Host Community Respecting Refugee Housing

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We propose a novel model for refugee housing respecting the preferences of the accepting community and refugees themselves. In particular, we are given a topology representing the local community, a set of inhabitants occupying some vertices of the topology, and a set of refugees that should be housed on the empty vertices of the graph. Both the inhabitants and the refugees have preferences over the structure of their neighbourhood.

We are specifically interested in the problem of finding housing such that the preferences of every individual are met; using game-theoretical words, we are looking for housing that is stable with respect to some well-defined notion of stability. We investigate conditions under which the existence of equilibria is guaranteed and study the computational complexity of finding such a stable outcome. As the problem is NP-hard even in very simple settings, we employ the parameterised complexity framework to give a finer-grained view of the problem's complexity with respect to natural parameters and structural restrictions of the given topology.

CCS Concepts: • **Mathematics of computing** → Graph theory; • **Theory of computation** → *Parameterized complexity and exact algorithms*; **Algorithmic game theory and mechanism design**.

Additional Key Words and Phrases: Refugee Housing, Social Choice, Computational Complexity, Stability Concepts, Fixed-parameter tractability

1 INTRODUCTION

According to the last report of the United Nations High Commissioner for Refugees (UNHCR), there were 89.3 million forcibly displaced persons at the end of 2021 [69]. It is the highest number since the aftermath of World War II and it is for sure that these numbers will even grow. In May 2022, UNHCR announced that a tragic milestone of 100 million displaced persons was reached. They identified the war in Ukraine as the leading cause of the dramatic growth in the last year [60]. Russian aggression not only forced many Ukrainians to leave their homes, but even caused food insecurity and related population movement in many parts of the world, since Ukraine is among the fifth largest wheat exporters in the world [13].

It should be mentioned that political and armed conflicts are not the only causes of forced displacement [69]. One of the most common reasons for fleeing is due to natural disasters. To name just a few, in August 2022, massive floods across Pakistan affected at least two-thirds of the districts and displaced at least 33 million people [56, 57]; the numbers are not yet final. At the same time, a devastating drought in Somalia caused the internal displacement of at least 755,000 people [69]. Furthermore, it is expected that, due to climate change, extremes of the climate will become even more common in the near future [39].

Arguably, the best prevention against the phenomenon of forced displacement is not allowing it to appear at all; however, the aforementioned numbers clearly show that these efforts are not very successful. Therefore, in practise, three main solutions are assumed [45]. Voluntary *repatriation* is the most desirable but not very successful option. In many situations, repatriation is not even possible due to ongoing conflicts or a completely devastated environment. *Resettlement* and *integration* in the country of origin or abroad are more common. These two solutions require considerable effort from both the newcomers and the host community sides.

The very problematic part of forced displacement is the fact that 38% of all refugees[1] are hosted in only five countries [69]. And these are only the absolute numbers. For example, in Lebanon, every one in four people is a refugee [62]. The redistribution of refugees seems to be a natural solution to this imbalance; however, not all countries are willing to accept all people. One such example can be the Czech Republic, which refuses to accept any Syrian refugees during the 2015 European migrant crisis, currently hosting the largest number of Ukrainian refugees per capita [70].

Even with working and widely accepted redistributing policies, there is still a need to provide housing in specific cities and communities. From the good examples of such integration strategies [63] it follows that one of the most important characteristics is that members of the accepting community do not feel threatened by newcomers.

Inspired by this, we propose a novel computational model for refugee housing. Our ultimate goal is to find an assignment of displaced persons into empty houses of a community such that this assignment corresponds to the preferences of the inhabitants about the structure of their neighbourhoods and, at the same time, our model also takes into consideration the preferences of the refugees themselves, as refugees dissatisfied with their neighbourhood have a strong intention to leave the community. More precisely, in our model, we are given a topology of the community, which is an undirected graph, a set of inhabitants together with their assignment to the vertices of the topology and preferences over the shape of their neighbourhood, and a set of refugees with the same requirements on the neighbourhood shape. We want to find a housing of refugees in the empty vertices of the topology such that the housing satisfies a certain criterion, such as stability.

Refugee redistribution has gained the attention of mathematicians and computer scientists very recently. The formal model for capturing refugee resettlement is double-sided matching [3, 7, 28, 29]. That is, on the input we are given a set of locations with multidimensional constraints and a set of refugees with multidimensional features. An example of a constraint can be the number of refugees the locality can accept on one side and the size of family on the refugee side. The question then is whether there exists a matching between localities and refugees respecting all constraints. According to us, this formulation of the refugees resettlement problem more concerns the global perspective of refugee redistributing, not the local housing problem, as we do in our paper. Aziz et al. [7] study mostly the complexity of finding stable matching with respect to different notions of stability, and it turns out that for most of the stability notions finding a stable matching is computationally intractable (NP-hard, in fact). Kuckuck et al. [51] later refined the model of Aziz et al. [7] in terms of hedonic games.

Our Contribution. Partly continuing the line of research in refugee resettlement, we introduce a novel model focused on the local housing of new refugees. Previous models [3, 7, 29] can be seen and used as a very effective model on the (inter-) national level to distribute refugees to certain locations, such as states or cities[2]. However, our model can be assumed as the second level of refugee redistribution; once refugees are allocated to some community, we want to house them in a way that respects preferences of both inhabitants and refugees.

In particular, we introduce three variants of refugee housing, each targeting a certain perspective of this problem. Our simplest model, introduced in Section 3, completely eliminates the preferences of refugees and studies only the stability of the housing with respect to the preferences of the inhabitants. We call this variant *anonymous* housing. Since refugees are assumed to be indistinguishable, inhabitants have preferences over the number of refugees in their neighbourhood.

[1]From the strict sociological point-of-view, not all forcibly displaced persons are classified as refugees. Slightly abusing the notation, we will use the term refugee and displaced person interchangeably.

[2]In fact, the American resettlement agency HIAS use the matching software *Annie*^{TM} Moore which was later improved by Ahani et al. [3].

As stated above, the most successful refugee integration projects have the following property in common; they try to make both inhabitants and refugees as satisfied as possible by various activities to ensure that both groups get to know each other. We believe that our *hedonic* model, where the preferences of both inhabitants and refugees are based on the identity of particular members of the second group, supports and leads to more stable and acceptable housing. This model is formally defined and studied in Section 4.

The two introduced models have some disadvantages. The first is disrespectful to the refugees' preferences, while the second is not very realistic, as it is hard to make all inhabitants familiar with all refugees and the other way around. Therefore, our last model can be seen as a compromise between these two extremes. In the *diversity* setting, introduced in Section 5, all agents (union of inhabitants and refugees) are partitioned into k types, and preferences are over the fractions of agents of each type in the neighbourhood of each agent. Another advantage of this approach is that it nicely captures also the settings where we already have number of integrated refugees and the newcomers want to have some of them in the neighbourhood, or the case of an internally displaced person.

In all the aforementioned variants of the refugee housing problem, agents have dichotomous preferences; that is, they approve some set of alternatives and do not distinguish between them. It can be seen as if the neighbourhood of some agent does not comply with his approval set, he would rather leave the local community, which is very undesired behaviour on both sides.

For all assumed variants, we show that an equilibrium is not guaranteed to exist even in very simple instances. Thus, we study the computational complexity of finding an equilibrium or deciding that no equilibrium exists. To this end, we provide polynomial-time algorithms and complementary NP-hardness results. In order to paint a more comprehensive picture of the computational tractability of the aforementioned problems, we employ a finer-grained framework of parameterised complexity to give tractable algorithms for, e. g., instances where the number of refugees or the number of inhabitants is small, or for certain structural restrictions of the topology. Additionally, we complement many of our algorithmic results with conditional lower-bounds matching the running-time of these algorithms.

Related Work. Our model is influenced by a game-theoretic reformulation of the famous Schelling's model [64, 65] of residential segregation introduced by Agarwal et al. [2]. Here, we are given a simple undirected graph G and a set of selfish agents partitioned into k types. Every agent wants to maximise the fraction of agents of her own type in her neighbourhood. The goal is then to assign agents to the vertices of G so that no agent can improve her utility by either jumping to an unoccupied vertex or swapping positions with another agent. Follow-up works include those that study the problem from the perspective of computational complexity [33, 50] and equilibrium existence guarantees [16, 17, 47].

The second main inspiration for our model is the Hedonic Seat Arrangement problem and its variants recently introduced by Bodlaender et al. [18]. Here, the goal is to find an assignment of agents with preferences to the vertices of the underlying topology. The desired assignment should then meet specific criteria such as different forms of stability, maximising social welfare, or being envy-free. In our model, compared to Hedonic Seat Arrangement of Bodlaender et al. [18], inhabitants already occupy some vertices of the topology and we have to assign refugees to the remaining (empty) vertices in a desirable way.

