Spring 2002 Name: ______________________

Com S 641 — Semantic Models of Programming Languages

Test on Back and von Wright Chapters 1-6

Special Directions for this Test

This test has 7 questions and pages numbered 1 through 5.
This test is open book and notes.
If you need more space, use the back, but let us know when you do that.
This test is timed. We will not grade your test if you try to take more than the time allowed. Therefore, before you begin, please take a moment to look over the entire test so that you can budget your time.
For formal proofs, clarity and correct syntax are important.
You can use (without proof) the axioms and inference rules for HOL in chapters 2 through 6 of Back and von Wright's book, the theorems and lemmas proved there, and any of the lemmas from Cohen's book that were included in the lecture notes. You may of course use other lemmas that you state and prove yourself.

1. (5 points) Briefly explain what it means for contract S to be refined by a contract P. Don't give a formal definition, but an informal explanation.

2. (5 points) Is the set of predicate transformers a bounded lattice? Why or why not?

3. (20 points) Consider the set of rational numbers $\mathbb{Q}$ ordered by $\leq$. For example, $2/4 \leq 5/8$. Answer each of the following with "yes" or "no" and a brief (no more than a sentence) justification.

   (a) Is $\mathbb{Q}$ ordered by $\leq$ a lattice?

   (b) Is $\mathbb{Q}$ ordered by $\leq$ bounded?

   (c) Is $\mathbb{Q}$ ordered by $\leq$ complete?

   (d) Is the function space $\text{Bool} \to \mathbb{Q}$ a lattice?

4. (15 points) Formally prove $\vdash Q \wedge (Q \Rightarrow P) \equiv Q \wedge P$, without using case analysis.

5. (15 points) Formally prove $\vdash (\forall x : x \vee y : x \wedge y \equiv x \equiv y)$, without using case analysis. In your proof, you may use the rule for dropping a vacuous quantifier (the constant term rule) without proving that.

6. (15 points) Formally prove that var $r, x : \mathrm{Bool} \vdash (r := \neg x; r := r \vee x) = (r := \mathrm{T})$.

7. (25 points) Formally prove that

$$\text{var } a : \text{Bool} \vdash (\text{begin var } x := a \text{ end}) = (\text{begin var } x := a;\ a := a \vee x \text{ end}).$$

In your proof, you may assume that $a \circ \text{end} = a$ (for the variable a in the formula) and you may use the fact that disjunction is idempotent on truth values. (Hint: you may also find a lemma helpful.)