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An improved nonlinear control design for series DC motors

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Abstract

A series DC motor must be represented by a nonlinear model when nonlinearities such as magnetic saturation are considered. To provide effective control, nonlinearities and uncertainties in the model must be taken into account in the control design. In this paper, the recursive design method is applied to generate nonlinear control, nonlinear PI control, and robust control, and these controls are shown to be efficient and robust in the simulation study compared to existing results.

Keywords: Machines; DC motor; Lyapunov stability; Recursive design; Robust control

1. Introduction

The problem of controlling a series DC motor has been studied using different techniques. Several results are cited here for our synopsis, and more can be found in the references cited therein. Motor control using traditional control techniques is discussed in detail in Ref. [9]. A good overview of the application of modern control techniques to motor control can be found in Ref. [5]. Most recently, the nonlinear differential-geometric technique, feedback linearization method, has been used to design control for both series and shunt DC motors [13,3,4]. In spite of this progress, further study is needed to develop a straightforward design and to yield a more effective control and better robustness.

In this paper, it will be shown that the dynamics of a series DC motor can be easily transformed into a cascaded structure (which includes feedback linearizable systems as a special case). Analysis and control design for a system in the cascaded form can be easily handled by the recursive design

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approach. Using this approach, nonlinear speed tracking control and nonlinear PI control (for better tracking under constant but unknown load) are designed. In the presence of parametric as well as dynamic uncertainties, the dynamics of the series DC motor satisfy the generalized matching conditions [14] under which nonlinear robust control can be designed. Specifically, unknown variations in load torque and armature inductance are considered in the paper. It has been shown that, compared with feedback linearization methods, the recursive design method is flexible in handling nonlinearities and uncertainties so that the singularity problem can be avoided, that conditions on feedback linearization can be relaxed, and that useful (or stabilizing) dynamics are not cancelled [16]. It is because of these advantages that a recursive design often yields a smoother and more stabilizing control, especially under uncertainties.

The paper is organized as follows. In Section 2, the model of the series DC motor is reviewed. In Section 3, the recursive design technique is introduced. In Section 4, nonlinear control is generated using the recursive design for the case that all dynamics are known. Nonlinear PI control is designed in Section 5 for the case that the load torque is unknown but constant. Finally, in Section 6, robust control is designed for the case that the load torque is dynamically perturbed and that some of the parameters in the system dynamics are not known. Simulation results for the three cases are presented and compared in Section 7.

2. Model for series DC motors

Series DC motors are often used in applications where high starting torque is required and an appreciable load torque exists under normal operation. Such applications include traction drives, locomotives, trolley buses, cranes, and hoists. In such a motor, the field circuit is connected in series with the armature circuit. Parallel with the field resistance (R_f), there is a by-passing circuit which contains resistance R_p, which is controlled by a switch. By turning on and off the switch, R_p is included or removed from the circuit. Resistance R_p provides field-weakening, which is used to raise the motor speed at reduced loads. Except for saturation, the electromagnetic torque produced by the motor is proportional to the square of the current. This motor produces more torque per Ampere of current than any other DC motor.

From Ref. [9] we note that the dynamics of a series DC motor can be represented by two sets of differential equations depending on the motor's operating condition. Operating conditions of a DC motor are defined in terms of motor speed, and they are divided into two cases. In the first case, the motor operates above base speed with the switch closed, by-passing the field winding to the armature (that is, $R_p < \infty$), and the system equations are:

$$L_a \mathrm{d}i_a/\mathrm{d}t = V - R_a i_a - R_p(i_a - i_f) - K_m \phi_f(i_f)\omega, \tag{1}$$

$$\mathrm{d}\phi_f/\mathrm{d}t = -R_f i_f + R_p(i_a - i_f), \tag{2}$$

$$J \mathrm{d}\omega/\mathrm{d}t = K_m \phi_f(i_f) i_a - B\omega - \tau_L. \tag{3}$$

In the second case, the motor operates below base speed. The switch in the by-passing circuit is open (that is, $R_p \to \infty$), and field weakening is not present. Therefore $i_f = i_a = i$, and the system equations become:

$$L_a \mathrm{d}i/\mathrm{d}t = V - R_a i - K_m \phi_f(i)\omega, \tag{4}$$

$$\mathrm{d}\phi/\mathrm{d}t = -R_f i, \tag{5}$$

$$J \mathrm{d}\omega/\mathrm{d}t = K_m \phi(i) i - B\omega - \tau_L. \tag{6}$$

It is obvious that the system equations for a series DC motor are nonlinear. Symbols in the equations are self-explanatory. For a detailed discussion of electric machines, one may refer to Refs. [6,8,11].

3. Recursive design and robust control

Stability concepts, analysis tools, and control design methods for nonlinear systems can be found in standard textbooks such as [7,10,17]. The common nonlinear design methods include Lyapunov direct method, feedback linearization, singular perturbation, etc. Lyapunov direct method is the universal technique because of its applicability. However, it is often difficult to find a proper Lyapunov function, especially for high order systems. One way to find a Lyapunov function is through the use of the so-called recursive design method.

The method is intuitively simple: find a sub-Lyapunov function for one of the system equations, relate the equation to the rest of the systems by a state transformation and by a design of so-called fictitious control, and repeat this process until all equations are considered and a Lyapunov function, formed from the sum of all sub-Lyapunov functions, is found. Control design using the recursive method for systems which do and do not meet the generalized matching conditions can be found in Refs. [14,15], respectively. Other work based on the recursive design can be found in Refs. [1,2,19] and the references cited therein.

