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Tracking control of rigid-link electrically-driven robot manipulators

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This paper illustrates a simple, hand-crafted approach which can be used to design tracking controllers for rigid-link electrically-driven (RLED) robot manipulators. The control methodology is intuitively simple since it is based on concepts readily identified by most control engineers. To illustrate the approach, we develop a corrective tracking controller for the RLED robot dynamics which yields global exponential stability for the link tracking error under the assumption of exact model knowledge. To compensate for the uncertainties in the rigid-link electrically-driven robot model, we then design a corrective robust tracking controller which yields global uniform ultimate bounded stability of the link tracking error. The proposed controller is robust with regard to parametric uncertainties and additive bounded disturbances while correcting for the typically ignored electrical actuator dynamics.

1. Introduction

In recent years, control engineers have become increasingly interested in the robot tracking problem. As a result many controllers have been developed which compensate for uncertainty in the nonlinear second-order dynamics commonly used to represent rigid-link robots. Most of the more rigorously developed nonlinear controllers for rigid-link robots fall into two categories, indirect adaptive control and robust nonlinear control. The interested reader is referred to Abdallah *et al.* (1991) and Ortega and Spong (1988) for review papers in these two areas.

A deficiency associated with many of the controllers represented in Abdallah *et al.* (1991) and Ortega and Spong (1988) is that these controllers have been designed at the torque input level. Therefore, any dynamics associated with the joint actuators (e.g. electrical effects) have been neglected. Several researchers have postulated that the detrimental effects of neglected actuator dynamics are preventing the development of high-performance motion and/or force tracking controllers (Eppinger and Seeing, 1987). Therefore, it is believed that additional progress can be made by including the effects of actuator dynamics in the control synthesis.

Some recent work regarding the compensation of electrical actuator dynamics is now summarized. In Ilic'-Spong *et al.* (1987) a detailed nonlinear model of the switched reluctance motor is developed and an electronic commutation strategy

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is established. The motor dynamics are then feedback linearized via a nonlinear controller and the controller's robustness is tested versus critical parameter uncertainties such as stator resistance. Taylor (1989) extends this work by formulating a composite control based on a singularly perturbed rigid-link eletrically-driven (RLED) robot model with switched reluctance actuator dynamics. Marino *et al.* (1990) develop a feedback linearizing control for an induction motor which includes both electrical and mechanical dynamics. The proposed controller contains a nonlinear adaptation scheme which is capable of identifying the motor load torque and rotor resistance (which are assumed to be unknown constants). Tarn *et al.* (1991) develop a feedback linearizing control that requires acceleration measurements for RLED manipulators with direct-current actuators. A feedback linearizing and decoupling transformation is presented by Tzafestas *et al.* (1984) to produce a reduced-order model of the rigid-link robot with direct-current actuator dynamics. The proposed controller reduces to a standard computed torque algorithm for rigid-link robots under specified conditions. Robust stability for the full-order RLED model is then considered and stability conditions are derived. Note, all of these controllers require exact knowledge of some (if not all) model parameters.

It is also important to note that many of the controllers proposed in the literature compensate for the fast actuator dynamics by using a singular perturbation approach (Kokotovic *et al.* 1986). Since the theory of singular perturbation can be applied to many general classes of systems, it is very useful for the control synthesis of both linear and nonlinear systems. The singular perturbation approach involves the use of concepts such as time-scale separation, integral manifolds, and power series expansions. While these concepts are powerful and elegant, it may be possible to use a simpler approach for the control synthesis in many specific applications. This paper illustrates how a simple hand-crafted approach can be used to analyse the stability of rigid-link electrically-driven robots.

Using this simplified approach and the assumption of exact RLED model knowledge, we develop a corrective tracking controller to achieve global exponential stability of the link tracking error. The term corrective controller (Kokotovic *et al.* 1986) is used to emphasize that the controller corrects for the typically ignored electrical actuator dynamics. To compensate for uncertainties in the RLED robot model, we then design a corrective robust tracking controller to achieve global uniform ultimate bounded stability of the link tracking error. The proposed controller is robust with regard to parametric uncertainties and additive bounded disturbances while correcting for the electrical actuator dynamics. While this paper focuses on the development of hand-crafted controllers for the RLED robot, it should be noted that similar results have also been obtained for the rigid-link flexible-joint (RLFJ) robot (Dawson *et al.* 1991).

