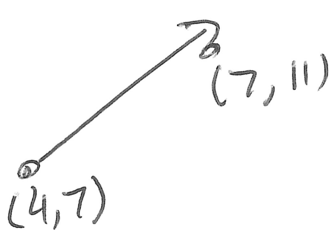
COP 4516 - 2D Geometry

✓ Vec Eq Line
✓ Line Intersect
✓ Line Seg Inter
Areas - parallelogram ✓, tria ✓, polygons ✓
Angles btw Vectors ✓, lines ✓
pt to line distance ✓
Intersections w/ circles, lines ✓
Convex Hull

* DIFF BTW THEORY + PRACTICE ⇒ FLOATING PT PRECISION

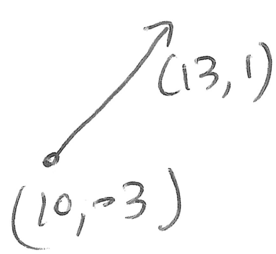
for 2 doubles a, b use fabs(a-b) < 1e-9 instead of a == b

Vector direction of movement. Not fixed in space



$3\vec{i} + 4\vec{j}$

$\vec{i}$: unit vector x dir
$\vec{j}$: unit vecto y dir



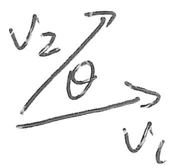
pt 1 (x_1, y_1)

pt 2 (x_2, y_2)

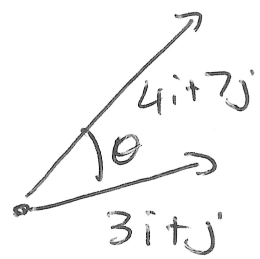
vector $(x_2-x_1)i + (y_2-y_1)j$

Dot Product

$$V_1 \cdot V_2 = |V_1|\,|V_2| \cos\theta$$



$$|ai + bj| = \sqrt{a^2 + b^2}$$



$$\begin{array}{c} (ai + bj) \cdot \\ (ci + dj) \\ \hline ac + bd \end{array}$$

$$\begin{array}{c} 4i + 7j \\ 3i + j \\ \hline 4 \times 3 + 7 \times 1 = 19 \end{array}$$

$$\sqrt{4^2+7^2}\,\sqrt{3^2+1^2}\cos\theta = 19$$

$$\cos\theta = \frac{19}{\sqrt{65}\,\sqrt{10}}$$

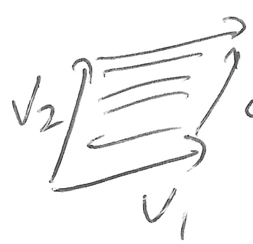
θ = math.acos(stuff)

Cross Product

$V_1 \times V_2$ = vector has magnitude $|V_1||V_2| \sin\theta$
direction is $\perp$ to both V_1, V_2

$$(ai + bj) \times (ci + dj) = \underline{(ad - bc)}\,\vec{k}$$

unit vector
z direction



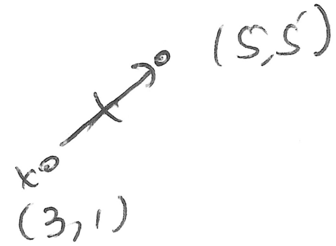
area of parallelogram
= cross product magnitude

$$= \frac{1}{2}(ad - bc)$$

Line Intersection

$dir = (5-3)i + (5-1)j$

Vector eq of line $(3,1)$ and $(5,5)$



dire

$$r = \underbrace{(3i+j)}_{\text{pt on line}} + \underbrace{\lambda}_{\text{param}}(2i+4j)$$

$$L_1 \quad \left.\begin{aligned} x &= 3+2\lambda \\ y &= 1+4\lambda \end{aligned}\right] \text{Parametric}$$

$$L_2 \quad \begin{aligned} x &= -5+\alpha \\ y &= 10-3\alpha \end{aligned}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$$