It is easy to see that the system described by Eqs. (1)–(6) satisfies the generalized matching conditions or, equivalently, has the cascaded structure. In this case, the recursive design takes a simpler form, that is, it consists of a sequence of nonlinear mappings. Specifically, the recursive design starts with the first subsystem and works through all subsystems one-by-one until the last one. In each step, the subsystem of state x_i, excluding dynamics associated with x_{i+1}, is stabilized by a fictitious control denoted by x_{i+1}^d, and a state transformation $z_{i+1} = x_{i+1} - x_{i+1}^d$ is formed to generate a dynamic equation for the next subsystem. Generation of x_{i+1}^d is facilitated by picking a Lyapunov function L_i. At the end (when $i = n$), recursive design is completed by setting the control to be x_{n+1}^d.

It should be noted that control designs developed in the references cited earlier in this section are for the so-called robust control. The robust control is a fixed control system designed to guarantee the design requirements in the presence of significant, bounded uncertainties. Its design usually involves three parts: (i) develop or assume bounding functions on uncertainties; (ii) differentiate Lyapunov function, bound the terms associated with uncertainties, and replace their magnitude by the corresponding bounding functions; and (iii) design a control in terms of bounding functions. It is obvious that, if everything is known, robust control design reduces to the conventional nonlinear design in which the operation of bounding the uncertainties and replacing them by bounding functions is no longer needed. In this paper, we shall use the same recursive design for both the case in which the system is perfectly known and the case in which the system contains uncertainties.

4. Application of recursive design under perfect knowledge

In this section, control design is pursued under the assumption that all variables and quantities in the model of the series DC motor are known. Load and parameter variations will be considered in the subsequent sections. In all of the cases, our control problem is to design an effective control under which motor speed tracks a constant desired speed w_0. The design is done by simply selecting state transformations $z_1 = x_1 - w_0$ and $z_2 = x_2 - x_2^d$ for a properly chosen x_2^d and by choosing Lyapunov function $L(z)$ to be a quadratic function of both z_1 and z_2. The transformations map the system into the proper cascaded form, and $L(z)$ is then used to design first fictitious control x_2^d and then actual control V.

First we consider the case when the motor operates above base speed with the switch closed ($R_p < \infty$, $i_f < i_a$), the so-called *field-weakening* region. Note that the system in Eqs. (1)–(3) may be written in a cascaded form by simply selecting the state variables $x_1 = \omega$, $x_2 = \phi_f(i_f)L_a i_a$, and $u = V$. After taking the derivatives of x_1 and x_2, with $\dot{x}_2 = L_a i_a(\mathrm{d}\phi_f(i_f)/\mathrm{d}t) + L_a\phi_f(i_f)(\mathrm{d}i_a/\mathrm{d}t)$, the system equations become:

$$\dot{x}_1 = \frac{K_m}{JL_a}x_2 - \frac{B}{J}x_1 - \frac{\tau_L}{J}$$
$$\dot{x}_2 = -L_a R_f i_a i_f + R_p L_a(i_a^2 - i_a i_f) - K_m\phi_f^2(i_f)x_1 + R_p\phi_f(i_f)i_f - \frac{R_a + R_p}{L_a}x_2 + \phi_f(i_f)u.$$

where τ_L is a constant load torque.

The above system is the cascaded form for which recursive design is readily applicable. Specifically, we shall design our control equation by equation. For the first equation, let $z_1 = x_1 - w_0$. It follows that

$$\dot{z}_1 = \frac{K_m}{JL_a}x_2 - \frac{B}{J}z_1 - \frac{B}{J}w_0 - \frac{\tau_L}{J}.$$

If x_2 were a controller, the first subsystem of state z_1 could be stabilized by setting $x_2^d = \frac{L_a}{K_m}(\tau_L + B\omega_0)$. This can be verified by using the Lyapunov function $L_1 = 0.5z_1^2$. The symbol x_2^d is used instead of x_2 due to the fact that x_2 is not a control variable. The problem that $x_2 \neq x_2^d$ can be resolved by setting $z_2 = x_2 - x_2^d$ and by forcing z_2 to converge to zero.

To this end, one must first derive the dynamic equation for z_2 through differentiation and then derive control u by employing Lyapunov function $L(z) = L_1(z_1) + L_2(z_2) = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$. Through simple algebraic computation, one can solve the control law u as, in terms of the original variables,

$$u = \frac{1}{\phi_f(i_f)}\left[\frac{R_a + R_p}{K_m}(\tau_L + B\omega_0) + L_a R_f i_a i_f - R_p L_a(i_a^2 - i_f i_a) - R_p\phi_f(i_f)i_f - \frac{K_m}{JL_a}(\omega - \omega_0) + K_m\phi_f^2(i_f)\omega\right], \tag{7}$$

under which $\dot{L}(z) = -(B/J)z_1^2 - (R_a + R_p/L_a)z_2^2 \leqslant 0$. It is obvious that the system is globally uniformly asymptotically stable.

For the second case, the switch is open ($R_p \to \infty, i_f = i_a = i$) and the system equations are of the form (4), (5) and (6). In this case, the design process is conceptually identical to that of the first case. First, define x_1, x_2, and z_1 as before. Second, set $L_1(z_1) = 0.5z_1^2$. Third, put x_2^d in the place of x_2 and then choose it to stabilize the first subsystem by studying $\dot{L}_1(z_1)$. It follows that the previous choice for x_2^d is also valid for this case. Now, define $z_2 = x_2 - x_2^d$ and derive its dynamics. Finally, use $L(z) = (1/2)z_1^2 + (L_a/2)z_2^2$ to derive the actual control law. One can show that, under the control

$$u = -\frac{K_m}{FJL_a^2}(\omega - \omega_0) + K_m\phi_f(i)\omega + R_a i - \frac{BG_1}{FJL_a}(\phi_f(i)L_a i - x_2^d), \tag{8}$$

the stability of the system is guaranteed since $\dot{L}(z) = -(B/J)z_1^2 - G_1(B/J)z_2^2 \leqslant 0$. In control Eq. (8), $F(i, \phi_f(i), \partial\phi_f(i)/\partial i) = (\partial\phi_f(i)/\partial i)i + \phi_f(i)$, and G_1 is a positive control gain for the designer to choose. Note that L_a is introduced into $L(z)$ so that the Lyapunov function can be used for the case of partial knowledge as well.

