COP4516 4/18/2025

2D Geometry

* - Vector Eqn vs Cartesian
  Dot, Cross Product
  Line Eq, Line Intersection
  Pt - Line Dist
- Circle-Circle
- Circle-Line
- Area Stuff Circles
- Poly Area
- Pick's Thm
- Pt in Poly
* Graham Scan - Convex Hull

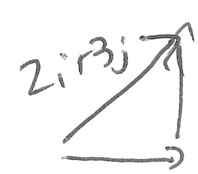
Vector

Direction Magnitude not fixed in space

$\xrightarrow{1}$ unit vector +x dir ($\vec{i}$)

$1\uparrow$ unit vector +y dir ($\vec{j}$)

$2\vec{i} + 3\vec{j}$



$\vec{a} \cdot \vec{b} = x_1 \cdot x_2 + y_1 y_2$ $\qquad |\vec{a}| = \sqrt{x_1^2 + y_1^2}$

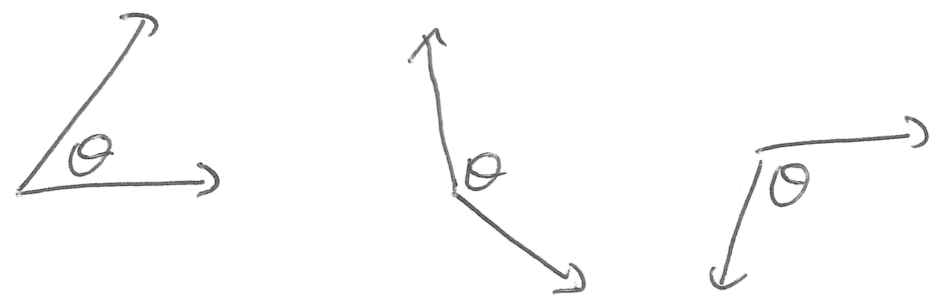
$a = x_1 \vec{i} + y_1 \vec{j}$
$b = x_2 \vec{i} + y_2 \vec{j}$

Dot Product
Ans = Scalar (number)

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

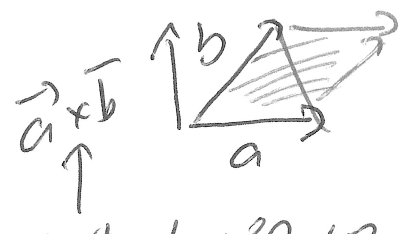
$$\boxed{\cos\theta = \frac{\cancel{x_1 x_2 + y_1 y_2}\ \vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}}$$

θ is angle btw 2 vectors



Cross Product

$\vec{a} \times \vec{b}$ = a vector $\perp$ to both $\vec{a}$ and $\vec{b}$ with a magnitude equal to that of the parallelogram the vectors define



