

## COP 3503 Honors – Midterm Take Home Exam (Non-Collaborative)

Due Date: October 16, 2024

1. Solve the following recurrence using the substitution method (10 pts)

$$T(n) = 4T(n/2) + n, T(1) = 1 \text{ ( Guess } T(n) = \Theta(n^2)\text{)}$$

2. Solve the following recurrence relation using the iteration method (find a tight upper bound) (10 pts)

$$T(n) = 4T(n/2) + n^2$$

3. Which sorting algorithm would you use in each of the following cases and why: (10 pts)

- Sorting  $n$  floating point numbers that have values between 0 and 1 and 5 significant digits
- Sorting  $n$  floating point numbers in the range of  $[0, \dots, 100]$ .
- Sorting  $n$  integers in the range  $[0, \dots, k]$  where  $k$  is much smaller than  $n$ .
- Sorting  $n$  floating point numbers that are generated with a uniform distribution in the range  $[0, \dots, 100]$
- Sorting  $n$  integers in the range  $[0, \dots, n^5 - 1]$

4. Use Strassen's algorithm to compute the following product (show all work and intermediate steps) (15 pts)

$$\begin{pmatrix} 1 & 2 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 4 & 9 \\ 2 & 3 \end{pmatrix}$$

5. (a) You are given a set  $S$  containing  $n$  integers in the range  $[1, \dots, k]$ . You want to preprocess the input so you can answer queries of the type, "How many elements in  $S$  fall in the range  $[a, \dots, b]$ " (where  $a, b$  are real numbers) as fast as possible. Give both the preprocessing and query answering algorithm and analyze their running time. (15 pts)

(b) What if instead of integers the set  $S$  contains floating point numbers? How would we preprocess the set and what would be an efficient algorithm for answering the queries? (10 pts)

6. A thief is in a room that contains  $n$  items. Each item has a price  $p$  and a weight  $w$  associated with it. The thief can only carry up to  $b$  pounds (she can either take or leave the item, it is not possible to take a fraction of it). She wants to choose a subset of the items so that the weight of all of them is less than or equal to  $b$  and her profit is maximized.
- Write an algorithm that, given an array  $W[1\dots n]$  containing the weights of the items, an array  $P[1\dots n]$  containing their prices, and the bound  $b$ , will output the maximum profit possible. Analyze the time and space requirements of your algorithm (20 pts).
  - Briefly describe what changes you would make to the above algorithm if you wanted to also compute which items she would take. (10 pts)