The controls derived under perfect knowledge of the system provide a baseline with which other controls such as PD/PID and robust control laws can be compared.

5. Nonlinear PI control

We now consider the situation when the load torque for the DC motor is unknown. By using a nonlinear PI control, we eliminate the need to know the load torque explicitly. In the case that motor velocity is not measured directly but computed on-line (for example, from reading an optical encoder), the control can also be viewed as a nonlinear PID control. Since many steps in the control are similar or identical to those in the previous section, only those which are different will be discussed in detail. All variables are the same as before unless defined otherwise.

For the purpose of designing a nonlinear PI control, a new state variable x_0 is introduced as $\dot{x}_0 = x_1$. This definition augments both systems above and below the base speed. Consider the system when the motor operates above base speed. It follows that the differential equation for x_1 can be written as

$$\dot{x}_1 = -k_0x_0 - k_1x_1 - \frac{\tau_L}{J} + \frac{K_m}{JL_a}\left[x_2 + \frac{JL_a}{K_m}\left(k_0x_0 + k_1x_1 - \frac{B}{J}x_1\right)\right],$$

in which the same terms k_0x_0 (integral part) and k_1x_1 (proportional part) are added and subtracted, and their sum serves as the fictitious control for x_2.

Define the state transformations $z_0 = [x_0 + (1/k_0)(\tau_L/J)]$, $z_1 = x_1$, and $z_2 = x_2 + (JL_a/K_m) \times (k_0x_0 + k_1x_1 - (B/J)x_1)$. The new state z is used to generate Lyapunov function and control law. It follows that, if $z_2 = 0$, the subsystem of state $[z_0\ z_1]^T$ is stable by choosing gains k_1 and k_2 properly. This can be seen from the fact that

$$\begin{bmatrix} \dot{z}_0 \\ \dot{z}_1 \end{bmatrix} = A\begin{bmatrix} z_0 \\ z_1 \end{bmatrix} + Bz_2,$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -k_0 & -k_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{K_m}{JL_a} \end{bmatrix}.$$

Stability of the subsystem can be shown using the sub-Lyapunov function $L_1(z) = 0.5 \times [z_0, z_1]P[z_0, z_1]^{\mathrm{T}}$ where P is the positive definite solution to $PA + A^{\mathrm{T}}P = -I$. In the case that $z_2 \neq 0$, it follows that $\dot{L}_1 = -z_0^2 - z_1^2 + [z_0, z_1]Pz_2$.

To ensure that z_2 is stable and to design a nonlinear PI control, choose the Lyapunov function $\dot{L} = L_1(z_1) + L_2(z_2)$ where $L_2(z_2) = (1/2)z_2^2$. It follows that, under the control

$$\begin{aligned} u = \frac{1}{\phi_f(i_f)} \Bigg[& -(k_1+k_2)L_a i_a \phi_f(i_f) + L_a R_f i_a i_f - R_p L_a (i_a^2 - i_f i_a) - R_p \phi_f(i_f) i_f - k_0 \frac{J}{K_m} \\ & \times (R_a + R_p)x_0 - k_0(k_1+k_2)\frac{JL_a}{K_m}x_0 + \left(k_2 \frac{BL_a}{K_m} + \frac{B}{K_m}(R_a+R_p) - k_1 k_2 \frac{JL_a}{K_m} - k_1 \frac{J}{K_m}(R_a+R_p) \right. \\ & \left. - k_0 \frac{JL_a}{K_m} \right)(\omega - \omega_0) + \left(\frac{JL_a}{K_m}k_1 - \frac{BL_a}{K_m} \right)\dot{\omega}_0 + K_m \phi_f^2(i_f)\omega + \left(k_1 \frac{BL_a}{K_m} - \frac{B^2 L_a}{JK_m} \right)\omega_0 \Bigg], \end{aligned} \tag{9}$$

the time derivative of the Lyapunov function becomes

$$\dot{L}_1 + \dot{L}_2 \leqslant -\|z_0\|^2 - \|z_1\|^2 + 2\sqrt{z_0^2 + z_1^2}\|z_2\|\sigma_{\max}(PB + R) - \left(k_2 + \frac{R_a + R_p}{L_a} + \frac{B}{J} \right)\|z_2\|^2,$$

where

$$R = \begin{bmatrix} \frac{1}{2}\left(\frac{BL_a}{K_m}k_0 - \frac{JL_a}{K_m}k_0 k_1 \right) \\ 0 \end{bmatrix}.$$

Thus, we know that $\dot{L}_1 + \dot{L}_2 < 0$ if the gains are chosen such that $k_0 > 0$, $k_1 > 0$, and $k_2 > \sigma_{\max}^2$.

We now turn our attention to the case when the motor is operating below base speed. The analysis for the case of motor operating above base speed can be duplicated here to yield

$$\begin{aligned} u = \frac{1}{F} \Bigg[& -(k_1+k_2)\phi_f(i)L_a i - k_0(k_1+k_2)\frac{JL_a}{K_m}x_0 + FR_a i + FK_m \phi_f(i)\omega \\ & + \left(-k_1 k_2 \frac{JL_a}{K_m} + \frac{BL_a}{K_m}k_2 + \frac{JL_a}{K_m}k_0 \right)(\omega - \omega_0) + \left(\frac{JL_a}{K_m}k_1 - \frac{BL_a}{K_m} \right)\dot{\omega}_0 + \left(\frac{BL_a}{K_m}k_1 - \frac{B^2 L_a}{JK_m} \right)\omega_0 \Bigg], \end{aligned} \tag{10}$$

under which the time derivative of Lyapunov function is negative definite as

$$\dot{L}_1 + \dot{L}_2 \leqslant -\|z_0\|^2 - \|z_1\|^2 + 2\sqrt{z_0^2 + z_1^2}\|z_2\|\sigma_{\max}(PB + R) - \left(k_2 + \frac{B}{J} \right)\|z_2\|^2,$$

provided that the gains are chosen as before.

