COP3502 11/14/23

Base Conversion

$279 = 2 \times \underline{\underline{10}}^2 + 7 \times 10^1 + 9 \times 10^0$

base = # symbols used to represent numbers

10 fingers ⟷ base 10?

Exam 1 - Thursday 2 Parts

Part A - announcement topics

Part B - " "

~~NOLC~~ NO CALC

NO NOTES

Memorize various alg.

1 symbol

|
||

Unary (base 1)

large quantities → unwieldy!

2 symbols : 0, 1 (binary)

8 symbols : 0,1, ..., 7 (octal)

16 symbols : 0,1, .. 9, a, b, c, d, e, f (hexadecimal)

binary : $\underline{10}110100_2$

base

$= 1\times2^7 + 0\times2^6 + 1\times2^5 + 1\times2^4$

$+ 0\times2^3 + 1\times2^2 + 0\times2^1 + 0\times2^0$

$= 2^7 + 2^5 + 2^4 + 2^2 = \boxed{180_{10}}$

Convert to base 10

$d_{k-1} d_{k-2} \cdots d_1 d_0$

most sig sym

least sig sym

$\to d_{k-1} \times b^{k-1} + d_{k-2} \times b^{k-2} + \cdots d_1 \times b + d_0$

000
001
010
011
100
101
110
11ϕ (7)

$1000 \to 8^{th}$ #
2^3

$\underset{2}{\_\_} \times \underset{2}{\_\_} \times \underset{2}{\_\_} = 2^3$ possible numerals

in base b we can form b^k ~~digits~~ symbols numerals starting at 0 with k

$\underbrace{10000\ldots0}_{k\ 0s}$ $\to$ b^k items preceding me. These are 0 to b^k-1

$C25_{16} = \underline{3109}_{10}$

$$\begin{array}{r} 256 \\ \times\ 12 \\ \hline 512 \\ 256 \\ \hline 3072 \end{array}$$

$12 \times 16^2 + 2 \times 16^1 + 5 \times 16^0 = 3072$

$$\begin{array}{r} 3072 \\ 32 \\ 5 \\ \hline 3109 \end{array}$$

$1323_4 = \underline{\qquad}_{10}$

$1 \times 4^3 + 3 \times 4^2 + 2 \times 4^{\phi} + 3 \times 4^0 = 64 + 48 + 8 + 3$
$= \boxed{123}$

Opposite Task

base 10 $\to$ base b

$123_{10} \to$ base 4

$123 = \underbrace{d_3 \times 4^3 + d_2 \times 4^2 + d_1 \times 4^1 + \boxed{d_0 \times 4^0}}$ mod 4

123 mod 4

$\boxed{123 \equiv 3 \bmod 4}$

$123 \% 4 = d_0$

1. Reveal last symbol via mod base

$$\frac{123}{4} = \frac{d_3\times 4^3 + d_2\times 4^2 + d_1\times 4^1 + 3\times 4^0}{4}$$ (int div by 4)

$$\underbrace{30 = d_3\times 4^2 + d_2\times 4^1 + d_1\times 4^0}$$ (Same problem, Smaller #

% base reveals last digit
/ base chops off last digit

4 | 123
4 | 30 R3
4 | 7 R2
4 | 1 R3
0 R1

Quotient

$= 1323_4$

$3826_{10} = \underline{\qquad}_9$

9 | 3826
9 | 425 R1
9 | 47 R2
9 | 5 R2
0 R5

5221_9

~~$246B_{12}$~~

$2961_{10} = \underline{1869}_{12}$

12 | 2961
12 | 246 R9
12 | 20 R6
1 R8

12 | 2963
12 | 246 R11 (B)
12 | 20 R6
1 R8

Shortcut

Converting btw bases powers of 2

base 16 → base 2   base 8 → base 2

base 2 → base 16   base 2 → base 8

$$C25_{16} = \underline{\underline{12}}\times 16^2 + 2\times 16^1 + 5\times 16^0$$

$$(2^3+2^2)\times 2^8 + (2^1)\times(2^4) + (2^2+2^0)\times 2^0$$

$$= 2^{11} + 2^{10} + 2^5 + 2^2 + 2^0$$

Just replace each hex sym with 4 bits

$$C25_{16} = \underbrace{1100}_{C}\ \underbrace{0010}_{2}\ \underbrace{0101}_{5}{}_2$$

$$374_8 = \overbrace{011}^{3}\ \overbrace{111}^{7}\ \overbrace{100}^{4}{}_2$$

remove

$$325_8 = \overbrace{011}^{3}\ \overbrace{010}^{2}\ \overbrace{101}^{5}{}_2$$

don't remove

$$\underline{1}\,\underline{0111}\,\underline{0101}_2 = 175_{16}$$

$$2^8 + 2^6 + 2^5 + 2^4 + 2^2 + 2^0$$

$$= \underbrace{1}_{base16}\times \underbrace{\underline{\underline{2^8}}}_{16^2} + \underbrace{(2^2+2^1+2^0)}_{b16}\times \underbrace{\underline{\underline{2^4}}}_{16^1} + \underbrace{(2^2+2^0)}_{conv\ b16}\times \underbrace{\underline{\underline{2^0}}}_{16^0}$$

$3FAC4_{16} = \underline{\qquad\qquad}_8$

$\underline{\qquad\qquad}_2$

$\underbrace{11}_{3}\,\underbrace{1111}_{F}\;\underbrace{1010}_{A}\;\underbrace{1100}_{C}\;\underbrace{0100}_{4}\,{}_2 = \underline{775304}_8$

faster then going through base 10