$$3+2\lambda = -5+\alpha$$
$$1+4\lambda = 10-3\alpha$$

$$2\lambda - \alpha = -8$$
$$4\lambda + 3\alpha = 9$$

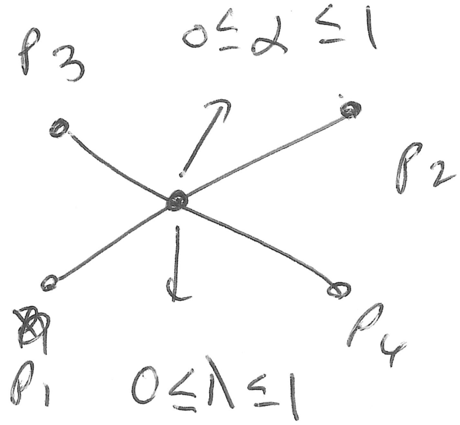
Kramer's Rule

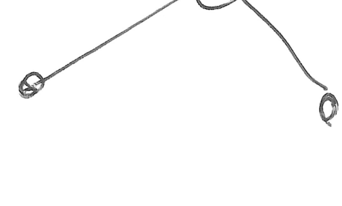
$$\lambda = \frac{\begin{vmatrix} -8 & -1 \\ 9 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 4 & 3 \end{vmatrix}} = \frac{-24+9}{6+4} = \frac{-15}{10} = -\frac{3}{2}$$