Next, the problem of house allocation [1] or housing market [67] has been extensively studied in the area of mechanism design. Here, each agent owns a house and the objective is to find a socially efficient outcome using reallocations of objects. Later, You et al. [72] introduced house allocation over social networks that follows current trend in mechanism design initiated by Li et al. [53], where each individual can only communicate with his neighbours. As stated before, the house allocation is studied mainly from the viewpoint of mechanism design and as such is far from our model.

Finally, hedonic games [21, 22, 32] are a well-studied class of coalition formation games where the goal is to partition agents into coalitions and where the utility of every agent depends on the identity of other agents in his coalition. In anonymous games [12, 21], the agents have preferences over the sizes of their coalition. The most recent variants of hedonic games are the so-called hedonic diversity games [19, 23] where agents are partitioned into k types and preferences are over the ratios of each type in the coalition. The main difference between our model and (all variants of) hedonic games is that in the latter model all coalitions are pairwise disjoint; however, in our case, each agent has his own neighbourhood overlapping with neighbourhoods of other agents.

2 PRELIMINARIES

Let $\mathbb{N}$ denote the set of positive integers. Given two positive integers $j, j' \in \mathbb{N}$, with $j \leq j'$, we call the set $[j, j'] = \{j, \ldots, j'\}$ an *interval*, and let $[j] = [1, j]$ and $[j]_0 = [j] \cup \{0\}$. Let S be a set. By 2^S we denote the set of all subsets of S and, given $k \leq |S|$, we denote by $\binom{S}{k}$ the set of all subsets of S of size k.

Let $R = \{r_1, \ldots, r_m\}$ be a non-empty set of *refugees* and $I = \{i_1, \ldots, i_\ell\}$ be a set of *inhabitants*. The set of all *agents* is defined as $N = R \cup I$. A *topology* is a simple undirected graph $G = (V, E)$, where $|V| \geq |N|$. For a vertex v, we denote by $N(v)$ the set of its *neighbours*, formally $N(v) = \{u \mid \{u, v\} \in E\}$. The size of the neighbourhood of a vertex v is called its *degree* and is defined as $\deg(v) = |N(v)|$. The *closed neighbourhood* of vertex v is defined as $N[v] = N(v) \cup \{v\}$. In this work, we follow the basic graph-theoretical terminology [30].

An *inhabitants assignment* is an injective function $\iota \colon I \to V$.
A set of vertices occupied by the inhabitants is denoted V_I and, given an inhabitant $i \in I$, we denote the set of unoccupied vertices in his neighbourhood $U_i = N(\iota(i)) \setminus V_I$. The set of all vertices that are not occupied by inhabitants is denoted $V_U = V \setminus V_I$.

The goal of every variant of our problem is to find a mapping of refugees to vertices that are not occupied by inhabitants. Formally, *housing* is an injective mapping $\pi \colon R \to V_U$. A set of vertices occupied by refugees with respect to housing π is denoted $V_\pi = \{\pi(r) \mid r \in R\}$. We denote by $\Pi_{G,\iota}$ the set of all possible housings, and we drop the subscript whenever G and ι are clear from the context.

Parameterised Complexity. We study the problem in the framework of parameterised complexity [26, 31, 61]. Here, we investigate the complexity of the problem not only with respect to an input size n, but even assuming some additional *parameter* k. The goal is to find a parameter which is small and the "hardness" can be confined to this parameter. The most favourable outcome is an algorithm with running time $f(k) \cdot n^{O(1)}$, where f is any computable function. We call this algorithm *fixed-parameter tractable* and the complexity class containing all problems that admit algorithms with such running time is called FPT.

Not all combinations of parameters yield to fixed-parameter tractable algorithms. A less favourable outcome is an algorithm running in $n^{f(k)}$ time, where f is any computable function. Parameterised problems admitting such algorithms belong to complexity class XP. To exclude the existence of a fixed-parameter tractable algorithm, one can show that the parameterised problem is W[t]-hard for some $t \geq 1$. This can be done via a *parameterised reduction* from any problem known to be W[t]-hard.

It could also be the case that a parameterised problem is NP-hard even for a fixed value of k; we call such problems para-NP-hard and, assuming P $\neq$ NP, such problems do not admit XP algorithms.

Our running-time lower-bounds are based on the well-known Exponential Time Hypothesis (ETH) of Impagliazzo and Paturi [43]; see also Impagliazzo et al. [44] and the survey of Lokshtanov et al. [54]. This conjecture states that, roughly speaking, there is no algorithm solving 3-SAT in time sub-exponential in the number of variables.

Structural Parameters. Let $G = (V, E)$ be a graph. A set $C \subseteq V$ is *vertex cover* of G if $G \setminus C$ is an edgeless graph. The *vertex cover number* $\mathrm{vc}(G)$ is the minimum size vertex cover in G.

The *tree-depth* of a graph $G = (V, E)$, denoted $\mathrm{td}(G)$, is defined as follows. It is 1 if G has only a single vertex, $1 + \min_{v \in V} \mathrm{td}(G - v)$ if G is connected, and otherwise it is the maximum tree-depth of connected components of G.

Definition 2.1 (Tree decomposition). A *tree decomposition* of a graph $G = (V, E)$ is a triple $\mathcal{T} = (T, \beta, r)$, where T is a tree rooted at node r and $\beta\colon V(T) \to 2^V$ is a mapping that satisfies:

(1) $\bigcup_{x \in V(T)} \beta(x) = V$;
(2) For every $\{u, v\} \in E$ there exists a node $x \in V(T)$, such that $u, v \in \beta(x)$;
(3) For every $u \in V$ the nodes $\{x \in V(T) \mid u \in \beta(x)\}$ form a connected subtree of T.

To distinguish between the vertices of a tree decomposition and the vertices of the underlying graph, we use the term *node* for the vertices of a given tree decomposition.

The *width* of a tree decomposition $\mathcal{T}$ is $\max_{x \in V(T)} |\beta(x)| - 1$. The *tree-width* of a graph G, denoted $\mathrm{tw}(G)$, is the minimum width of a tree decomposition of G over all such decompositions.

3 ANONYMOUS REFUGEES

In our simplest model of refugee housing, we assume refugees to be non-strategic and concern only the preferences of inhabitants. In this sense, the refugees are, from the viewpoint of inhabitants, anonymous, and the preferences only takes into account the number of refugees in the neighbourhood of each inhabitant. Similar preferences have already been studied in different problems such as anonymous hedonic games [11, 12, 21].

We formally capture this setting in the computational problem called the Anonymous Refugees Housing problem (ARH for short). A preference of every inhabitant $i \in I$ is a set $A_i \subseteq [\deg(\iota(i))]_0$ of the approved numbers of refugees in the neighbourhood. Our goal is to decide whether there is a housing $\pi\colon R \to V_U$ that respects the preferences of all inhabitants.

Definition 3.1. A housing $\pi\colon R \to V_U$ is called *inhabitant-respecting* if for every $i \in I$ we have $|N(\iota(i)) \cap V_\pi| \in A_i$.

If the approval set A_i for an inhabitant $i \in I$ consists of consecutive numbers, we say that the inhabitant i approves an interval.

Example 3.2. Let the topology be a cycle with four vertices. There are two inhabitants assigned to neighbouring vertices. One of these inhabitants, call her h_1, has approval set $A_{h_1} = \{0, 1\}$, and the second one, say h_2, is not approving any refugees in his neighbourhood, that is, $A_{h_2} = \{0\}$. We have $R = \{r\}$. The only valid housing is next to the inhabitant h_1 as housing r in the neighbourhood of h_2 clearly does not respect his preference. Also note that in this particular example, all the inhabitants approve intervals.

As our first result, we observe that even in a very simple settings, it is not guaranteed that any inhabitant-respecting refugees housing exists.

Proposition 3.3. *There is an instance of the ARH problem with no inhabitant-respecting refugees housing even if all inhabitants approve intervals.*

To prove Proposition 3.3, assume an instance with one inhabitant i and two refugees r_1 and r_2. Let the topology be K^3, the inhabitant i be assigned to an arbitrary vertex, and let $A_i = \{0\}$. There are exactly two possible refugees housings and in any of them the inhabitant i has two neighbouring refugees; therefore, there is no inhabitant-respecting housing.

In the previous example, we used the fact that the inhabitant i does not approve any refugees in his neighbourhood. We call such inhabitants *intolerant*. Despite the fact that the instance does not have an inhabitant-respecting housing even if $A_i = \{1\}$, we observe that intolerant inhabitants can be safely removed.

Proposition 3.4. *Let $\mathcal{I} = (G, I, R, \iota, (A_i)_{i\in I})$ be an instance of the ARH problem, $j \in I$ be an inhabitant with $A_j = \{0\}$, and $F_j = \{\iota(j)\} \cup U_j$. $\mathcal{I}$ admits an inhabitant-respecting housing iff the instance $\mathcal{I}' = (G \setminus F_j, I \setminus \{j\}, R, \iota, (A_i)_{i\in I\setminus\{j\}})$ admits an inhabitant-respecting housing.*

Proof. Let $\mathcal{I}$ be a *yes*-instance, let $j \in I$ be an inhabitant with $A_j = \{0\}$, and let π be an inhabitant-respecting refugees housing. Since π is an inhabitant-respecting housing, there is no refugee in the neighbourhood of j, so π is a solution even for $\mathcal{I}'$.

