COT 3100 Fall 2022 Homework #6
Please Consult WebCourses for the due date/time

1) (6 pts) Let F_i denote the ith Fibonacci number. Prove by induction on n that, for all positive integers n:

$$\sum_{i=1}^{n} F_{2i} = F_{2n+1} - 1$$

2) (12 pts) Define a sequence, a_i, as follows:

$$a_0 = 0, a_1 = 1, a_2 = 3, a_n = 3a_{n-1} + 2a_{n-2}, for\ all\ ints\ n > 2$$

Using induction on n, prove for all positive integers, n, that

$$\begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} a_{n+1} & 2a_n \\ a_n & 2a_{n-1} \end{pmatrix}$$

3) (12 pts) Let $a > 1$ be a positive integer. Using induction on n, prove for all positive integers n,

$$(a^2 - a + 1)|((a - 1)^{n+1} + a^{2n-1}).$$

4) (15 pts) Using mathematical induction on n, prove for all positive integers n, that

$$\sum_{i=1}^{n^2} \sqrt{i} \geq \frac{n(4n^2 - 3n + 5)}{6}$$

Hint: In bounding the second sum, you'll have to use two separate lower bounds to replicate the result; one for most of the terms and a different bound for the very last term. (This hint is intentionally cryptic, so don't ask me what it means. If you get to a particular point in the problem, then this hint is more likely to seem relevant.)

5) (5 pts) Give a summary of the academic contributions of Grigori Perelman. Please aim for a length of roughly 200 - 400 words. **_Your summary must be typed._** Please state the sources you used in writing your summary.