coeff 2 var

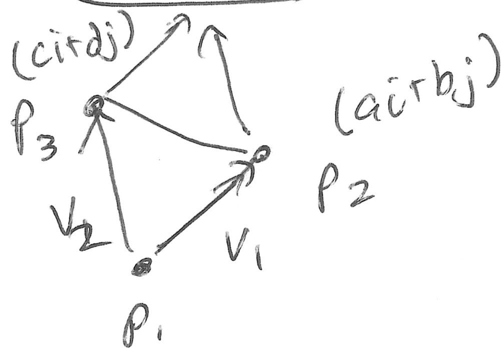
$$\lambda = -\frac{3}{2}, \alpha = 5$$

$$3+2\left(-\tfrac{3}{2}\right) = 0$$
$$1+4\left(-\tfrac{3}{2}\right) = -5$$





Areas



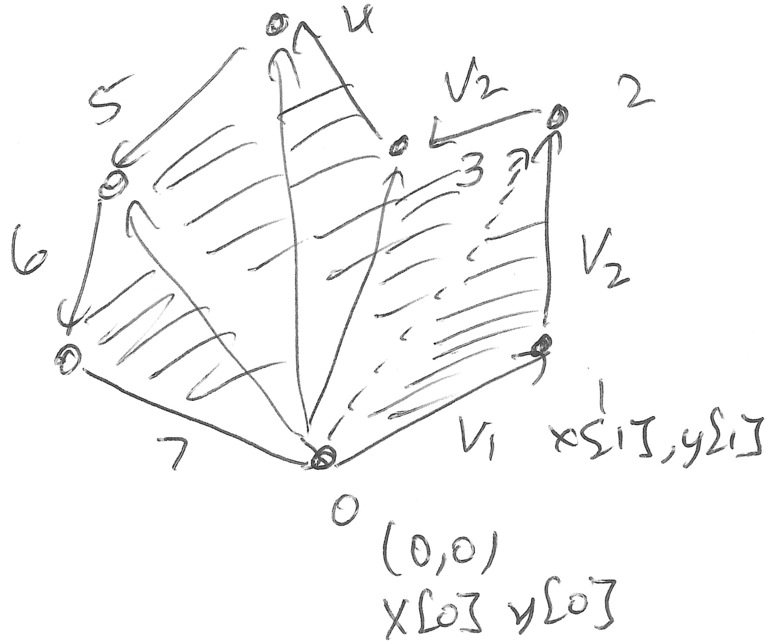
$V_1 = p_2 - p_1$

$V_2 = p_3 - p_1$

$|V_1 \times V_2|$ = parallelogram area

triarea = $\frac{1}{2} \underline{|V_1 \times V_2|}$

$\frac{1}{2} |ad - bc|$



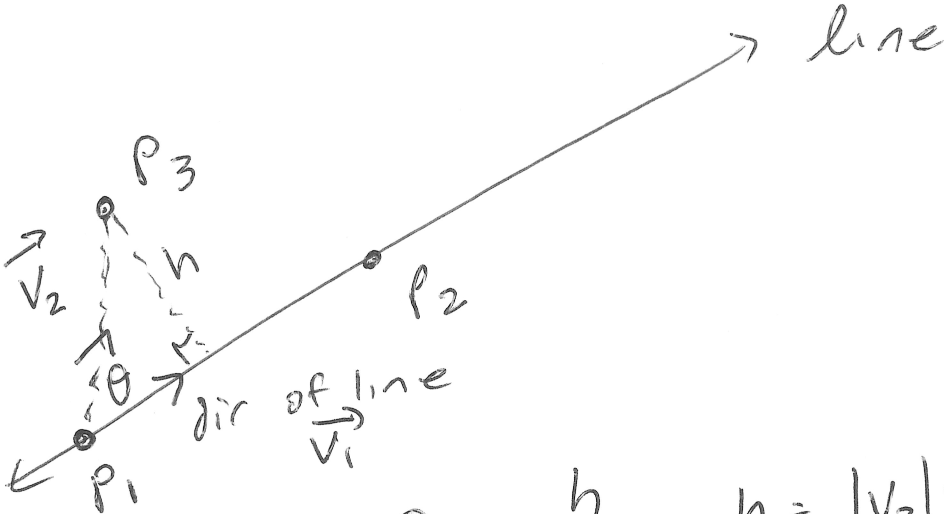
$\frac{1}{2} |V_1 \times V_2|$

$\frac{1}{2} |V_2 \times V_3|$

```
int Doublearea(int* x, int* y, int n) {
    int res = 0;
    for (int i = 0; i < n; i++) {
        int tmp = x[i]*y[(i+1)%n] -
                  x[(i+1)%n]*y[n];

        res += tmp;
    }
    return abs(res);
}
```

