Spring 2002 Name: _____

Com S 641 — Semantic Models of Programming Languages

Test on Back and von Wright Chapters 7–9 and 11

Special Directions for this Test

This test has 6 questions and pages numbered 1 through 7.
This test is open book and notes.
If you need more space, use the back, but let us know when you do that.
This test is timed. We will not grade your test if you try to take more than the time allowed. Therefore, before you begin, please take a moment to look over the entire test so that you can budget your time.
For formal proofs, clarity and correct proof format are important. You can abbreviate steps as long as they are clear.

1. (10 points) Let Σ be a state space and p and q be predicates in $\mathcal{P}(\Sigma)$. Prove that the coerced relations $|p|$ and $|q|$ satisfy the following formula:

$$(|p|; |q|) = |p \cap q|.$$

2. (15 points) Back and von Wright give the following definition of a deterministic if statement:

$$\textbf{if } b \textbf{ then } S_1 \textbf{ else } S_2 \textbf{ fi} \;\hat{=}\; ([b]; S_1) \sqcap ([\neg b]; S_2) \tag{1}$$

where b is a Boolean expression and S_1 and S_2 are contracts.

Prove that for all Boolean expressions b and contracts S_1, S_2, and S_2,

$$(\textbf{if } b \textbf{ then } S_1 \textbf{ else } S_2 \textbf{ fi}); S_3 = \textbf{if } b \textbf{ then } S_1;\ S_3 \textbf{ else } S_2;\ S_3 \textbf{ fi}.$$

3. (15 points) We can define a one-armed if statement by the following equation.

$$\textbf{if } b \textbf{ then } S \textbf{ fi } \hat{=} \textbf{ if } b \textbf{ then } S \textbf{ else skip fi} \tag{2}$$

where b is a Boolean expression and S is a contract. Derive the weakest precondition of the one-armed if statement, $\mathsf{wp}.(\textbf{if } b \textbf{ then } S \textbf{ fi}).q$. Your result should be expressed as a simple formula involving implications ($\Rightarrow$), negations ($\neg$), and intersections ($\cap$) of predicates, including $\mathsf{wp}.S.q$, but not involving $\mathsf{wp}.\textbf{skip}$ or states.

4. (10 points) Prove

$$\text{var } x, y \vdash [x := x' \mid x' > y + 2] \sqsubseteq \langle x := y + 5 \rangle.$$

5. (20 points) The previous problem is a special case of Carroll Morgan's refinement law "Following assignment 3.5", from p. 32 of *Programming from Specifications* (2nd ed., Prentice Hall). Your task in this problem is to prove the translation of this law into Back and von Wright's notation.

   Let e be an expression that does not contain any free occurrences of x'. Prove that

   $$\text{var } x \vdash [x := x' \mid p[x := x']] \sqsubseteq ([x := x' \mid p[x := e[x := x']]]; \langle x := e \rangle).$$

   In doing this proof, you may use as a lemma (without proof) the following formula, for all functions f.

   $$\vdash (\forall v :: t) \Rightarrow (\forall v :: t[v := f.v]) \tag{3}$$

6. (30 points) Prove that angelic update is monotonic. That is, for all relations P and Q in $\Sigma \leftrightarrow \Gamma$, $(P \subseteq Q) \equiv (\{P\} \sqsubseteq \{Q\})$.