In the opposite direction, let $\mathcal{I}'$ be a *yes*-instance and π' be an inhabitant-respecting housing in $\mathcal{I}'$. As π' houses all refugees to $V' = V \setminus \{N(\iota(j))\}$ the π' is also a solution for $\mathcal{I}$. □

Due to the definition of approval sets, inhabitants without unoccupied neighbourhood are necessarily assumed intolerant and therefore can be safely removed by Proposition 3.4. Hence, we assume only instances without intolerant inhabitants where every inhabitant has at least one unoccupied vertex in her neighbourhood.

Proposition 3.5. *Let $\mathcal{I} = (G, I, R, \iota, (A_i)_{i\in I})$ be an instance of the ARH problem and $\{u, v\} \in E(G)$ be an edge such that either $u, v \in V_I$ or $u, v \in V_U$. Then $\mathcal{I}$ admits an inhabitant-respecting housing iff the instance $\mathcal{I}' = ((V(G), E(G) \setminus \{\{u, v\}\}), I, R, \iota, (A_i)_{i\in I})$ admits an inhabitant-respecting housing.*

Proposition 3.5 directly implies that all graphs assumed in this section are naturally bipartite.

Theorem 3.6. *Every instance of the ARH problem where the topology is a graph of maximum degree 2 can be solved in polynomial-time.*

Proof. Our algorithm is based on the dynamic programming approach combined with the gradual elimination of inhabitants' approval sets and exhaustive application of Proposition 3.4. Observe that graph of maximum degree 2 is a collection of paths and cycles [30]. We first introduce an algorithm that solves the problem on paths and then show how to improve the algorithm to solve cycles.

Let the topology be a path $P = v_1 v_2 \ldots v_k$, $k \geq 3$, and suppose that the vertex v_1 is occupied by an inhabitant $i \in I$. We distinguish two cases based on A_i and show how the algorithm proceed. If $A_i = \{1\}$, then we have to house a refugee on v_2. However, this adds one refugee in the neighbourhood of the inhabitant j occupying the vertex v_3. To capture this, we decrease the value of all elements in A_j. If there are any negative numbers in A_j after this operation, we remove all of them from the list. Then we delete v_1 and v_2 from P, decrease $|R|$ by one, and solve the problem for $P' = v_3 \ldots v_k$. If $A_i = \{0, 1\}$, we have to try both possibilities. That is, we run the algorithm once with $A_i = \{0\}$ and once with $A_i = \{1\}$. If any run of the algorithm finds a solution, we also have a solution for the original instance.

The described algorithm is exponential in the worst case. To improve the running time, we tabularise the computed partial solutions. Our dynamic programming table *DP* has three dimensions: the first for an inhabitant, the second for an actual value of $|R|$, and the third for a shape of approval set. The stored value is either *yes* or *no* depending on whether the combination of indices yields to a inhabitant-respecting housing. There are $O(n)$ inhabitants, the value of $|R|$ is also in $O(n)$, and there are at most 2 different approval sets possible for each inhabitant on the path. Therefore, the size of the table is at most $O(n^2)$, which is also the running time of our algorithm.

If the vertex v_1 is unoccupied, we can add the dummy vertex v_0 as a neighbour of v_1, make it occupied by an inhabitant with approval set $\{0, 1\}$ and run the preceding algorithm.

For cycles, we identify, based on approval sets, the best vertex to break this cycle and turning it into a path. In particular, if there is an inhabitant with the approval set $\{2\}$, we delete this vertex together with his two neighbours and obtain a path. In all other cases, we try all (at most three) ways of resolving his neighbourhood. To do so, we run an algorithm similar to the one from the beginning of this proof. □

Unfortunately, as the following theorem states, the bounded-degree condition from Theorem 3.6 cannot be relaxed any more.

Theorem 3.7. *The ARH problem is NP-complete even if the topology is a graph of maximum degree* 3 *and all inhabitants approve intervals.*

Proof. Given a housing π, it is easy to verify in polynomial time whether π is inhabitant-respecting by enumerating all inhabitants and comparing their neighbourhood with approval lists. Thus, ARH is indeed in NP.

For NP-hardness, we present a polynomial-time reduction from a variant of the 2-Balanced 3-SAT problem which is known to be NP-complete [15, 38, 68]. In this variant of 3-SAT, we are given a propositional formula φ with n variables $x_1, \ldots, x_n$ and m clauses $C_1, \ldots, C_m$ such that each clause contains at most 3 literals and every variable appears in at most 4 clauses – at most twice as a positive literal and at most twice as a negative literal. Later in this paper, we will refer to this reduction as *basic reduction*.

We construct an equivalent instance $\mathcal{I}$ of ARH as follows. We represent every variable x_i by a single *variable gadget* X_i that is a path $t_i v_i f_i$. The vertex v_i is occupied by an inhabitant with approval set $\{1\}$. All other vertices are empty and we call the vertex t_i the *t-port* and the vertex f_i the *f-port*. Every *clause* C_j is represented by a single vertex c_j occupied by an inhabitant h_j who approves the interval $[1, |C_j|]$ and is connected to the t-port of the variable gadget X_i if the variable x_i occurs as a positive literal in C_j and to the f-port of X_i if x_i occurs as a negative literal in C_j, respectively. To complete the reduction, we set $|R| = n$.

For the correctness of the construction, let φ be a satisfiable 2-Balanced 3-SAT formula and α be a truth assignment. For every variable x_i, we house a refugee at v_i^1 if $\alpha(x_i) = 1$ and at v_i^3 if $\alpha(x_i) = 0$, respectively. This housing is clearly a solution of $\mathcal{I}$ as every inhabitant assigned to the variable gadget is clearly satisfied, every inhabitant h_j is also satisfied since α is a satisfying assignment, and we housed 1 refugee for every variable gadget.

In the opposite direction, observe that there is exactly one refugee assigned to every variable gadget and, thus, in every assignment π there is no variable gadget X_i such that the t-port and the f-port are occupied at the same time. Hence, we can set $\alpha(x_i)$ equal to 1 if and only if the t-port is occupied by a refugee. Clearly, α is a truth assignment as π has to satisfy each inhabitant occupying clause vertex.

By definition, every clause contains at most 3 literals, and thus the degree of every vertex c_j, $j \in [m]$, is at most 3. For every variable gadget X_i, $i \in [n]$, the vertex v_i^2 has degree 2 and both t-port and f-port have degree at most 3 –

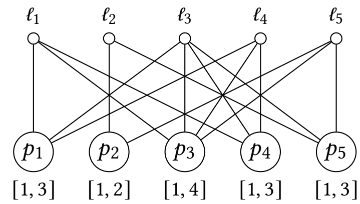

Fig. 1. An illustration of the construction used in the proof of Theorem 3.8.

they are adjacent to v_i^2 and at most two vertices representing clauses. Hence, the bounded-degree condition holds and the construction can be clearly done in polynomial time, finishing the proof. □

Since the above results clearly show that the problem is very hard even in simple settings, we turn our attention to the parameterised complexity of the ARH problem. In particular, we study the problem's complexity from the viewpoint of natural parameters, such as the number of refugees, the number of inhabitants, the number of empty vertices, and various structural parameters restricting the shape of the topology.

Theorem 3.8. *The ARH problem is W[2]-hard parameterised by the number of refugees $|R|$ even if all inhabitants approve intervals. Moreover, unless ETH fails, there is no algorithm that solves ARH in $f(|R|) \cdot n^{o(|R|)}$ time for any computable function f.*

Proof. We reduce from the Dominating Set problem, which is known to be W[2]-complete [31] and, unless ETH fails, cannot be solved in $f(k) \cdot n^{o(k)}$ time for any computable function f [25]. The instance $\mathcal{I}$ of Dominating Set consists of a simple undirected graph $G = (V, E)$ and an integer $k \in \mathbb{N}$. The goal is to decide whether there is a set $D \subseteq V$ of size at most k such that each vertex $v \in V$ is either in D or at least one of its neighbours is in D.

We construct an equivalent instance $\mathcal{I}'$ of the ARH problem as follows. We start by defining the topology. For each vertex $v \in V$ we add two vertices ℓ_v and p_v. The vertex ℓ_v represents the original vertex and is intended to be free for refugees. The vertex p_v is occupied by an inhabitant h_v with $A_{h_v} = [1, |N[v]|]$. This inhabitant ensures that there is at least one refugee housed in the closed neighbourhood of p_v. The edge set of the topology G' is $\bigcup_{v \in V} \{\{p_v, \ell_w\} \mid w \in N_G[v]\}$. To complete the construction, we set $|R| = k$. For an overview of our construction, we refer the reader to Figure 1.