Pt Line Distance

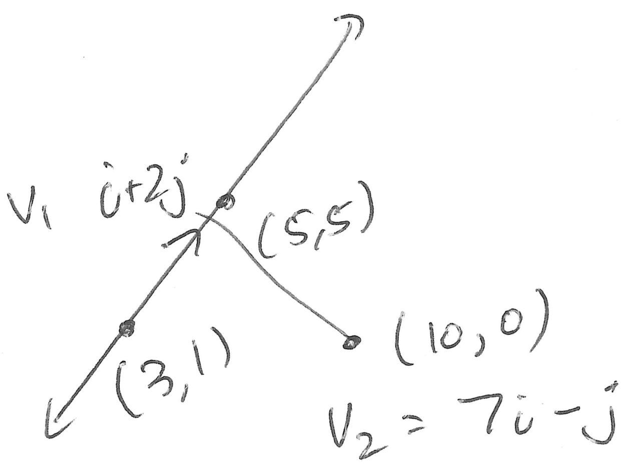


$$\sin\theta = \frac{h}{|V_2|} \qquad h = |V_2|\sin\theta$$

$$|\vec{V_1} \times \vec{V_2}| = |V_1||V_2|\sin\theta$$

$$d(\text{pt to line}) = \frac{|V_1 \times V_2|}{|V_1|}$$

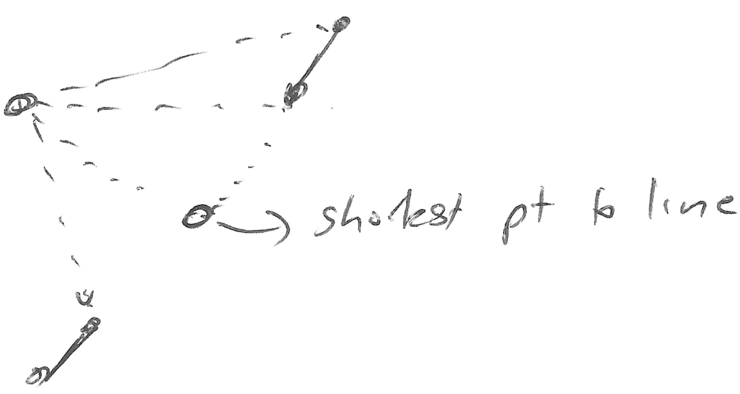
dir vector



$$\begin{vmatrix} 1 & 2 \\ 7 & -1 \end{vmatrix} = 1(-1) - 2(7) = -15$$

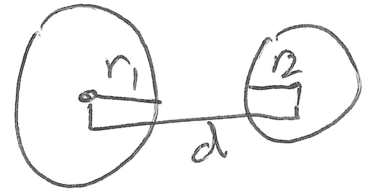
$$\frac{|V_1 \times V_2|}{|V_1|} = \frac{|-15|}{\sqrt{5}} = \frac{15\sqrt{5}}{5} = \boxed{3\sqrt{5}}$$

pt line seg



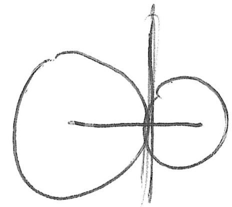
If ⊥ not on seg the min of dist to the 2 end pts

Circles



$d > r_1 + r_2$

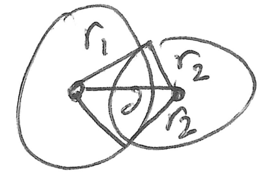
0 inter



$d == r_1 + r_2$

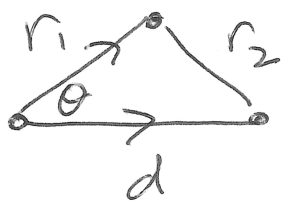
1 inter

tangent pt



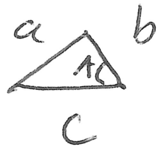
$d < r_1 + r_2$

2 inter



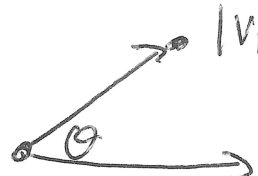
Use law of cosines to figure out θ

Law of cosines



$a^2 = b^2 + c^2 - 2bc\cos A$

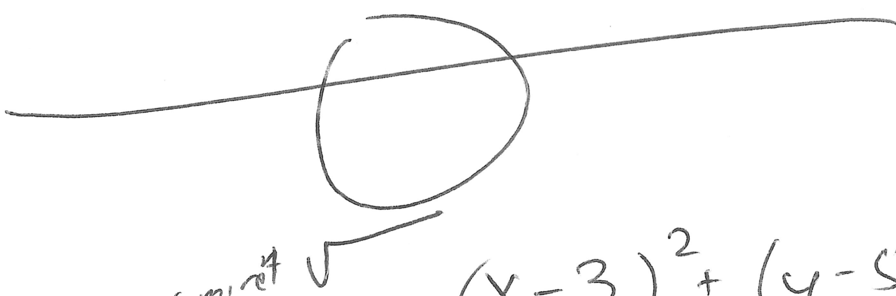
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$



Circle r Line

$x = 1 + \lambda$

$y = 3 - 2\lambda$



O

if discriminant $\sqrt{\ }$

$(x-3)^2 + (y-5)^2 = 4^2$

$x_c \quad y_c \quad r$

$\sqrt{\ }$

$b^2 - 4ac < 0$ no inter

$b^2 - 4ac == 0$ 1 inter

$b^2 - 4ac > 0 \rightarrow$ 2 inter

$(1+\lambda-3)^2 + (3-2\lambda-5)^2 = 16$

quadratic in λ

Convex Hull

I didn't get to!