Summations 2/25/2022

When I look at code with loops, if I want to add up the # of simple statements $\Rightarrow$ corresponds to the run time, I need to know how to add lots of numbers quickly.

Sum is "zipped" info. (LOSSLESS)

$$4+5+6+7+8+9 \Rightarrow \sum_{i=4}^{9} i$$

9 upper

$i=4$ lower

$i \to$ formula in terms of loop index.

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + \dots + f(b)$$

HOW TO UNZIP! $\to$ easier than condensing

$$8+11+14+\dots+98 = \sum_{i=2}^{32} (3i+2)$$

$\Rightarrow$ how to zip harder

Quick Review

$$\sum_{i=a}^{b} c = \underbrace{c(b-a+1)}_{\text{repeated addition}}$$

$a \le b$ const

= multiplication

$$\sum_{i=a}^{b} c \cdot f(i) = c \sum_{i=a}^{b} f(i)$$

factor out mult const

$$\sum_{i=a}^{b}\left[f(i)+g(i)\right] = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$

using commutative property of addition

$$\sum_{i=a}^{b} f(i) = \left[\sum_{i=1}^{b} f(i) - \sum_{i=1}^{a-1} f(i)\right]$$

$1 < a \leq b$

$$f(a)+f(a+1)+\dots f(b) = \cancel{f(1)}+\cancel{f(2)}+\cancel{f(3)}+\cancel{f(a-1)}+f(a)+f(a+1)+\dots f(b)$$
$$- \left\{\cancel{f(1)}+\cancel{f(2)}+\cancel{f(3)}+\dots \cancel{f(a-1)}\right]$$

$$f(a)+f(a+1)+\dots f(b)$$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

memorize

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}}$$

no need mem

$$\boxed{\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}}$$

no need mem

$$S = 1 + 2 + 3 + \dots + n$$
$$S = n + (n-1) + (n-2) + \dots + 1$$

$$2S = \underline{(1+n)} + \underline{(1+n)} + \underline{(1+n)} + \dots (n+1)$$

$$2S = n(n+1)$$

$$\boxed{S = \frac{n(n+1)}{2}}$$

Arith Series $5, 8, 11, \dots 101$

common difference btw terms

$$\left[\begin{array}{l} a_i = a_{i-1} + d \\ a_n = a_1 + \underbrace{(n-1)}_{\#\text{ hops}}\underbrace{d}_{\text{how far per hop}} \end{array}\right.$$

$$S = a_1 \xrightarrow{+d} a_2 + a_3 + \dots a_n$$
$$S = a_n \xrightarrow{-d} a_{n-1} + a_{n-2} + \dots a_1$$

$$\boxed{\frac{\text{SUM}}{\#\text{ TERM}} = \text{AVG}}$$

$$2S = (a_1 + a_n)n$$
$$S = \frac{(a_1+a_n)n}{2} = (\text{AVG}) \cdot (\#\text{ TERMS})$$

For me: OKAY

$$\sum_{i=50}^{150}(3i-4) \Rightarrow \text{arith seq} \quad ai+b$$

const

$$\begin{array}{r} 446 \\ 146 \\ \hline 592 \end{array}$$

$a_1 = 3(50)-4 = 146$

$a_{101} = 3(150)-4 = 446$

$$S = \frac{(146+446)}{2} \times 101$$

$$= \frac{592}{2} \times 101$$

$$= 296 \times 101$$

$$= 29,896$$

$$\begin{array}{r} 296 \\ 296 \\ \hline 29896 \end{array}$$

$$\sum_{i=50}^{150}(3i-4) = 3\sum_{i=50}^{\cancel{100}\ 150} i - \sum_{i=50}^{\cancel{100}\ 150} 4$$

$$= 3\left[\sum_{i=1}^{150} i - \sum_{i=1}^{49} i\right] - 4(150-50+1)$$

$$= 3\left[\frac{150\times151}{2} - \frac{49\times50}{2}\right] - 404$$

$$= 3\left[75\times151 - 49\times25\right] - 404$$

$$= 3\times25\left[\overset{151\times3}{\cancel{151}} - 49\right] - 404$$

$$= 75\left[453-49\right] - 404$$

$$= 75\left[404\right] - 404$$

$$= 404\left[75-1\right]$$

$$= 404\times74$$

$$= 29,896$$

$$\begin{array}{r} 404 \\ \times\ 74 \\ \hline 1616 \\ 2828\ \\ \hline 29896 \end{array}$$

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

$$-\frac{1}{2}S = \cancel{\frac{1}{2}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{8}} + \cdots$$

$$S - \frac{S}{2} = 1$$

$$\frac{S}{2} = 1$$

$$\boxed{S = 2}$$

$$S = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \cdots$$

$$-\ rS = \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \cdots$$

$$S(1-r) = a_1$$

Sum of infinite geo sequence

$$\boxed{S = \frac{a_1}{1-r}, \ |r| < 1 \quad \text{memorize}}$$

$$S = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cdots + \cancel{a_1 r^{n-1}}$$

$$-\ rS = \cancel{a_1 r} + \cancel{a_1 r^2} + \cdots + \cancel{a_1 r^{n-1}} + a_1 r^n$$

$$S(1-r) = a_1 - a_1 r^n$$

$$S(1-r) = a_1(1-r^n)$$

$$\boxed{S = \frac{a_1(1-r^n)}{1-r} \quad \text{memorize}} = \frac{a_1(r^n-1)}{r-1}, \ \boxed{r \neq 1}$$

$$S = \underline{1 \times 2^0} + \underline{2 \times 2^1} + \underline{3 \times 2^2} + 4 \times 2^3 + \cdots + n \times 2^{n-1}$$

$$-2S = \quad 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + (n-1) 2^{n-1} + n \cdot 2^n$$

$$-S = \underbrace{1 + 2^1 + 2^2 + 2^3 + \quad 2^{n-1}} - n \cdot 2^n$$

$$\boxed{\sum_{i=1}^{n} i \cdot 2^{i-1}}$$

$$-S = \frac{2^n - 1}{2 - 1} - n \cdot 2^n$$

$$S = -(2^n - 1) + n \cdot 2^n$$

$$S = n \cdot 2^n - 2^n + 1$$

$$\boxed{S = (n-1) 2^n + 1}$$