  $a[\text{part}]$  and  $a[j]$



quicksort (int arr, int low, int high) {

where  
partition  
elem  
ended up

if (low >= high) return;  
int part = partition (arr, low, high);  
quicksort (arr, low, part-1);  
quicksort (arr, part+1, high);

}

Why on earth is quicksort faster  
if it often splits unevenly???

A: works in place. No extra  
allocating or copy back of items

Run-Time Analysis

two even  
recursive  
cases

best case  $T(n) = 2T(\frac{n}{2}) + O(n) \rightarrow O(n \log n)$

all items  
on one side

worst case  $T(n) = T(n-1) + O(n) \rightarrow O(n^2)$

Intuitive proof avg case  $O(n \log n)$

$\frac{1}{2}$  time we split at least  $\frac{1}{4}n$   $\frac{3}{4}n$

$T(n) = T(\frac{3n}{4}) + T(\frac{n}{4}) + O(n)$  ugly

$\leq 2T(\frac{3n}{4}) + O(n)$  proof  $O(n \log n)$   
via iteration  
use master