6. Robust control

In the third case we address the situation more commonly encountered in practical applications, that is, the system under study contains significant but bounded uncertainties. Specifically, we assume that L_a is uncertain but bounded and that τ_L is dynamically perturbed. Possible uncertainties in other parameters or functions can be treated in a similar fashion.

As before, we first consider the situation when the motor is operating above base speed. We may define x_1 and z_1 as in the case of perfect knowledge; however, since L_a is not known exactly, we must remove L_a from the definition of x_2 and redefine x_2 as $\phi_f(i_f)i_a$.

In the design of robust control, one must define nominal values and ranges for unknown parameters or dynamics. In the subsequent analysis, the nominal values are chosen to be $L_a \in [L_{a_0} - \kappa_1 L_{a_0}, L_{a_0} + \kappa_1 L_{a_0}]$ and $\tau_L \in [\tau_{L_0} - \kappa_2 \tau_{L_0}, \tau_{L_0} + \kappa_2 \tau_{L_0}]$. Later in the simulation, a 10% variation in the nominal value of armature inductance and a 10% variation in load-torque are used (that is $\kappa_1 = \kappa_2 = 0.1$).

To design the robust control, we chose the Lyapunov function $L(z) = (1/2)z_1^2 + (L_a^2/2)z_2^2$ where $z_2 = x_2 - x_2^d$. x_2^d is the fictitious control to be designed. By letting $x_2^d = (1/K_m)(B\omega_0 + \tau_{L_0}) + u_{R_{11}}$, we can show that

$$z_1\dot{z}_1 = -\frac{B}{J}z_1^2 + \frac{K_m}{J}\left[\frac{1}{K_m}(\tau_{L_0} + B\omega_0) + u_{R_{11}}\right]z_1 - \frac{B}{J}\omega_0 z_1 - \frac{\tau_L}{J}z_1 + \frac{K_m}{J}z_1 z_2,$$

in which $u_{R_{11}}$ is to be designed to compensate for all the terms except for $(K_m/J)z_1z_2$ (which will be considered in the design of u).

Selecting the bounding function ρ_1 to be equal to $\rho_1 = (\tau_{L_0}/J)\kappa_2$, letting $u_{R_{11}} = -(1/K_m) \times ((1/\epsilon_1)\rho_1^2)z_1$, and dropping the $(K_m/J)z_1z_2$ term we have

$$z_1\dot{z}_1 \leqslant -\frac{B}{J}z_1^2 + \frac{\epsilon_1}{4J},$$

where design parameter ϵ_1 determines the accuracy of the control.

Once x_2^d is found explicitly, differential equation for z_2 can be found. It can be shown that the terms in $\dot{z}_2$ associated with the uncertainties can be bounded by function ρ_2 where

$$\rho_2 = L_{a_0}\kappa_1\left[R_f i_a i_f + R_p(i_a^2 - i_a i_f) + \frac{B}{JK_m}\left(\frac{1}{\epsilon_1}\right)\rho_1^2|x_1| + \frac{1}{J\epsilon_1}\rho_1^2|x_2|\right] + L_{a_0}(1+\kappa_1)\frac{\tau_{L_0}(1+\kappa_2)}{JK_m}\left(\frac{1}{\epsilon_1}\right)\rho_1^2 + \frac{K_m\kappa_1}{JL_{a_0}(1-\kappa_1)}|z_1|.$$

The nonlinear control u is used to cancel the known terms and to compensate for the uncertain terms. Let

$$u = \frac{1}{\phi_f(i_f)}\left[K_m\phi_f^2(i_f)\omega + \frac{R_a + R_p}{K_m}(B\omega_0 + \tau_{L_0}) - \frac{(R_a + R_p)}{K_m}\left(\frac{1}{\epsilon_1}\right)\rho_1^2(\omega - \omega_0) - R_p\phi_f(i_f)i_f + R_f L_{a_0} i_a i_f - \frac{K_m}{JL_{a_0}}(\omega - \omega_0) + \frac{BL_{a_0}}{JK_m}\left(\frac{1}{\epsilon_1}\right)\rho_1^2\omega - \frac{L_{a_0}}{J}\left(\frac{1}{\epsilon_1}\right)\rho_1^2\phi_f(i_f)i_a - R_p L_{a_0}(i_a^2 - i_a i_f) + u_{R_{12}}\right], \quad (11)$$

where $\mu = \rho_2 z_2$, and

$$u_{R_{12}} = -\frac{\mu^2 + \epsilon_2^2}{|\mu|^3 + \epsilon_2^3}\mu\rho_2.$$

Under the control, we have

$$\dot{L}(z) \leqslant -\frac{B}{J}z_1^2 - (R_a + R_p)L_a z_2^2 + L_a\left[\frac{\epsilon_2}{4} + \frac{\epsilon_2^2|\mu| - \epsilon_2|\mu|^2}{|\mu|^3 + \epsilon_2^3}\right]\epsilon_2 + \frac{\epsilon_1}{4J}.$$

By Holder's inequality [12] $(ab \leqslant (a^p/p) + (b^q/q))$, we can show that

$$\frac{\epsilon_2^2|\mu| - \epsilon_2|\mu|^2}{|\mu|^3 + \epsilon_2^3}\epsilon_2 \leqslant \frac{2}{C}\epsilon_2,$$

where $C = 3\left(\frac{1}{2}\right)^{2/3}$. Therefore, it follows that

$$\dot{L}(z) \leqslant -\frac{B}{J}z_1^2 - (R_a + R_p)L_a z_2^2 + L_a\left[\frac{\epsilon_2}{4} + \frac{2}{C}\epsilon_2\right] + \frac{1}{4J}\epsilon_1.$$

Since $(R_a + R_p)L_a \ll (B/J)$, we can rewrite the above inequality as

$$\dot{L}(z) \leqslant -2(R_a + R_p)L_a L(z) + \left(\frac{1}{4} + \frac{2}{C}\right)L_a\epsilon_2 + \frac{1}{4J}\epsilon_1.$$

Solving the above inequality, we can easily show that the system is globally, uniformly ultimately bounded.