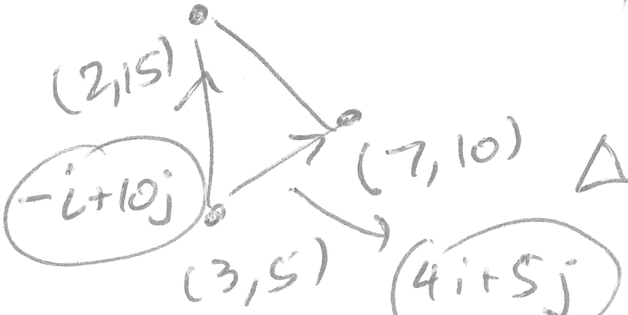
vertical up in air

$a = x_1 \vec{i} + y_1 \vec{j}$
$b = x_2 \vec{i} + y_2 \vec{j}$

$\vec{a} \times \vec{b} = (x_1 y_2 - x_2 y_1) \vec{k}$

$|\vec{a} \times \vec{b}| = x_1 y_2 - x_2 y_1$

$4i + 5j$
$-i + 10j$

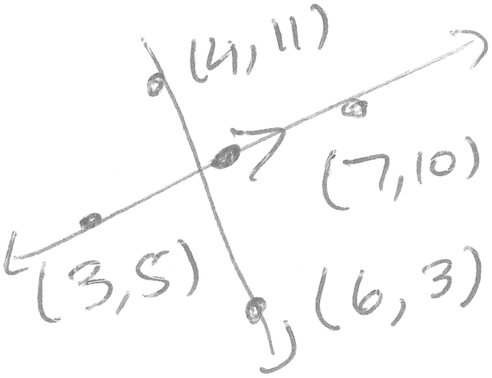


$\Delta\text{area} = |\cancel{7 \times 5 - 4 \times 10}|$

$|4 \times 10 - (-1)(5)| / 2$

$= \boxed{\frac{45}{2}}$

Vector Eqn Line



$y = \frac{5}{4}x + \frac{5}{4}$

$r = p + \lambda d$

pt on line — parameter — dir vector of line

$r = (3\vec{i} + 5\vec{j}) + \lambda(4i + 5j)$

$x = (4 + 2\beta)\bar{i}$
$y = (11 - 8\beta)\vec{j}$

$x = (3 + 4\lambda)\vec{i}$
$y = (\cancel{4}\,5 + 5\lambda)\vec{j}$

plug in $\lambda = .5 \rightarrow$ $x = 5$, $y = \cancel{6}.5$ 7

$$\begin{aligned} 4 + 2\beta &= 3 + 4\lambda \\ 11 - 8\beta &= 5 + 5\lambda \end{aligned}$$

$$\begin{aligned} 2\beta - 4\lambda &= -1 \\ -8\beta - 5\lambda &= -6 \\ 8\beta - 16\lambda &= -4 \\ \hline -21\lambda &= -10 \\ \lambda &= \frac{10}{21} \end{aligned}$$

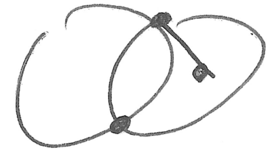
$ax + by = c$
$dx + ey = f$

Cramer's Rule

$$x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

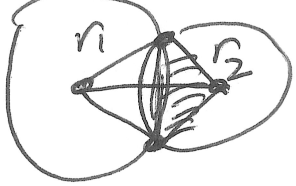
$\boxed{\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \text{ determinant}}$

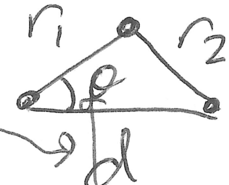
Circle Stuff



$d((x_1,y_1),(x_2,y_2)) > r_1 + r_2$ NO intersection

$$(x-x_1)^2+(y-y_1)^2=r_1^2$$
$$(x-x_2)^2+(y-y_2)^2=r_2^2$$





Law of Cosines $\quad c^2=a^2+b^2-2ab\cos C$ $\quad$ Use this to get θ

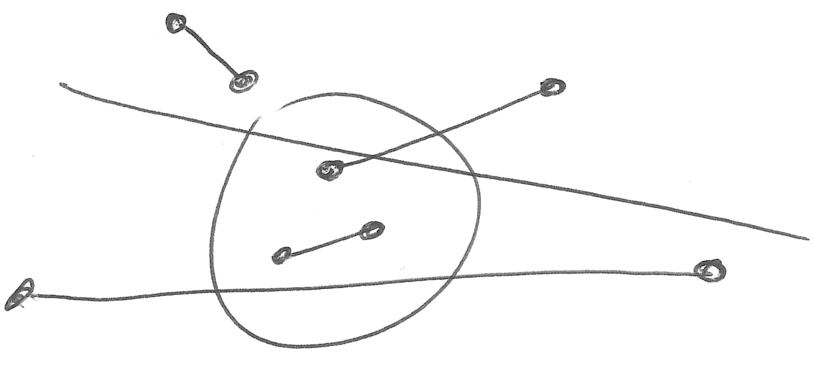
for any vector $x\vec{i}+y\vec{j}$ it's corresponding angle is atan2(y, x).

for any angle θ, the unit vector in that direction is $\cos\theta\,\vec{i}+\sin\theta\,\vec{j}$.

Once you get angle β direction r_1 in picture then $(x_1,y_1)+(r_1\cos\beta, r_1\sin\beta)$.

Triangle Area $= \frac{1}{2}bc\sin A$ (included angle)

Circle - Line



$$x = x_2 + \lambda d_x$$
$$y = y_2 + \lambda d_y$$

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

$$(x_2 + \lambda d_x - x_1)^2 + (y_2 + \lambda d_y - y_1)^2 = r^2$$

λ is unknown

quadratic set up quadratic formula

$$\underbrace{\left(d_x^2 \lambda^2 + d_y^2 \lambda^2\right)}_{a} + \underbrace{\left(2(x_2 - x_1)d_x + 2(y_2 - y_1)d_y\right)}_{b}\lambda + \underbrace{x_1^2 + y_1^2 - r^2}_{c} = 0$$

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disc = b*b - 4*a*c;
if (disc < 0)
    // NO SOLS
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Polygon Area