

Fall 2022 COT 3100 Homework #1

1) (6 pts) Joanna and Ahmed are running a 13 mile race. For the first three miles, both run at the same speed. Unfortunately, Ahmed got a little tired at that point, so for the last ten miles, he ran 2 miles per hour slower than his original speed while Joanna continued to run at the same speed she ran the first three miles. If Ahmed finished the race exactly 25 minutes after Joanna did.

- (a) At what speed (in miles per hour), did both start running at the beginning of the race?
- (b) How long did Joanna take to finish the race (answer in hours:minutes:seconds)?
- (c) How long did Ahmed take to finish the race (answer in hours:minutes:seconds)?

2) (6 pts) Determine the ordered pair, (x, y) , which satisfies the following pair of equations:

$$\log_2(x^2) + \log_4 y = 7$$

$$\log_4(8x) + \log_2(2y^2) = 8$$

3) (8 pts) Create a truth table for the following logical expression: $\neg(p \wedge q) \vee (\bar{r} \oplus s)$. Please make sure your first four columns are for p , q , r , and s , respectively, and that you have appropriate intermediary columns.

4) (4 pts) Use the truth table method to prove the following two expressions are logically equivalent:

$$\frac{\overline{(p \wedge q)} \rightarrow \bar{q}}{p \vee \bar{q}}$$

5) (6 pts) Use the laws of logic to show that two following expressions are logically equivalent:

$$\frac{\overline{(p \wedge q)} \rightarrow \bar{q}}{p \vee \bar{q}}$$

6) (5 pts) Use the rules of inference to prove the following argument:

$$\begin{array}{l} p \rightarrow r \\ s \\ \bar{q} \\ s \rightarrow (p \vee q) \\ \hline \therefore r \end{array}$$

7) (5 pts) Use the rules of inference to prove the following argument:

$p \rightarrow t$
 \bar{u}
 $\bar{r} \vee s$
 $\bar{q} \rightarrow \bar{t}$
 $u \vee (p \vee r)$

 $\therefore q \vee s$

8) (5 pts) Prove or disprove the following claim over the universe of all real numbers for x and y.

$$\forall x[\exists y[xy - x - y = -1]]$$

If the claim is false, find a value of x for which it is false. If it is true, show which value(s) of y exist to make the claim true for all x.

9) (5 pts) The last question of each homework assignment will be to write up a two paragraph summary of a topic from the history of mathematics. The idea here is that rarely is any of this history taught in mathematics classes and while I don't have class time to teach it, I thought it would be nice if students learned a bit for each homework assignment. There's no need to use fancy sources, websites will do, but please site which websites you pulled your information from.

Give a summary of the life and mathematical contributions of **Girolamo Cardano**. Note: in some sources, his first name is spelled "Gerolamo."