The case when the motor operates below base speed can be analyzed in exactly the same manner. That is, consider first Lyapunov function $L(z) = (1/2)z_1^2 + (L_a^3/2)z_2^2$; let $x_2^d = (1/K_m) \times (B\omega_0 + \tau_{L_0}) + u_{R_{21}}$ where $u_{R_{21}} = u_{R_{11}} = -(1/K_m)((1/\epsilon_1)\rho_1^2)z_1$. A bounding function for the uncertainties in $\dot{z}_2$ is

$$\rho_2 = \frac{BL_{a_0}\kappa_1}{FJK_m}\left(\frac{1}{\epsilon_1}\right)\rho_1^2|x_1| + \frac{L_{a_0}\kappa_1}{FJ}\left(\frac{1}{\epsilon_1}\right)\rho_1^2|x_2| + \frac{K_m(2\kappa_1 + \kappa_1^2)}{FJL_{a_0}^2(1-\kappa_1)^2}|z_1|$$
$$+ L_{a_0}(1+\kappa_1)\frac{1}{FJK_m}\tau_{L_0}(1+\kappa_2)\left(\frac{1}{\epsilon_1}\right)\rho_1^2,$$

and the robust control is

$$u = R_a i + K_m\phi_f(i_f)\omega + \frac{BL_{a_0}}{FJK_m}\left(\frac{1}{\epsilon_1}\right)\rho_1^2\omega - \frac{L_{a_0}}{FJ}\left(\frac{1}{\epsilon_1}\right)\rho_1^2(\phi_f(i)i) - \frac{K_m}{FJL_{a_0}^2}(\omega - \omega_0) + u_{R_{22}} - \frac{G}{F}z_2, \tag{12}$$

where

$$u_{R_{22}} = -\frac{\mu^2 + \epsilon_2^2}{|\mu|^3 + \epsilon_2^3}\mu\rho_2.$$

Stability and its proof are identical to those shown in the first case. Robust controls equivalent to $u_{R_{22}}$ can be found in Ref. [16].

7. Simulation and comparison

The results from the three cases were simulated. Note that the control law changes as the motor moves from below base speed to above base speed. Base speed was chosen as $\omega_{\text{base}} = 200$ rad/s.

The load torque, τ_L, was simulated as described by Chiasson [4] as

$$\tau_L = \begin{cases} 0 & \text{Nm} \quad 0 \leqslant t \leqslant 5, \\ 1250(t-5)/5 & \text{Nm} \quad 5 \leqslant t \leqslant 10, \\ 1250 & \text{Nm} \quad 10 \leqslant t. \end{cases}$$

The parameters related to this motor, also from Chiasson [4] are the armature inductance ($L_a = 0.0014$ H), the resistance of the field windings ($R_f = 0.01485\ \Omega$), the parallel resistance of field weakening ($R_p = 0.01696\ \Omega$), the resistance of the armature windings ($R_a = 0.00989\ \Omega$), the viscous friction ($B = 0.1$ Nm/rd/s), the torque/back-emf ($K_m = 0.04329$ (Nm)/(WbA)), and the moment of inertia ($J = 3.0$ Kgm^2). For all cases, the reference speed was chosen to start from 0 and go up to 520 rad/s in 20 s and is plotted in Fig. 1. The flux, $\phi_f(i_f)$, was derived from Fig. 4 of Chiasson [4].

Several different simulations were attempted by varying the value of the control gain constant, G_1. As G_1 was increased, the error during the first few seconds settled down and the control law became smoother. Past a certain value, however, the error began to increase during the first few seconds without any improvement in the control law. The results for the best choice of G_1 are presented in Figs. 2 and 3.

The PID control law was simulated under the assumption that all quantities are known except the load torque. Using the relationships developed previously, values of k_0 and k_1 were chosen and

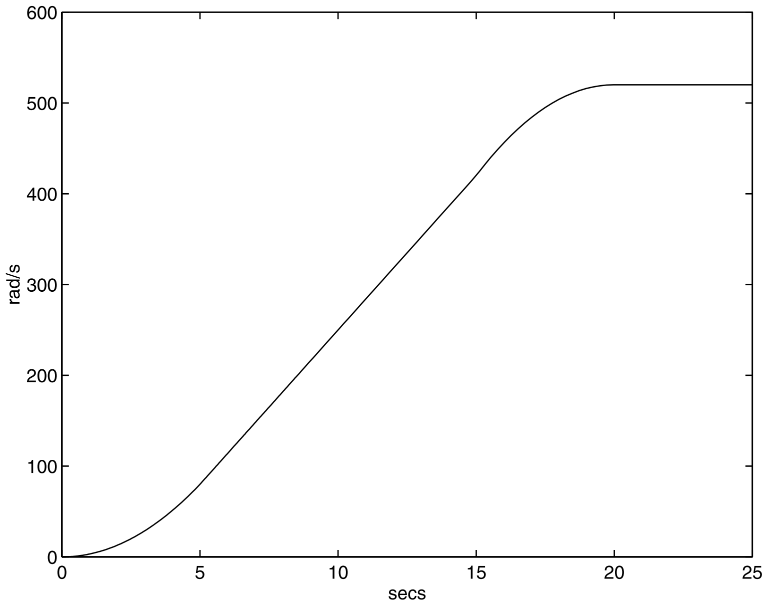

Fig. 1. Plot of reference speed for the motor.