The paper is organized as follows. In § 2, we design a corrective tracking controller for RLED robots under the assumption of exact model knowledge. We extend these results in § 3 by designing a corrective robust tracking controller which compensates for uncertainty in the RLED model. In § 4, we present simulation results for a two-degree of freedom RLED robot with direct-current actuators to illustrate the performance of the proposed corrective tracking and corrective robust tracking controllers.

2. Corrective tracking in the presence of electrical dynamics

2.1. *Mathematical preliminaries*

Before presenting the RLED robot dynamics, we introduce two stability lemmas which will be exploited later in the development.

Lemma 2.1 (See the Appendix): *If the $n \times 1$ state vector $x(t)$ in the continuous system*

$$\dot{x}(t) = f(x(t), t) \tag{1}$$

has an associated Lyapunov function (Corless and Leitmann 1981) *$V(x, t)$ with the following properties*

$$\lambda_1 \|x(t)\|^2 \leq V(x, t) \leq \lambda_2 \|x(t)\|^2 \qquad \forall (x, t) \in \mathbb{R}^n \times \mathbb{R} \tag{2}$$

$$\dot{V}(x, t) \leq -\lambda_3 \|x(t)\|^2 + \varepsilon \qquad \forall (x, t) \in \mathbb{R}^n \times \mathbb{R} \tag{3}$$

where λ_1, λ_2, λ_3, and ε are positive scalar constants, then the state $x(t)$ is globally uniform ultimately bounded (GUUB) (Corless and Leitmann 1981) *in the sense that*

$$\|x(t)\| \leq \left[\frac{\lambda_2}{\lambda_1} \|x(0)\|^2 \mathrm{e}^{-\lambda t} + \frac{\varepsilon}{\lambda_1 \lambda} [1 - \mathrm{e}^{-\lambda t}]\right]^{1/2} \tag{4}$$

where $\lambda = \lambda_3/\lambda_2$, and e is the natural logarithm exponential. The notation $\|\{\cdot\}\|$ is used to represent the euclidean norm (Barnett 1984) *of the vector $\{\cdot\}$ and a 'dot' is used to designate differentiation with respect to time throughout the development.*

Lemma 2.2 (See the Appendix): *Let $y(t)$ be a $n \times 1$ vector defined in terms of a $n \times 1$ vector $z(t)$ as shown*

$$y(t) = \dot{z}(t) + \alpha z(t) \tag{5}$$

where α is a positive scalar constant. If $y(t)$ is upper bounded by the expression

$$\|y(t)\| \leq \sqrt{\mathcal{A}} + \sqrt{|\mathcal{B}|}\mathrm{e}^{-\lambda t/2} \tag{6}$$

where $\mathcal{A}$ is a non-negative scalar constant, and $|\mathcal{B}|$, α, and λ are positive scalar constants, then $z(t)$ can be upper bounded as shown

$$\|z(t)\| \leq n\mathrm{e}^{-\alpha t}\|z(0)\| + n\frac{\sqrt{\mathcal{A}}}{\alpha}[1 - \mathrm{e}^{-\alpha t}] + \frac{2n\sqrt{|\mathcal{B}|}}{2\alpha - \lambda}[\mathrm{e}^{-\lambda t/2} - \mathrm{e}^{-\alpha t}] \tag{7}$$

2.2. *RLED model development and associated properties*

For simplicity, we assume that the actuator is a permanent magnet direct-current motor. It should be noted that the following analysis can also be extended to other motors commonly used in robotics, such as brushless direct-current motors. The model (Taylor 1989) for an n-link RLED robot is taken to be

