COT 3100 Fall 2022 Homework #10
Please Consult WebCourses for the due date/time

1) (14 pts) Let R_1 and R_2 be relations on a set A = {1, 2, 3, 4}.
In particular, let R_1 = {(1, 3), (2, 2), (2, 4), (3, 1), (4, 2)} and
R_2 = {(1, 1), (1, 3), (2, 2), (2, 3), (3, 3), (3, 4), (4, 4)}.

Determine the following:

a) Whether or not R_1 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
b) Whether or not R_2 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
c) The relation $R_1 \circ R_2$.
d) The relation $R_2 \circ R_1$.
e) $R_1 \cup R_2$
f) $R_1 \cap R_2$
g) The reflexive, symmetric and transitive closures of both R_1 and R_2.

2) (5 pts) Let R be a relation over the positive integers defined as follows:

R = {(a,b) | a, 2a and b form side lengths of a triangle with positive area }

Determine whether or not R satisfies the following properties. Give a brief justification for each of your answers.

(i) reflexive
(ii) irreflexive
(iii) symmetric
(iv) anti-symmetric
(v) transitive

3) (6 pts) How many anti-symmetric relations on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} contain the ordered pairs (1, 1), (3, 3), (4, 4), (5, 8), and (9, 1)?

4) (4 pts) Let $P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$. P(4) = P(5) = P(6) = P(7) = P(8) = 0. What is the value of a - b + c - d + e?

5) (4 pts) Let $f(x) = x^2 + 8x - 9$ with a domain of all real $x \in [-\infty, -4]$. Prove that f is injective. What is the range of f? (You may either use Calculus or complete the square to prove your answers.)

6) (4 pts) Find $f^{-1}(x)$ for the function given in question #5.

7) (8 pts) Let $f(x) = ax^3 + bx^2 + cx + d$ and $f(-1) = -6$, $f(0) = 4$ and $f(1) = 12$.

(1) (1 pt) What is the value of d?
(2) (3 pts) What is the value of b?
(3) (2 pts) What is the value of a+c?
(4) (2 pts) Prove that the value of a is not uniquely determined by finding two sets of ordered quadruplets (a_1, b_1, c_1, d_1) and (a_2, b_2, c_2, d_2) with $a_1 \neq a_2$ that is consistent with all of the given information above.

8) (5 pts) Please give a summary of the life and mathematical contributions of Srinivasa Ramanujan. Please aim for a length of roughly 200 - 400 words. ***Your summary must be typed.*** Please state the sources you used in writing your summary. If you are so inclined, after finals, go watch the movie "The Man Who Knew Infinity."