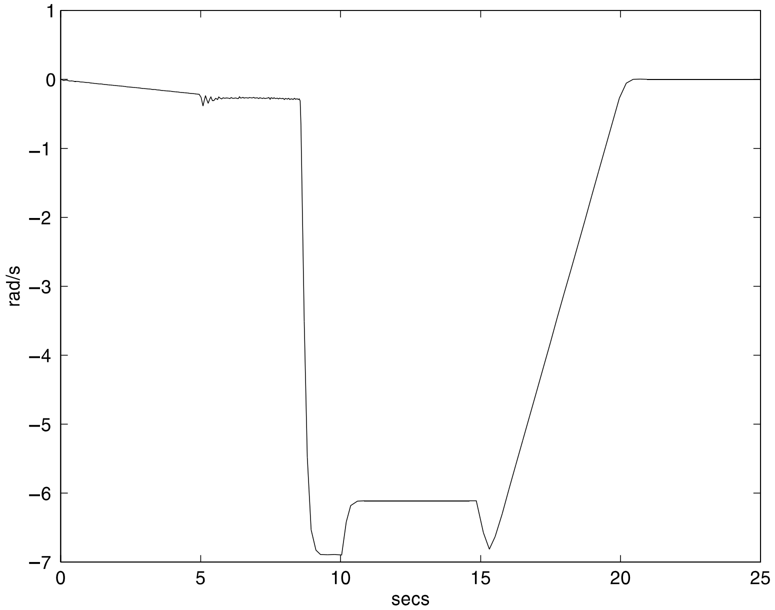

Fig. 2. Plot of error for $G_1 = 20.0$.

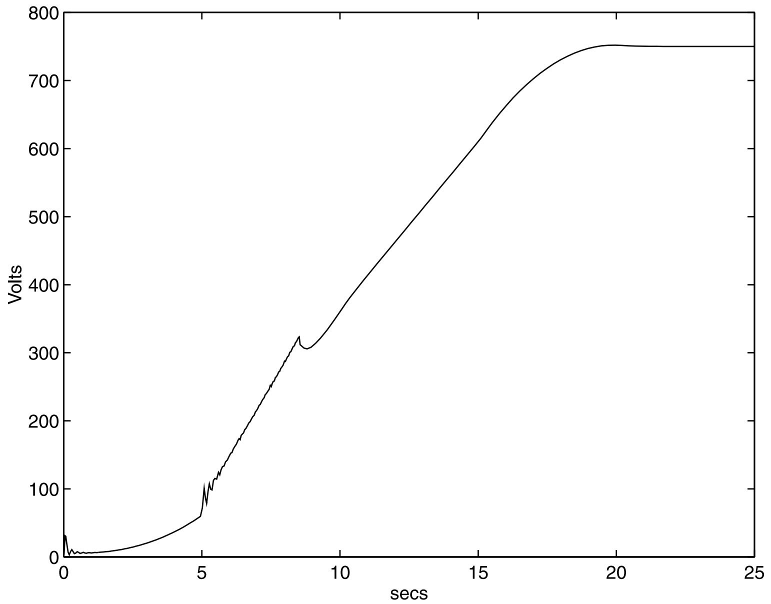

Fig. 3. Plot of the combined control law for $G_1 = 20.0$.

then the appropriate range of values for k_2 was calculated. For example, for the choices of $k_0 = 7$ and $k_1 = 16$, we found that k_2 must be chosen greater than 44.

Simulations were attempted for several different values of k_0, k_1, and k_2, and simulation results corresponding to the best choices are shown in Figs. 4 and 5. Generally, the gains should be chosen in the range of 1–50 for the purposes of actual physical implementation.

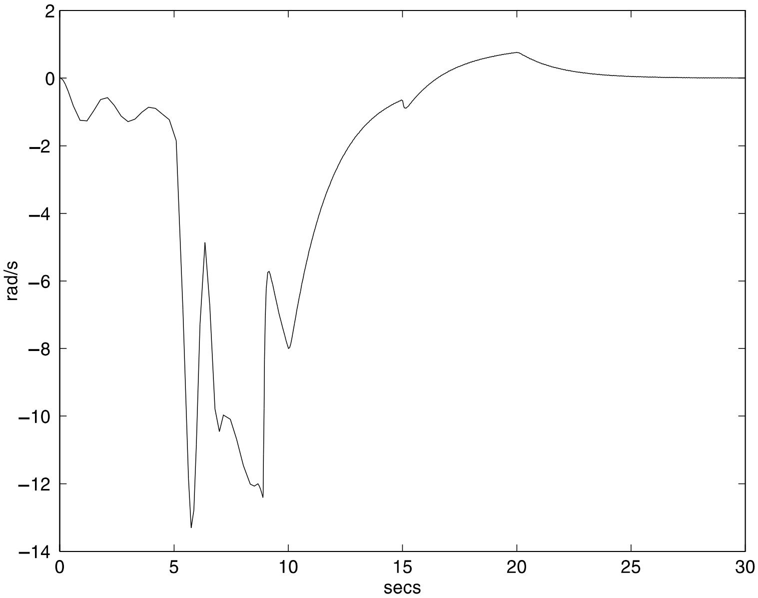

Fig. 4. Error plot for $k_0 = 7.0$, $k_1 = 16.0$, $k_2 = 50.0$.