Let $\mathcal{I}$ be a *yes*-instance and D be a dominating set of size k. For every vertex $v \in D$, we house a refugee in the vertex ℓ_v. Since D was a dominating set of size k, in the closed neighbourhood of every $v \in V$ in G there is at least one vertex $u \in D$. Therefore, for every inhabitant h_v, there is at least one refugee in his neighbourhood and $\mathcal{I}'$ is indeed a *yes*-instance.

In the opposite direction, let $\mathcal{I}'$ be a *yes*-instance and π be a solution housing. We set D to $\{v \in V \mid \pi^{-1}(\ell_v) \neq \emptyset\}$. Due to the definition of approved intervals of the inhabitants, it holds for every $v \in V$ either v or at least one of his neighbours is in D, as otherwise the inhabitant h_v would not be respected.

To complete the proof, we recall that $|R| = k$ and, hence, the presented reduction is indeed a parameterised reduction. Moreover, assume that there is an algorithm $\mathcal{A}$, that solves ARH in $f(|R|) \cdot |\mathcal{I}'|^{o(|R|)}$ time. Then we can reduce an instance of the Dominating Set problem to the instance of the ARH problem, solve the reduced instance using algorithm $\mathcal{A}$ and reconstruct a solution of the original instance. This is an algorithm for Dominating Set running in $f(k) \cdot n^{o(k)}$ time, which contradicts ETH. □

We complement Theorem 3.8 with an algorithm that runs in time matching the lower-bound given in this theorem.

Proposition 3.9. *The ARH problem can be solved in $n^{O(|R|)}$ time, where $n = |V(G)|$. That is, ARH is in XP parameterised by the number of refugees.*

Proof. Our algorithm is a simple brute-force. Let $V_U = V(G) \setminus V_I$ be the number of empty vertices and let $n = |V|$. Note that $|V_U| \leq n$. We try all subsets of V_U of size $|R|$ and for each such subset, we check in linear time whether the housing is inhabitant-respecting. This gives us the total running time $n^{O(|R|)}$. □

As the number of refugees is not a parameter promising tractable algorithms even if all inhabitants approve intervals, we turn our attention to the case where the number of inhabitants is small. Our algorithm is based on integer linear programming formulation of the problem and we use the following result of Eisenbrand and Weismantel [34].

Theorem 3.10 ([34, Theorem 2.2]). *Integer linear programme $\mathbb{A}x \leq b$, $x \geq 0$, with n variables and m constraints can be solved in*

$$(m\Delta)^{O(m)} \cdot ||b||_\infty^2$$

time, where Δ is an upper-bound on all absolute values in $\mathbb{A}$.

Theorem 3.11. *If all inhabitants approve intervals, then the ARH problem is fixed-parameter tractable parameterised by the number of inhabitants $|I|$.*

Proof. We solve the ARH problem using an integer linear programming formulation of the problem. We introduce one binary variable x_v for every empty vertex $v \in V_U$ representing if a refugee is housed on v or not. Next, we add the following constraints.

$$\forall i \in I\colon \sum_{v \in N_G(\iota(i))} x_v \geq \mathrm{low}(i) \tag{1}$$

$$\forall i \in I\colon \sum_{v \in N_G(\iota(i))} x_v \leq \mathrm{high}(i) \tag{2}$$

$$\sum_{v \in V_U} x_v = |R|, \tag{3}$$

where for an inhabitant $i \in I$ the value $\mathrm{low}(i)$ stands for lower-end and $\mathrm{high}(i)$ stands for upper-end of the approved interval by inhabitant i, respectively. Equations (1) and (2) ensure that the number of refugees in the neighbourhood of each inhabitant is in its approved interval, while Equation (3) ensures that all refugees are housed somewhere. Using Theorem 3.10 we see that the given integer programme can be solved in time $|I|^{O(|I|)} \cdot n^{O(1)}$, as $m = 2|I| + 1$, $\Delta = 1$, and $||b||_\infty \leq n$. That is, ARH is in FPT parameterised by the number of inhabitants $|I|$. □

Note that it would be possible to provide a different ILP formulation of the problem and use the famous theorem of Lenstra Jr. [52] to show membership in FPT, however, this would yield to an algorithm with much worse (i. e., doubly-exponential) running-time.

The result from Theorem 3.11 cannot be easily generalised to the case with inhabitants approving general sets. However, we can show that if the number of intervals in each approval set is bounded, the problem is still fixed-parameter tractable.

Theorem 3.12. *The ARH problem is fixed-parameter tractable when parameterised by the combined parameter the number of inhabitants $|I|$ and the maximum number of disjoint intervals δ in the approval sets.*

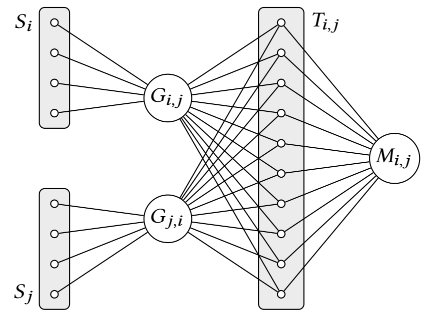

Fig. 2. An overview of the construction used in the proof of Theorem 3.13. The sets S_i, S_j, and $T_{i,j}$ consist of unoccupied vertices, and there are sequentially n, n, and mn^2 of them in each set. The vertex $G_{i,j}$ is occupied by an inhabitant with an approval set $\{\epsilon_{i,j}(\{u,v\})\cdot n^2+\nu_i(u) \mid \{u,v\}\in E_{i,j}\wedge u\in V_i\}$ and the vertex $G_{j,i}$ is occupied by an inhabitant with an approval set $\{\epsilon_{i,j}(\{u,v\})\cdot n^2+\nu_j(v) \mid \{u,v\}\in E_{i,j}\wedge v\in V_j\}$. The inhabitant occupying the vertex $M_{i,j}$ approves the set $\{t\cdot n^2 \mid t\in[m]\}$.

Proof. We start our algorithm by guessing an effective interval for every inhabitant $i\in I$. There are at most $\delta^{|I|}$ such guesses and for each guess we run the integer programme from the proof of Theorem 3.11. The overall running time of this algorithm is $\delta^{|I|}|I|^{O(|I|)}\cdot n^{O(1)}$. □

In our next result, we show that the parameter δ from Theorem 3.12 cannot be dropped while keeping the problem tractable.

Theorem 3.13. *The ARH problem is W[1]-hard parameterised by the number of inhabitants $|I|$.*

Proof. Our reduction is very similar to the one of Knop et al. [48, Theorem 3.1]. We reduce from the Multicoloured Clique problem where we are given a k-partite graph $G=(V_1\cup\cdots\cup V_k,E)$ and the goal is to find a complete subgraph with k vertices such that it contains a vertex from every V_i, $i\in[k]$. Multicoloured Clique is known to be W[1]-hard with respect to k [37]. We may assume that every colour class V_i, $i\in[k]$, is of size n. By $E_{i,j}$ we denote the set of edges between colour classes V_i and V_j, that is, $E_{i,j}=\{\{u,v\}\mid u\in V_i\wedge v\in V_j\}$ and we may assume that for every pair of distinct $i,j\in[k]$ we have $|E_{i,j}|=m$.

We begin the reduction by fixing a bijection $\nu_i\colon V_i\to[n]$ for every $i\in[k]$ and a bijection $\epsilon_{i,j}\colon E_{i,j}\to[m]$ for every pair of distinct $i,j\in[k]$. Next, for every colour class V_i we add a *vertex-selection gadget* S_i consisting of n vertices and leave these vertices empty. The number of refugees housed on the vertices of S_i will correspond to a vertex in V_i that is part of the clique. Then, for every $E_{i,j}$, $i,j\in[k]$, we add a set $T_{i,j}$ with $m\cdot n^2$ empty vertices and connect these to the vertex $M_{i,j}$. The vertex $M_{i,j}$ is occupied by an inhabitant with approval set $\{tn^2\mid t\in[m]\}$ and we call $T_{i,j}$ an *edge-selection gadget*. Similarly to the vertex selection gadget, the number of refugees assigned to $T_{i,j}$ will correspond to the edge selected for the solution. To ensure that the choice performed in the vertex-selection gadgets and the edge-selection gadgets is compatible, we introduce two vertices $G_{i,j}$ and $G_{j,i}$ for every $E_{i,j}$, $i,j\in[k]$ and $i\neq j$. The vertex $G_{i,j}$ is occupied by an inhabitant with approval set $\{\epsilon_{i,j}(e)\cdot n^2+\nu_i(v)\mid e\in E_{i,j}\wedge v\in e\wedge v\in V_i\}$ and is adjacent to every vertex in S_i, and the vertex $G_{j,i}$ is occupied by an inhabitant with approval set $\{\epsilon_{i,j}(e)\cdot n^2+\nu_j(v)\mid e\in E_{i,j}\wedge v\in e\wedge v\in V_j\}$ and is adjacent to every vertex in S_j. To complete the construction, we set $|R|=\binom{k}{2}\cdot m\cdot n^2+k\cdot n$ and introduce the same number of auxiliary vertices of degree 0 which are intended for the remaining refugees not assigned to vertex- and edge-selection gadgets. For an overview of our construction, please refer to Figure 2.

