

COT 3100 Fall 2022 Homework #9
Please Consult WebCourses for the due date/time

Note: Please *justify* your answers and why you use each formula. Please leave answers in factorials, combinations and powers. When possible, reduce fractions to lowest terms.

1) (5 pts) Johnny has rolled a pair of standard fair six-sided dice. Given that one of the two dice shows 3 dots, what is the probability that the sum of the number of dots showing on both dice is even?

2) (8 pts) In class, we played a game where 3 distinct digits were hidden behind 3 “doors.” The contestant would choose one of the doors and the number behind it was revealed. Based on this number, the contestant could either choose to stay with that door, or switch to another door, with the goal of selecting the door that had the lowest number behind it. We also showed in class that the optimal strategy for the game was to keep the same door if the number revealed was 3 or lower, and to switch otherwise. Change the game so that the numbers behind the 3 doors are distinct positive integers in between 1 and 30, inclusive. What is the largest integer k , such that if a contestant reveals k , their best chance of winning is staying with that door. For that value of k , what is the contestant’s chance of winning, given that they stay with the same door? For that value of k , what is the contestant’s chance of winning if they switch to a different door?

3) (6 pts) Consider the following discrete random variable X :

$X = 2$, with probability 0.1
= 8, with probability 0.2
= 12, with probability 0.5
= 16, with probability 0.2

Calculate both $E(X)$ and $\text{Var}(X)$.

4) (5 pts) Sam's probability of getting an A on an individual test is 75%. If he takes 12 tests, what is the probability he gets As on exactly 10 of those tests?

5) (10 pts) Here are the rules for Arup’s Game of Dice:

- 1) Roll a pair of dice.
- 2) If you roll a sum of 11 or 12, you win.
- 3) If you roll a sum of 2, you lose.
- 4) Otherwise, record what you've rolled. Let this sum be k ; also known as your point.
- 5) Roll one more time. If this roll exceeds your point(k), you win!
- 6) If this roll is the same as your point(k) or lower, you lose.

Calculate the probability that a player wins Arup’s Game of Dice.

6) (6 pts) Define a continuous random variable as follows:

$$f(x) = \frac{1}{9}x^2, \text{ for } 0 \leq x \leq 3, \text{ and} \\ = 0, \text{ otherwise}$$

Determine both the expectation and variance of the continuous random variable defined above.

7) (5 pts) Suppose E and F are events in a sample space and $p(E) = 1/3$, $p(F) = 2/5$, and $p(F | E) = 1/2$. Find $p(E | F)$.

8) (5 pts) Give a summary of the life (thus far!) and mathematical contributions of Jennifer Balakrishnan.