University of Central Florida
School of Computer Science
COT 4210 Spring 2004

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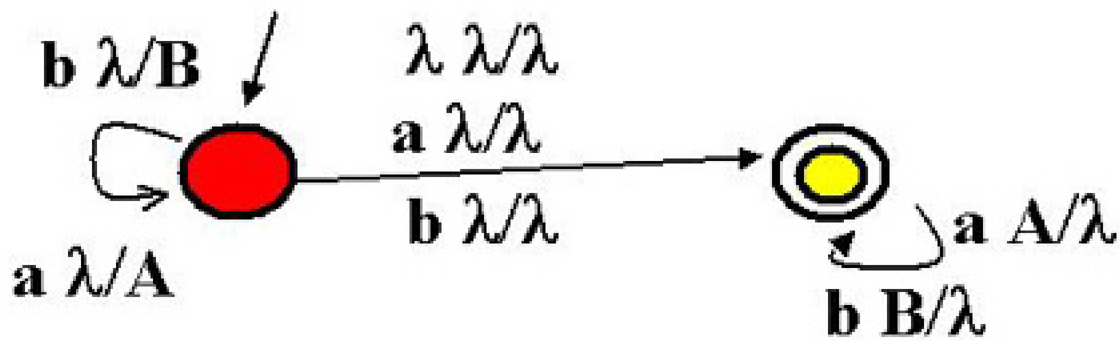
Answers to Homework 4

1. Construct a PDA that accepts the set of palindromes over $\Sigma = \{a, b\}$. Use the notation of section 8.1 of the text.

   answer:



2. The Pumping Lemma for Regular Languages, as stated in Theorem 7.6.3 of the text (Theorem 9 in Prof. Workman's notes), says that a long-enough word z in a regular language can be "pumped" somewhere inside a prefix of z. I have argued in class that restricting the lemma to a prefix is unnecessary. State the more general pumping lemma. (You do not need to prove it, but you must state it precisely).

   answer: Here is one possible wording:

   If

   - L is a regular language over the alphabet Σ; and
   - L is accepted by a DFA with α states; and
   - u, v, w are strings in Σ^* such that $uvw \in L$ and $\alpha \leq |v|$,

then there exists a decomposition rst of v such that $s \neq \lambda$ and $urs^itw \in L$ for all $i \geq 0$.

3. For each language in exercise 12, section E, of Prof. Workman's notes answer the following.

   (a) Can the language be proven non-regular by a direct application of the "pump at prefix-only" version of the Pumping Lemma?

   (b) Can the language be proven non-regular by a direct application of the more general version of the Pumping Lemma?

   answer: The only language in the list for which "pump at prefix" cannot be directly used is

   $$f) \quad L = (a+b)^* \cup c^+\{a^nb^n \mid n \geq 0\}.$$

   Suppose $x \in c^+\{a^nb^n \mid n \geq 0\}$. The Pumping Lemma says there exists a decomposition $x = uvw$, with $|uv| < \alpha$ and $v \neq \lambda$, such that uv^iw is in the language for all $i \geq 0$. If $u = \lambda$ and $v = c$, then $uv^iw = c^iw$. When $i > 0$ or w starts with c, we have $c^iw \in c^+\{a^nb^n \mid n \geq 0\}$. If $i = 0$ and w does not start with c, we have $c^0w = w \in (a+b)^*$. Thus, it is not possible to conclude that uv^iw is not in the language for some i.

   The more general version of the Pumping Lemma can be used as follows. Assume the language L is regular and is accepted by a DFA with α states. Let $x = ca^\alpha b^\alpha$. Then x is in L. But pumping in the a^α region of x yields a word not in L, contradiction. Hence, L is not regular.

   Problem b), $T = \{a^nb^m \mid n \neq m\}$, is difficult. At first glance it appears neither version of the Pumping Lemma is sufficient to prove T is not regular. A proof can be constructed using closure properties: assume T is regular. Then T^c (the complement of T in $(a+b)^*$) is regular. Also, $V = \{a^nb^m \mid n, m \geq 0\}$ is clearly regular. Therefore $T^c \cap V$ is regular. But $T^c \cap V = \{a^nb^n \mid n \geq 0\}$ which we know is not regular, contradiction.