For correctness, let $\mathcal{I} = (G, k)$ be a *yes*-instance and $v_i, \dots, v_k$, where $v_i \in V_i$, be vertices that form a clique in G. For every $i \in [k]$ we assign $\nu_i(v_i)$ refugees to empty vertices of S_i and for each pair of distinct $i, j \in [k]$, we house $\epsilon_{i,j}(\{v_i, v_k\}) \cdot n^2$ refugees on empty vertices of $T_{i,j}$. The remaining refugees are assigned to the auxiliary vertices. The only inhabitants occupy the vertices $M_{i,j}$, $G_{i,j}$, and $G_{j,i}$. The inhabitants of $M_{i,j}$ are easily satisfied since we assign some multiple of n^2 to every $T_{i,j}$. An inhabitant in $G_{i,j}$ is adjacent to $\epsilon_{i,j}(\{v_i, v_k\}) \cdot n^2$ refugees from $T_{i,j}$ and $\nu_i(v_i)$ refugees from S_i while $G_{j,i}$ is adjacent to $\epsilon_{i,j}(\{v_i, v_k\}) \cdot n^2$ refugees from $T_{i,j}$ and $\nu_j(v_j)$ refugees from S_j, respectively. This complies with their approval set.

In the opposite direction, let the equivalent ARH instance $\mathcal{I}'$ be a *yes*-instance and π be an inhabitant respecting housing in $\mathcal{I}'$. Due to the inhabitants of $M_{i,j}$, where $i, j \in [k]$, there is some positive multiple of n^2 refugees assigned to every $T_{i,j}$ that corresponds to some edge $e \in E_{i,j}$ in the original graph. Moreover, due to inhabitant on $G_{i,j}$, the number of refugees assigned to S_i corresponds to the identification of some vertex $v \in V_i$ that is necessarily incident to e. The same holds for $G_{j,i}$.

It is not difficult to see that the vertices $G_{i,j}$ and $G_{j,i}$ together with $M_{i,j}$, where $i, j \in [k]$ and $i \neq j$, form a vertex cover of the topology G' as S_i and $T_{i,j}$ for each pair of distinct $i, j \in [k]$ are sets of independent vertices. It follows that $\text{vc}(G') = \binom{k}{2} + k(k-1) = \mathcal{O}(k^2)$ and thus the presented reduction is indeed a parameterised reduction, finishing the proof. □

By careful guessing, we can prove that for the combined parameters the number of refugees and the number of inhabitants, we may obtain fixed-parameter tractable algorithm.

Lemma 3.14. *The ARH problem is fixed-parameter tractable when parameterised by the number of refugees $|R|$ and the number of inhabitants $|I|$ combined.*

Proof. We can guess a particular neighbourhood of every refugee, and for every such guess, verify whether it is correct. Let $I_{r_i} \subseteq I$ be a neighbourhood guessed for a refugee $r_i \in R$. If there is no empty vertex v with $\deg(v) = |I_{r_i}|$ at the intersection of neighbourhoods of vertices in I_{r_i}, we discard this guess. Otherwise, we house the refugee r_i on v and continue with r_{i+1}. If all refugees are housed, we simply check whether the housing respects the approval sets of all inhabitants.

Overall, there are $(2^{|I|})^{|R|} = 2^{|I| \cdot |R|}$ such guesses and every guess can be verified in polynomial-time. Therefore, the running time of our algorithm is $2^{|I| \cdot |R|} \cdot n^{\mathcal{O}(1)}$, that is, ARH is in FPT with respect to $|R|$ and $|I|$ combined. □

The last assumed natural parameter is the number of empty vertices the refugees can be assigned to. Note that $|V_U| \geq |R|$. This parameterisation yields, in contrast to Theorem 3.8, to a simple algorithm running in FPT time which is, although its simplicity, optimal assuming the Exponential Time Hypothesis.

Theorem 3.15. *The ARH problem can be solved in $2^{\mathcal{O}(|V_U|)} \cdot n^{\mathcal{O}(1)}$ time and, unless ETH fails, there is no algorithm solving ARH in $2^{o(|V_U|)} \cdot n^{\mathcal{O}(1)}$ time even if all inhabitants approve intervals.*

Proof. First, observe that if $|V_U| \leq |R|$, then the instance is trivially *no*-instance, as it is not possible to house all refugees. Hence, we can enumerate all subsets of $|V_U|$ of size $|R|$ and, for every guess, check whether all inhabitants are satisfied with this assignment. This can be done in $2^{\mathcal{O}(|V_U|)} \cdot n^{\mathcal{O}(1)}$ time, that is, ARH is in FPT with respect to the number of empty vertices $|V_U|$.

For the running-time lower-bound, we recall that 3, 4-SAT cannot be solved in $2^{o(n+m)}$ time unless ETH fails [43, 44]. Assume to the contrary, there is an algorithm $\mathcal{A}$ that solves ARH in $2^{o(|V_U|)} \cdot n^{\mathcal{O}(1)}$. Then, for every 3, 4-SAT formula

φ we can use the basic reduction from Theorem 3.7, which can be clearly done in polynomial time, to the equivalent instance $\mathcal{I}$ of the ARH problem, solve $\mathcal{I}$ in $2^{o(|V_U|)} \cdot n^{\mathcal{O}(1)}$ time and then reconstruct a solution for φ. Overall, this gives us an algorithm running in $2^{o(|V_U|)} \cdot n^{\mathcal{O}(1)} \subseteq 2^{o(n)} \subseteq 2^{o(n+m)}$ time for 3, 4-SAT which contradicts ETH. □

In the remainder of this section, we present complexity results concerning various structural restrictions of the topology. Arguably, the most prominent structural parameter is the tree-width of a graph that, informally speaking, expresses its tree-likeness, and which is usually small in real-life networks [58]. Unfortunately, we can show the following stronger intractability result.

Theorem 3.16. *The ARH problem is W[1]-hard parameterised by the tree-depth* $\mathrm{td}(G)$ *of the topology* G *and, unless ETH fails, there is no algorithm solving ARH in* $f(k) \cdot n^{o(k/\log k)}$ *time, where* $k = \mathrm{td}(G)$*, for any function* f*.*

Proof. We reduce from the Unary Bin Packing problem. Here, we are given a bin capacity B, a set of items $A = (a_1, \ldots, a_n)$ and number of bins k. Our goal is to decide whether there is an assignment β of all items to bins with respect to bin capacity. Unary Bin Packing is known to be W[1]-hard with respect to the number of bins and cannot be solved in time $f(k) \cdot n^{o(k/\log k)}$ for any function f [46].

Our construction of an equivalent instance $\mathcal{I}'$ of the ARH problem is as follows. First, for every $i \in [k]$ we create a *bin vertex* B_i and make it occupied by an inhabitant b_i with $A_{b_i} = [B]_0$. Note that these inhabitants approve the whole interval from 0 to B. Next, we add *items gadgets* to map the items to bins. The single item gadget for some $a_j \in A$ is a star with a_j leaves and centre occupied by an inhabitant c_j with $A_{c_j} = \{0, a_j\}$. This ensures that in the solution either all leaves are occupied by refugees or none of the leaves are. We add an item gadget X_j^i for every $i \in [k]$ and every $a_j \in A$ and add an edge connecting bin vertex B_i with all leaves of item gadgets X_j^i, where $i \in [k]$ and $j \in [n]$. Finally, we must ensure that every item is assigned to exactly one bag. This is ensured by a *guard vertex* G_i for every item $a_i \in A$. This vertex is occupied by an inhabitant g_i with $A_{g_i} = \{a_i\}$ and is connected to all leaves of item gadgets X_i^j, where $j \in [k]$. The number of refugees in our instance is $|R| = \sum_{a_i \in A} a_i$. For an illustration of our construction, we refer the reader to Figure 3.

For the correctness, let $\mathcal{I}$ be a *yes*-instance of the Unary Bin Packing problem and β be a solution assignment. For every $a_i \in A$, we house a_i refugees to the leaves of the item gadget $X_i^{\beta(a_i)}$. Since $\mathcal{I}$ is a *yes*-instance, every bin vertex has at most B refugees in the neighbourhood. Moreover, each item gadget is either empty or full, in the neighbourhood of every guard inhabitant g_i there is exactly a_i neighbouring refugees, and, finally, all refugees are housed. Thus, $\mathcal{I}'$ is also a *yes*-instance.