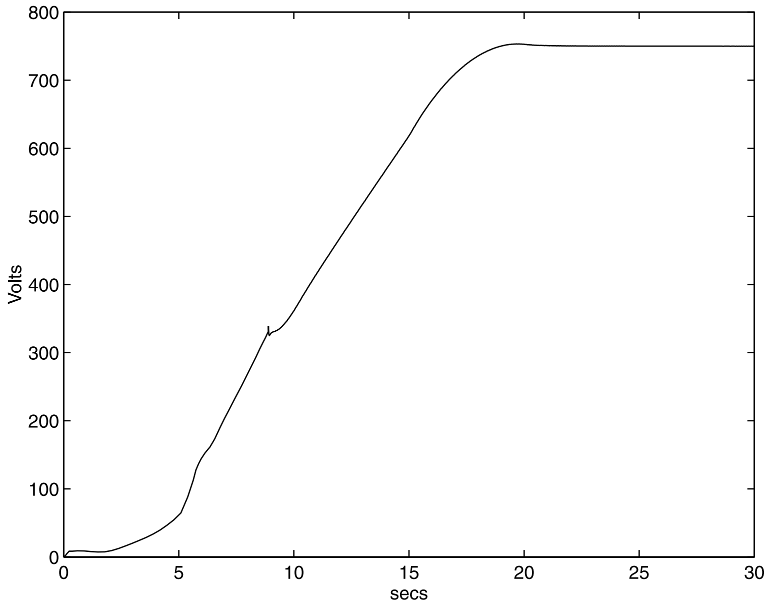

Fig. 5. Plot of the combined PID control law for $k_0 = 7.0$, $k_1 = 16.0$, $k_2 = 50.0$.

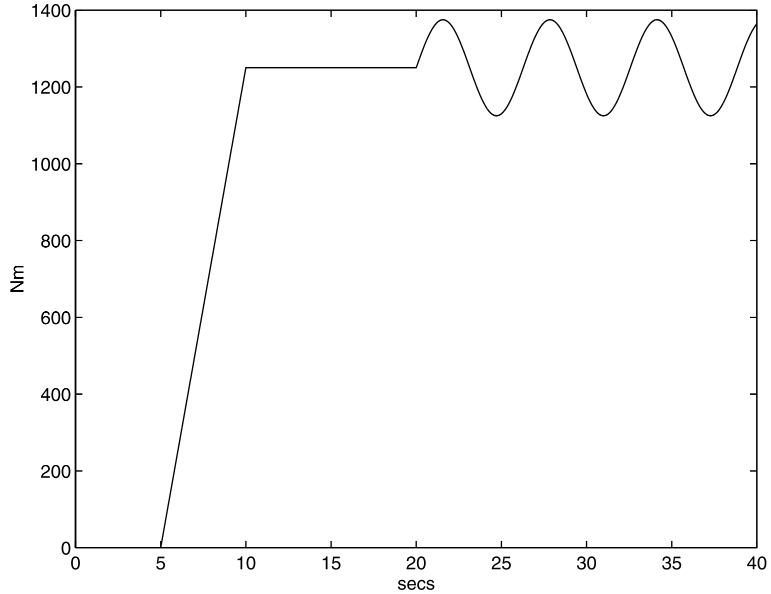

Fig. 6. Plot of simulated load torque with dynamic perturbation.

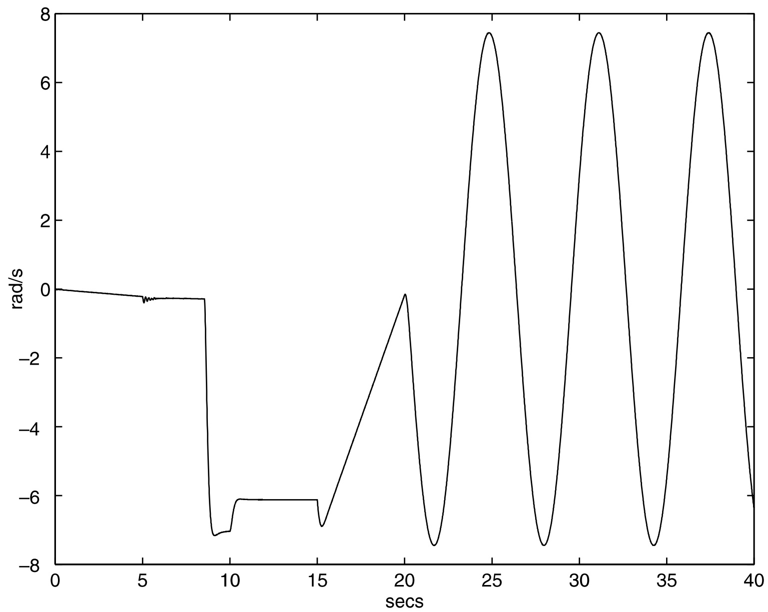

Fig. 7. Plot of error for perfect knowledge case.

The robust control law contains several gain parameters which must be varied to obtain the best results. In general, ϵ_1 should be chosen greater than ϵ_2 and the value of G should be chosen to be within a reasonable range. The simulation must also be altered to test the robustness of the

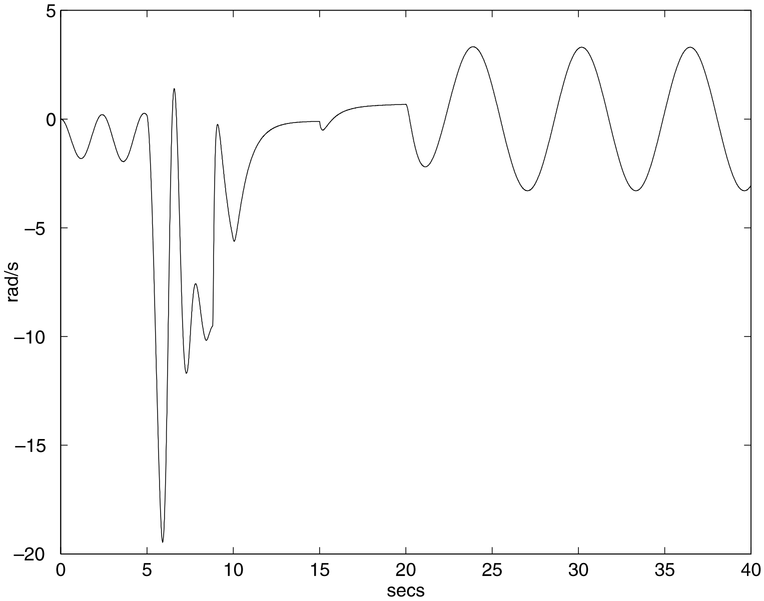

Fig. 8. Plot of error for PID control case.