   So, is T the first non-regular language we encounter for which we might actually need Myhill-Nerode? Not so: let $x = a^\alpha b^{\alpha+\alpha!}$. Then the Pumping Lemma says you can pump somewhere within the a^α prefix. This means

   $$(a^{ik})a^{\alpha-k}b^{\alpha+\alpha!} \in T$$

for all $i \geq 0$. Since k divides $\alpha!$, we can choose $i = \frac{\alpha!}{k} + 1$. But then $\alpha - k + ik = \alpha + \alpha!$, and therefore

$$(a^{ik})a^{\alpha-k}b^{\alpha+\alpha!} = a^{\alpha+\alpha!}b^{\alpha+\alpha!}$$

which is not in T, contradiction.[1]

Finally, I will just list strings which can be used, along with the Pumping Lemma with parameter α, to prove each language is not regular.

- a) choose $a^{\alpha}b^{\alpha+1}$.
- c) choose $a^{\alpha}\#c^{\alpha}$.
- d) choose $0^{\alpha}0^{\alpha}$.
- e) choose $a^{\alpha}b^{\alpha}$.
- g) choose $a^{\alpha}\#c^{2\alpha}$.
- h) choose $a^{\alpha}b^{\alpha}\#c^{7\alpha}$.
- i) choose $0^{\alpha}10^{\alpha}10^{2\alpha}$.
- j) choose $0^{\alpha}10^{2\alpha}10^{\alpha}$.
- k) choose $a^{2\alpha+1}b^{2\alpha+1}$.

4. Consider the languages e through k of the previous question. For each, either construct a PDA or prove that the language is not a CFL. Use the notation of section 8.1 of the text.

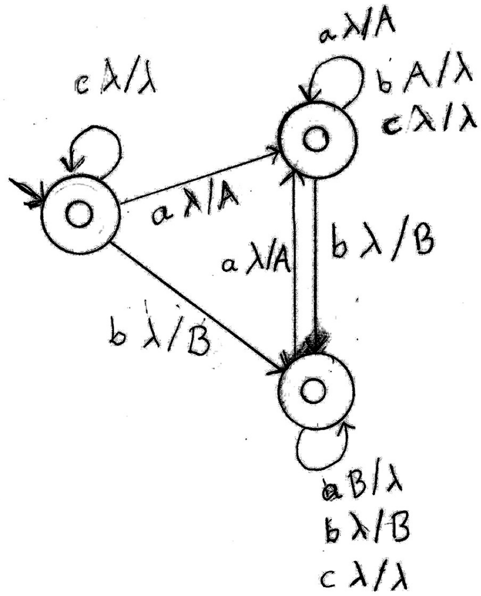
   answer: I think they are all context-free. Below are PDAs for e) and f).

5. Consider the language L over $\Sigma = \{a, b, c\}$ consisting of words with more a's than b's and more b's than c's. Prove or disprove L is a CFL.

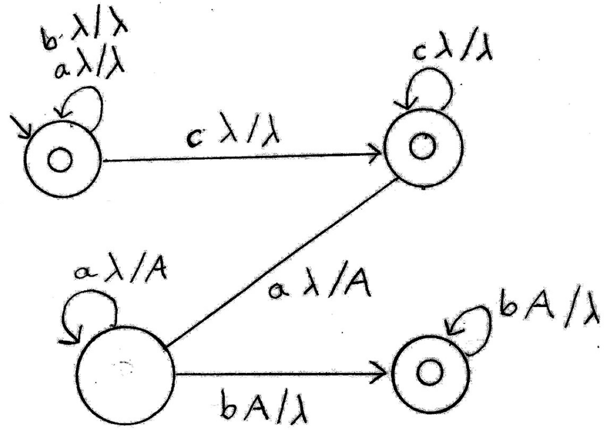
   answer: The language is not context-free. If it was context-free, then $T = L \cap a^*b^*c^*$ would also be context-free. Notice $T = \{a^n b^m c^k \mid n > m > k \geq 0\}$. The string $\tau = a^{\alpha+2}b^{\alpha+1}c^{\alpha}$ belongs to T. By the Pumping Lemma with parameter α, $\tau = uvwxy$ such that $uv^iwx^iy \in T \quad (vx \neq \lambda)$ for all $i \geq 0$. It is easy to see that v and x must belong to $a^* + b^* + c^*$. Pumping bs or cs in τ will cause the string to have more bs or cs than as, which is not allowed in T. Thus v and x belong to a^*. But then uv^0wx^0y is not in T (it has too few as), contradiction.

[1] This **really nice** argument is not mine. Prof. Workman taught me it. He says it appears in a textbook by Aho et. al.

e)


a λ/A
b A/λ
c λ/λ
c λ/λ
a λ/A
a λ/A
b λ/B
b λ/B
a B/λ
b λ/B
c λ/λ

f)


b λ/λ
a λ/λ
c λ/λ
c λ/λ
a λ/A
a λ/A
b A/λ
b A/λ