In the opposite direction, let $\mathcal{I}'$ be a *yes*-instance and π be a solution housing. Due to definition of the approval sets, every item gadget is either full or empty. In our construction, there are k copies of item gadgets for every item $a_i \in A$ and guard vertices secure that exactly one copy is full. Moreover, the bin vertices accept at most B refugees in their neighbourhood. We recall that $|R| = \sum_{a_i \in A} a_i$. Hence, we define the solution assignment β for $\mathcal{I}$ as $\beta(a_i) = j$, where $j \in [k]$ and X_i^j is full.

Finally, we discuss the bound on the parameter value. It is not hard to see that the construction have tree-depth $\mathcal{O}(k)$; By removing vertices $B_1, \ldots, B_k$, we obtain n connected components $C_1, \ldots, C_n$ and in every C_i it is enough to remove the vertex G_i to obtain a collection of stars, which have tree-depth at most 2. Also note that algorithm running in $f(k') \cdot |\mathcal{I}'|^{o(k'/\log k')}$ time for ARH would yield, together with our parameterised reduction, to an algorithm running in $f(k) \cdot n^{o(k/\log k)}$ time for Unary Bin Packing, which contradicts ETH. □

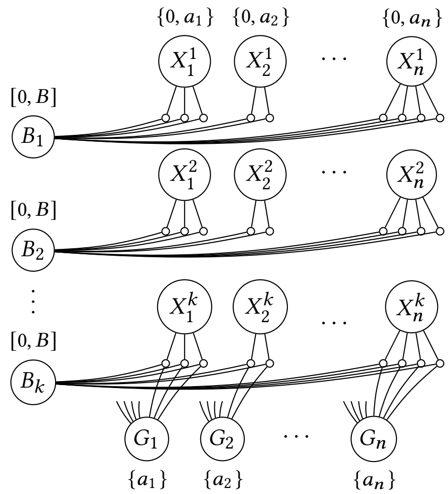

Fig. 3. Overview of the construction used in the proof of Theorem 3.16. Every guard vertex G_i is connected to the leaves of each element gadget X_i^j, where $j \in [k]$.

It is well known, and easy to see, that the tree-width of a graph is at most its tree-depth. Hence, we directly obtain the following result.

Corollary 3.17. *The ARH problem is W[1]-hard parameterised by the tree-width* $\mathrm{tw}(G)$ *of the topology* G.

It is well-known, an easy to see, that the tree-width of a graph is at most its vertex-cover number. Hence, due to Theorem 3.19, we directly obtain the following result for tree-width.

Corollary 3.18. *The ARH problem is W[1]-hard parameterised by the tree-width* $\mathrm{tw}(G)$ *of the topology* G.

Many problems that are computationally hard with respect to tree-width are studied from the viewpoint of less restricted parameters. Vertex cover number is a frequent representative of such parameters [24, 49, 55]; however, in the ARH problem, not even this restriction of the topology leads to a tractable algorithm.

Theorem 3.19. *The ARH problem is W[1]-hard parameterised by the vertex cover number* $\mathrm{vc}(G)$ *of the topology.*

Proof. Although it was not stated formally, it can be seen that the construction used to prove Theorem 3.13 has not only parameter-many inhabitants, but these agents also forms a vertex cover of the topology. Therefore, we directly obtain W[1]-hardness for the setting with bounded vertex cover number. □

Nevertheless, if we additionally restrict the approval sets, we obtain the following algorithmic result.

Theorem 3.20. *The ARH problem is fixed-parameter tractable parameterised by the vertex cover number* $\mathrm{vc}(G)$ *if all inhabitants approve intervals.*

Proof. Let $M \subseteq V$ be a minimum size vertex cover of G and let $k = |M|$. It is not hard to see that, due to Proposition 3.5, for each connected component C_i of G all vertices in $M \cup C_i$ are either empty vertices or occupied by inhabitants. By C_I we denote components with vertex cover occupied by inhabitants and by C_R the components where the vertex cover consists of empty vertices, respectively.

First, we guess the number $k' \leq k$ of refugees assigned to components of C_R. Now, we try all $2^{k'}$ subsets and check for each subset whether all refugees are housed and whether the housing is inhabitant-respecting in C_R. At the same time, we try to find a housing of $|R| - k'$ refugees to components in C_I. To do so, we can use the algorithm from Theorem 3.11 as the number of inhabitants is at most k. Altogether, we obtain that the ARH problem is in FPT when parameterized by the vertex cover number of the topology if all inhabitants approve intervals. □

By the same argumentation used in the proof of Theorem 3.20, we obtain the following last result of this section.

Corollary 3.21. *The ARH problem is fixed-parameter tractable when parameterised by the vertex cover number* $\mathrm{vc}(G)$ *and the maximum number of disjoint intervals* δ *combined.*

4 FULLY HEDONIC PREFERENCES

Our second model of refugee housing improves upon the previous model by introducing individual preferences of refugees. Naturally, refugees are no longer anonymous and the identity of every particular refugee matters. The preferences of the inhabitants are again dichotomous, and for every inhabitant $i \in I$ the approval set is a subset of 2^R. Similarly, for a refugee $r \in R$, the approval set A_r is a subset of 2^I. Our goal is to find housing that conforms to the preferences of both groups.

Definition 4.1. A housing $\pi\colon R \to V_U$ is called *respecting* if for every $i \in I$ we have $N(\iota(i)) \in A_i$ and for every $r \in R$ we have $N(\pi(r)) \in A_r$.

In other words, a housing π is respecting if every inhabitant and every refugee approves its neighbourhood. We study the problem of deciding whether there is a respecting housing in the instance with hedonic preferences under the name Hedonic Refugees Housing (HRH for short).

Example 4.2. Let the topology be a cycle with four vertices. There are two inhabitants h_1 and h_2 assigned to neighbouring vertices and two refugees r_1 and r_2 to house. The approval set of inhabitant h_1 is $A_{h_1} = \{\{r_1\}, \{r_2\}\}$, that is, h_1 approves only one refugee in her neighbourhood regardless of the identity. The second inhabitant approves the set $A_{h_2} = \{\{r_2,\}, \{r_1, r_2\}\}$. In other words, the inhabitant h_2 is dissatisfied with having only the refugee r_1 in the neighbourhood; however, he is fine with neighbouring with both the refugees. For the refugees, we have $A_{r_1} = \{\{h_1\}\}$ and $A_{r_2} = \{\{h_2\}\}$. Housing r_1 in the neighbourhood of h_1 and r_2 in the neighbourhood of h_2 is clearly respecting.

Observe that, since both inhabitants and refugees have preferences only over the other set of individuals, we can remove all edges between empty and occupied vertices, respectively. Hence, all graphs assumed in this section are again bipartite.

Now, we show how the results from Section 3 carry over to the hedonic setting studied in this section. Our first theorem shows that the hedonic setting is also computationally hard on graphs of constant degree. The construction is very similar to the one used to prove Theorem 3.7.

Theorem 4.3. *The HRH problem is NP-complete even if the topology is a graph of maximum degree* 3.

Proof. Our reduction is very similar to the reduction used to prove Theorem 3.7. We again reduce from the 2-Balanced 3-SAT problem where every variable appears at most twice as positive literal and at most twice as negative literal, respectively. However, due to the different nature of the problem, we have to slightly change the variable gadget and approval sets of agents.

For every variable $x_i \in \varphi$, we introduce one variable gadget X_i. The variable gadget again consists of a path $P_i = v_i^1 v_i^2 v_i^3$ that is intended to represent the assignment of a variable x_i. Together with P_i, we add a refugee r_i and an inhabitant h_i. Let $C_{x_i} = \{C_{j_1}, \ldots, C_{j_4}\}$, where $j_1, j_2, j_3, j_4 \in [m]$, be a set of clauses in which the variable x_i appears. The refugee r_i approves the set $\{S \cup \{h_i\} \mid S \subseteq 2^{C_{x_i}}\}$ and the inhabitant h_i approves the set $\{\{r_i\}\}$. Moreover, we set $\iota(h_i) = v_i^2$.

Every clause $C_j \in \varphi$ is represented by a single vertex c_j. Let C_j consist of variables $x_{i_1} x_{i_2} x_{i_3}$. The vertex c_j is occupied by an inhabitant H_j with approval set $2^{\{x_{i_1}, x_{i_2}, x_{i_3}\}} \setminus \emptyset$ and for every $i \in \{i_1, i_2, i_3\}$ the vertex c_j is connected to v_i^1 if the variable x_i occurs as a positive literal in C_j and to v_i^3 if the variable x_i occurs as a negative literal in C_j. This finishes the construction.