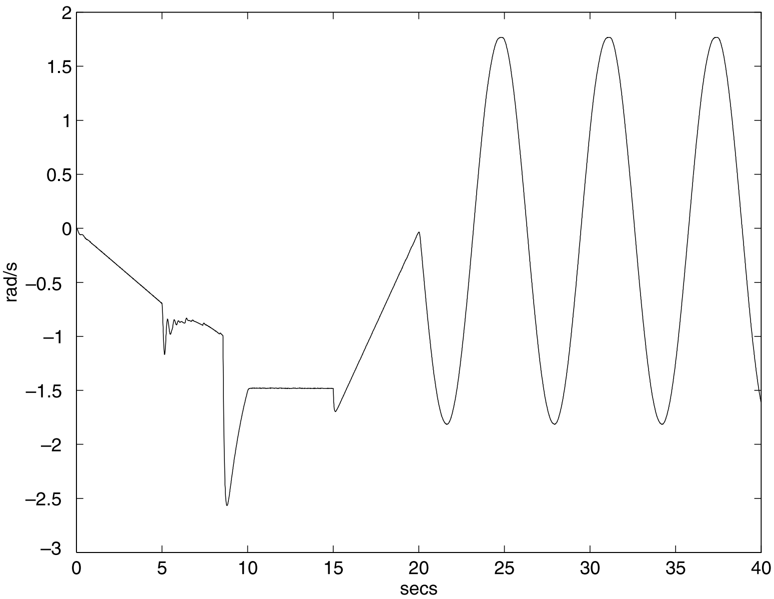

Fig. 9. Plot of error for robust control case.

control. After several simulations, the following values for gain were chosen: $\epsilon_{11} = 25.0$, $\epsilon_{12} = 0.1$, $\epsilon_{21} = 50.0$, $\epsilon_{22} = 0.3$, $G = 20.0$.

In order to demonstrate the true power of the robust control law, simulations were performed which included perturbations from the nominal values of two system parameters.

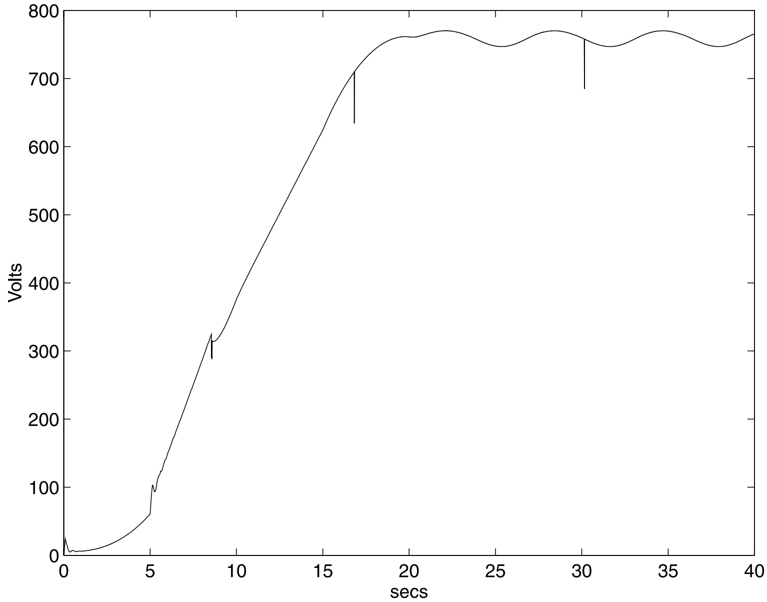

Fig. 10. Plot of combined control law for robust control case.

Many nonlinear systems are highly sensitive to changes in system parameters, as discussed in Ref. [18]. It is through the use of robust control, then, that we hope to compensate for this sensitivity.

A load torque with dynamic perturbation, shown in Fig. 6, was chosen. In addition to perturbing the load torque, the value of the armature inductance (L_a) was perturbed by 10% as well. Figs. 7–10 show the errors under the uncertainties (load change in Fig. 6 and parameter variation) and under various types of controls. It is apparent that robust control achieved the best result.

It should be noted that the spikes in the control law for the robust case are artifacts of the algorithms used to simulate the system and reduce the error in calculations, not an indication of an error in the equations of the control law.

8. Conclusions

In related work, Chiasson's use of the nonlinear-geometric technique produced generally good results, but did not include the possibility of uncertain terms [4]. His design also required a speed and load-torque observer. Compared to this and other techniques, robust control proves to be well suited to the task of handling the presence of dynamic perturbations in the system parameters. It does not require the use of estimators or observers. Using current and speed measurements along with the assumed function of flux, the input voltage is varied according to the control law.

We have seen that the recursive design approach may be successfully applied to the problem of designing a robust control for the nonlinear model of a series DC motor. When the only unknown parameter is the load torque, PI control may be applied to the system with generally good results. However, when other system parameters are unknown and/or dynamic perturbation is possible, the robust control approach provides the best results. A control based upon the assumption that all parameters are perfectly known fails when dynamic perturbation is present.

Although we only considered the cases when load torque and armature inductance were unknown, the approach as presented could be easily extended to handle additional uncertainties. Further research could be conducted by including additional nonlinear terms in the system equations. Or one might choose to consider the possibility of the existence of uncertainties in other system parameters.

As manufacturing standards continue to demand greater precision and performance from robots and other computer controlled mechanisms, the need for more precise, robust control laws becomes greater too. When the model of a system includes nonlinear dynamics and uncertain terms, the usefulness of robust control is apparent.

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