For the correctness, let α be a satisfying assignment for φ. For every $x_i \in \varphi$, we house the refugee r_i to vertex v_i^1 if $\alpha(x_i) = 1$ and to vertex v_i^3 otherwise. Observe that this housing clearly respects each h_i and, since α is satisfying, there is at least one refugee in the neighbourhood of each H_j, as prescribed. In the opposite direction, let π be a correct housing for the equivalent HRH instance $\mathcal{I}'$. Due to the approval sets of h_i, $i \in [n]$, every refugee r_i is housed to one of the two positions of the variable gadget. Moreover, the approval set of inhabitant H_j, $j \in [m]$, which represents the clause of the original formula, ensures that there is at least one refugee in his neighbourhood. Hence, the following assignment

$$\alpha(x_i) = \begin{cases} 1 & \text{if } \pi(r_i) = v_i^1 \\ 0 & \text{otherwise} \end{cases}$$

indeed satisfy φ and the reduction can be done in polynomial time. □

From the proof of Theorem 4.3, we can easily distil the following general reduction from the ARH problem to the HRH problem. Let $\mathcal{I}$ be an ARH instance. For every empty vertex $v \in V_U$ we add into R' one refugee r_v with approval set $A_{r_v} = N(v)$. Next, for every inhabitant $i \in I$ we add a new inhabitant h_i approving the set $A_{h_i} = \{\binom{R_{h_i}}{a} \mid a \in A_i\}$, where $R_{h_i} = \{r_v \mid v \in N(v) \cap V_U\}$. To ensure that only $|R|$ refugees are housed, we extend the construction by a single star with $|V_E| - |R|$ leaves and occupy the centre of the star with an inhabitant G approving the set $\binom{R'}{|V_E|-|R|}$. Moreover, we have to add $\{G\}$ to the approval set of every refugee $r_v \in R'$. It is not difficult to see that the instances are indeed equivalent; however, the reduction is not polynomial-time, the approval sets can be at worst exponential in the number of empty vertices. Hence, the reduction works only in cases where the number of empty vertices is at most logarithmic in the size of the topology. Unfortunately, this is not the case for most of our polynomial-time reductions, however, we are able to show similar results using different techniques.

Theorem 4.4. *The HRH problem is W[1]-hard when parameterised by the combined parameter the vertex cover number* vc(G) *of the topology and the number of inhabitants* $|I|$.

Proof. We reduce from the W[1]-hard Multicoloured Clique problem [37]. We recall that here we are given a k-partite graph $G = (V_1 \cup \cdots \cup V_k, E)$ and the goal is to find a complete sub-graph with k vertices such that it contains a vertex from every V_i, $i \in [k]$.

Let $G = (V_1 \cup \cdots \cup V_k, E)$ be an instance of the Multicoloured Clique problem such that all colour classes V_i are of the same size n_G. We construct an equivalent instance $\mathcal{I}'$ of the HRH problem as follows. For every vertex set V_i, where $i \in [k]$, we introduce a *vertex-selection gadget* S_i which is a star with n_G leaves. We call an arbitrary leaf a *selection leaf*. This selection leaf serves for a vertex of colour i that should be part of the clique and is the only connection of the vertex-selection gadget with the rest of the topology. Let $\{v_i^1, \ldots, v_i^{n_G}\}$ be a set of vertices in the colour class V_i.

We introduce one refugee r_i^p for every v_i^p, $p \in [n_G]$ and one inhabitant s_i which is assigned to the centre c_i of S_i and approves the set $\{\{r_i^1, \dots, r_i^{n_G}\}\}$. Every refugee r_i^p approves the set $\{\{s_i\}\}$.

Then, for every pair of distinct colours $i, j \in [k]$, we introduce one additional *guard vertex* $g_{i,j}$ securing that there is an edge between vertices selected in incident vertex-selection gadgets. We connect this guard vertex $g_{i,j}$ to selection leaves of vertex-selection gadgets V_i and V_j. Moreover, we introduce an inhabitant $h_{i,j}$ assigned to $g_{i,j}$ with approval set $\{\{r_i^p, r_j^q\} \mid \{v_i^p, v_j^q\} \in E \wedge p, q \in [n_G]\}$. That is, $h_{i,j}$ approves exactly those pairs of vertices from V_i and V_j that are connected by an edge.

To be able to house any refugee to selection leaves, we have to extend their approval sets. Thus, for every refugee r_i^p, where $i \in [k]$ and $p \in [n_G]$, we add to the approval set the set $\{s_i\} \cup \{h_{i,j} \mid j \in [k] \setminus \{i\}\}$. This finishes the construction.

For the correctness, let $K = \{v_1^{j_1}, \dots, v_k^{j_k}\}$ be a multicoloured clique in G. For every vertex-selection gadget S_i, we house to the selection leaf the refugee $r_i^{j_i}$ and the refugees r_i^p, where $p \in [n_G] \setminus \{j_i\}$ arbitrarily to the remaining leaves of S_i. As vertices of K form a clique in G, there is an edge between each pair of vertices assigned to selection leaves; hence, the housing is correct.

In the opposite direction, suppose that $\mathcal{I}'$ is a *yes*-instance, π is a respecting housing and there is a pair of refugees r_i^p and r_j^q, where $i, j \in [k]$ are distinct and $p, q \in [n_G]$, assigned to selection leaves such that there is no corresponding $\{v_i^p, v_j^q\} \in E(G)$. Then $\{r_i^p, r_j^q\}$ is not in the approval set of the inhabitant $h_{i,j}$, which is a contradiction with π being a respecting housing.

Clearly, the removal of the vertex set $\{c_i \mid i \in [k]\}$ together with the set $\{g_{i,j} \mid i, j \in [k]\}$ leads to an edgeless graph. As the size of both sets is polynomial in k and all inhabitants are assigned to these vertices, we obtain that the presented reduction is indeed a parameterised reduction and the theorem follows. □

As a final result of this section, we prove that the HRH is NP-hard even for graphs of tree-width at most 3.

Theorem 4.5. *The HRH problem is para-NP-hard parameterised by the tree-width* $\mathrm{tw}(G)$ *of the topology.*

Proof. Our reduction is from the Channel Assignment problem which is known to be NP-hard even for graphs of tree-width at most 3 [59]. Here, we are given an undirected graph $G = (V, E)$, a function $\ell \colon E \to \mathbb{N}$, and a positive integer λ. The goal is to decide whether there exists a function $f \colon V \to [\lambda]$ such that for every $\{u, v\} \in E$ we have $|f(u) - f(v)| \geq \ell(\{u, v\})$. It should be noted that the problem is NP-hard even if λ and $\max_{e \in E} \ell(e)$ are constants.

Let $\mathcal{I} = (G, \ell, \lambda)$ be a Channel Assignment instance. The topology of an equivalent HRH instance $\mathcal{I}'$ is a graph G' which arises from G by subdividing all edges. We call the vertex created by subdivision of the edge $\{u, w\} \in E(G)$ of the original graph $v_{u,w}$. Next, we introduce λ refugees $r_v^1, \dots, r_v^\lambda$ for every vertex $v \in V(G)$. Then, for every vertex $v_{u,w}$, where $\{u, w\} \in E(G)$, we introduce one inhabitant $h_{u,w}$ and assign it to $v_{u,w}$. The approval set for every refugee r_v^i is $\{\{h_{u,v}\} \mid v_{u,w} \in N_{G'}(v)\}$ and for every inhabitant $h_{u,w}$ it is $\{\{r_u^i, r_w^j\} \mid |i - j| \geq \ell(\{u, w\})\}$. In this construction, there are λ candidates for every empty vertex, hence, we have to make sure that there is a house where to house remaining $\lambda - 1$ candidates. To secure this, we introduce for every $v \in V(G)$ star S_v with $\lambda - 1$ leaves, make the centre of S_v occupied by an inhabitant g_v approving all subsets of size $\lambda - 1$ of the set $\{r_v^1, \dots, r_v^\lambda\}$, and we add set $\{g_v\}$ to the approval set of every refugee r_v^i, $i \in [\lambda]$.

For the correctness, let f be a proper channel assignment for $\mathcal{I}$. For every vertex $v \in V(G)$, we house the refugee $r_v^{f(v)}$ to v and we house all the remaining refugees r_v^i, $i \in [\lambda] \setminus \{f(v)\}$ arbitrarily to empty vertices of S_v. Clearly, all assigned refugees are satisfied with this housing as well as the inhabitant g_v assigned to the centre of S_v. The same is true for inhabitants $h_{u,w}$, as f is a proper channel assignment and $h_{u,w}$ accept only pairs $\{r_u^i, r_w^j\}$ where $|i - j| \geq \ell(\{u, w\})$.

In the opposite direction, let π be a correct housing. The number of refugees to house is equal to the number of empty vertices of G', hence, every empty vertex is occupied. The inhabitants occupying vertices $v_{u,w}$ secure that for refugees r_u^i and r_w^j assigned to u and w, respectively, it holds that $|i - j| \leq \ell(\{u, w\})$, hence, we can set $f(v) = i$, where $r_v^i = \pi^{-1}(v)$.

We already claimed that the Channel Assignment problem is NP-hard even for the graphs of tree-width at most 3. As the edge subdivision do not increase tree-width, we have that G' is also of tree-width at most 3. Moreover, the reduction can be done in polynomial-time. This finishes the proof. ◻

5 DIVERSITY PREFERENCES

In the anonymous refugee housing, we are not assuming the preferences of individual refugees. Thanks to this property, the model is as simple as possible. The fully hedonic setting from Section 4 precisely captures preferences of both the refugees and the inhabitants. On the other hand, the fully hedonic model is not very realistic, as it is hard to acquaint all inhabitants with all refugees.

Hence, we introduce the third model of refugees housing, where both the inhabitants and the refugees are partitioned into types and agents from both groups have preferences over fractions of agents of each type in their neighbourhood.

Such a diversity goals, where agents are partitioned into types and the preferences of agents are based on the fraction of each type in their neighbourhood or coalition, was successfully used in many scenarios such as school choice [8–10], public housing [14, 42], hedonic games [19, 20, 23, 27, 40], multi-attribute matching [4], or employee hiring [66].

Before we formally define the computational problem of our interest, let us introduce further notation. Let $N = I \cup R$ be a set of agents partitioned into k types $T_1, \ldots, T_k$. For a set $S \subseteq N$, we define a *palette* as a k-tuple $\left(\frac{|T_i \cap S|}{|S|}\right)_{i \in [k]}$ if $|S| \geq 1$ and k-tuple $(0, \ldots, 0)$ if $S = \emptyset$. Given an agent $a \in N$, her approval set is a subset of the set $\left\{\left(\frac{|T_i \cap S|}{|S|}\right)_{i \in [k]} \mid S \subseteq 2^N\right\}$.

Definition 5.1. A housing $\pi\colon R \to V_U$ is called *diversity respecting* if for every inhabitant $i \in I$ the palette for the set $\{h \in I \mid \iota(h) \in N(\iota(i))\} \cup \{r \in R \mid \pi(r) \in N(\iota(i))\}$ is in A_i, and for every refugee $r \in R$ the palette for the set $\{h \in I \mid \iota(h) \in N(\pi(r))\} \cup \{r' \in R \mid \pi(r') \in N(\pi(r))\}$ is in A_r.

The Diversity Refugees Housing problem (DRH for short) then asks whether there is a diversity respecting housing π. Note that this time, we are not allowed to drop edges between two inhabitants or two refugees and, thus, the graphs assumed in this section are no longer bipartite.

Example 5.2. Let the topology be a cycle with four vertices. There are two agents of type T_1. One of these agents is an inhabitant h_1 approving $\{(1, 0), (1/2, 1/2)\}$ and the second one is a refugee r approving only agents of his own type, that is, $A_r = \{(1, 0)\}$. The type T_2 contains one inhabitant h_2 approving the set $\{(1, 0)\}$. Inhabitants are assigned such that they are neighbours. There are two possible housings for the refugee r. She can be either neighbour of h_1 or h_2. Since she accepts only agents of her own type in the neighbourhood, the only diversity respecting housing is next to inhabitant h_1.

Despite that the topology can be much complicated, we can show, using similar construction as in Theorem 3.7, that the tractability condition based on the bounded degree cannot be surpassed even in this model.

Theorem 5.3. *The DRH problem is NP-complete even if the topology is a graph of maximum degree* 3.

Proof. We reduce from the 2-Balanced 3-SAT where each variable appears at most twice positive and at most twice negative. Let φ be a 3,4-SAT formula. In an equivalent instance $\mathcal{I}'$ of DRH, we have $n+1$ types – one for every variable and an extra one for agents representing clauses. Let T_{n+1} be a *clause type*. For every variable x_i, we create a *variable gadget* which is a path $v_i^1 v_i^2 v_i^3$ on 3 vertices. We also introduce two agents – one refugee r_i and one inhabitant h_i. Both of them belong to type T_i. The inhabitant h_i is then assigned to the middle vertex of a path and approves (and requires) only agents of his own type in his neighbourhood. This secure that the refugee r_i is necessarily assigned to his variable gadget in each housing. The refugee requires in his neighbourhood an agent of his own type and at most 2 agents of clause type.

For every clause C_j, we create one vertex c_j and occupy it with an inhabitant g_j of type T_{n+1}. For every variable x_i which is part of C_j, we connect c_j to v_i^1 if x_i is as positive literal in C_j, and to v_i^3 if x_i appears as a negative literal in C_j. Let x_{i_1}, x_{i_2}, and x_{i_3} be three variables forming C_j. The inhabitant g_j then approves to have monotype neighbourhood with types i_1, i_2, or i_3, and neighbourhoods where these types are represented equally – either two of them or all three.

For the correctness, let α be a satisfying assignment for φ. For every variable x_i we house the only refugee of type T_i to the vertex v_i^1 if $\alpha(x_i) = 1$, or to the vertex v_i^2 if $\alpha(x_i) = 0$. It is easy to see that this housing is diversity respecting as every h_i has only one neighbour of his own type and, since α is satisfying, every g_j, $j \in [m]$, has at least one neighbouring refugee. In the opposite direction, let π be a diversity respecting housing. Then every refugee is housed to the neighbourhood of the inhabitant of their own type. For every c_j we have also one inhabitant g_j who ensures that there is at least one refugee in the neighbourhood. Hence, we can directly derive the truth assignment α from π. ◻

In Theorem 5.3 we exploit the number of types to ensure that every refugee is housed in the right house. Therefore, the number of types was as large as the number of empty vertices. In the following result, we show that the DRH problem is computationally hard even if the number of types is small.

Theorem 5.4. *The DRH problem is NP-complete even if there are only two types of agents.*

Proof. We show the NP-hardness by a reduction from the Set Cover problem which is known to be NP-complete [41]. Here, we are given a universe $U = \{u_1, \ldots, u_n\}$, a family $\mathcal{F}$ of subsets of U, and an integer $k \in \mathbb{N}$. The goal is to decide whether there is a sub-family $C \subseteq \mathcal{F}$ of size at most k such that $\bigcup_{C \in C} C = U$.

Given an instance $\mathcal{I} = (U, \mathcal{F}, k)$, we construct an equivalent instance $\mathcal{I}'$ of DRH as follows. For every element $u_i \in U$ we add one vertex v_i and assign to it an inhabitant h_i. The inhabitant h_i is of type T_1 and his approval set is $\{(0,1)\}$. Next, for every subset $F \in \mathcal{F}$, we create one vertex v_F that is adjacent to every v_i such that $u_i \in F$. To finalise the construction, we add k refugees $r_1, \ldots, r_k$ of type T_2 approving the set $\{(1,0)\}$.

For the correctness, let $\mathcal{I}$ be a *yes*-instance and $C = \{C_1, \ldots, C_k\}$ be a solution for $\mathcal{I}$. For every $i \in [k]$, we house the refugee r_i on the vertex corresponding to the set C_i. In this housing, every refugee is satisfied and, since C is a set cover, every $h_i \in I$ neighbours with at least one refugee. In the opposite direction, let π be a diversity respecting housing. We add to C a set $C_i \in C$ if and only if there is a refugee housed on the corresponding vertex v_{C_i}. Suppose that there is an element $u \in U$ which is not covered by C. Then there is an inhabitant which is not neighbour of any refugee. However, this could not be the case as π is diversity respecting. Hence, the reduction is correct and clearly can be done in polynomial-time. ◻

Note that the NP-hardness proved in Theorem 5.4 can be strengthened to a single type of agents; however, we find this situation not very natural in the context of DRH.

Additionally, it is known that the Set Cover problem is W[2]-complete and cannot be solved in $f(k) \cdot n^{o(k)}$ time, unless the ETH fails [26]. Therefore, from the construction used to prove Theorem 5.4, we directly obtain the following corollary.

Corollary 5.5. *The DRH problem is W[2]-hard when parameterised by the number of refugees $|R|$ even if there are only two types of agents and, unless ETH fails, there is no algorithm solving DRH in $f(R) \cdot n^{o(R)}$ time for any computable function f.*

6 CONCLUSIONS

We initiated the study of a novel model of refugee housing. The model mainly targets the situations where refugees need to be accommodated and integrated in the local community. This distinguishes us from the previous settings of refugee resettlement.

Our results identify some tractable and intractable cases of finding stable outcomes from the viewpoint of both the classical computational complexity and the finer-grained framework of parameterised complexity. To this end, we believe that other notions of stability inspired, for example, by the model of Schelling games of Agarwal et al. [2] or by exchange-stability of Alcalde [5], should be investigated.

Natural way to tackle the intractability of problems in computational social choice is to restrict the preferences of agents [36]. One such restriction that should be investigated, especially in the case of anonymous setting, are the single-peaked preferences [6] that were successfully used in similar scenarios; see, e.g., [16, 23, 71] or the survey of Elkind et al. [36]. Beside that, we are interested in the anonymous setting in which every inhabitant $i \in I$ approves an interval $[0, u_i]$, where $u_i \geq 0$ is an inhabitant-specific upper-bound on the number of refugees in her neighbourhood.

Finally, there are many notions measuring quality of an outcome studied in the literature in both the context of Schelling and hedonic games [2, 7, 35], and we believe that these notions should be investigated even in the context of refugee housing. In this line, the most prominent notion is the social-welfare of an outcome.

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