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T_L = K_T I \tag{8}$$

and

$$L\dot{I} + R(I, \dot{q}) + T_E = u_E \tag{9}$$

where $M(q)$ is a $n \times n$ inertia matrix, $V_m(q, \dot{q})$ is a $n \times n$ matrix containing the centripetal and Coriolis terms, $G(q)$ is a $n \times 1$ vector containing the gravity terms, $F(\dot{q})$ is a $n \times 1$ vector containing the static and dynamic friction terms, $q(t)$ is a $n \times 1$ vector representing the link displacements, $I(t)$ is an $n \times 1$ vector used to denote the armature current in each joint actuator, K_T is a positive definite constant diagonal $n \times n$ matrix which characterizes the electromechanical conversion between current and torque, L is a positive definite constant diagonal $n \times n$ matrix used to represent the electrical inductance, $R(I, \dot{q})$ is a $n \times 1$ vector used to represent the electrical resistance and the motor back-electromotive force, u_E is an $n \times 1$ control vector used to represent the motor terminal voltages, T_L is a $n \times 1$ vector representing an additive bounded torque disturbance, and T_E is a $n \times 1$ vector representing an additive bounded voltage disturbance.

It is important to emphasize that constant bounds are assumed for each of the parametric quantities represented in (8) and (9). For example, the torque transmission matrix K_T is assumed to be bounded

$$k_1\|x\|^2 \leqslant x^{\mathrm{T}} K_T x \leqslant k_2\|x\|^2 \quad \text{for an arbitrary } n \times 1 \text{ vector } x \tag{10}$$

where k_1 and k_2 are positive scalar bounding constants. The inductance matrix L is also assumed to be bounded

$$l_1\|x\|^2 \leqslant x^{\mathrm{T}} L x \leqslant l_2\|x\|^2 \quad \text{for an arbitrary } n \times 1 \text{ vector } x \tag{11}$$

where l_1 and l_2 are positive scalar bounding constants. In addition, upper bounds are assumed for the remaining RLED model parameters.

Traditionally, the robotic control literature (Slotine 1988) has emphasized the use of the manipulator's physical properties to aid in the stability analysis. Therefore, we note the following useful robot properties

Property: Inertia. *The inertia matrix $M(q)$ defined in (8) is positive definite symmetric and is uniformly bounded as a function of q. Therefore, we can state for an arbitrary $n \times 1$ vector x*

$$m_1\|x\|^2 = \lambda_{\min}\{M(q)\}\|x\|^2 \leqslant x^{\mathrm{T}} M(q) x \leqslant \lambda_{\max}\{M(q)\}\|x\|^2 = m_2\|x\|^2 \tag{12}$$

where m_1 and m_2 are positive scalar constants that depend on the mass properties of the specific robot considered. In general, Property 1 only applies to revolute joint robots.

Property: Skew symmetry. *A useful relationship exists between the time derivative of the inertia matrix $M(q)$ and Coriolis/centrifugal matrix $V_m(q, \dot{q})$ as shown*

$$x^{\mathrm{T}}(\dot{M}(q) - 2V_m(q, \dot{q}))x = 0 \tag{13}$$

where x is an arbitrary $n \times 1$ vector.

2.3. *Formulation of the RLED error system*

Our objective is to synthesize a controller given exact RLED model knowledge which ensures global exponential link tracking in the presence of the typically ignored electrical actuator dynamics. Note, the formulation of the closed-loop error system is a non-trivial matter since the stability analysis is directly related to the developed error system. With this objective in mind, we define the tracking error to be

$$e = q_d - q \tag{14}$$

where q_d is a $n \times 1$ vector representing the desired link trajectory. We will assume that q_d and its first, second, and third derivatives are all bounded functions of time. This assumption on the 'smoothness' of the desired trajectory ensures that the controller, to be defined later, remains bounded (i.e. requires finite control energy). In addition, we define a $n \times 1$ filtered link tracking error (Slotine 1988) to be

$$r = \dot{e} + \alpha e \tag{15}$$

where α is a positive scalar constant.

We now write the RLED robot dynamics of (8) in terms of the filtered link tracking error of (15)

$$M(q)\dot{r} = w_L - V_m(q, \dot{q})r - K_T I \tag{16}$$

where

$$w_L = M(q)[\ddot{q}_d + \alpha\dot{e}] + V_m(q, \dot{q})[\dot{q}_d + \alpha e] + G(q) + F(\dot{q}) + T_L \tag{17}$$

The error dynamics of (16) clearly lack a torque level control input; therefore we add and subtract a 'fictitious' control term u_L to the right hand side of (16)

$$M(q)\dot{r} = w_L - V_m(q, \dot{q})r - K_T u_L + K_T \eta \tag{18}$$

where

$$\eta = u_L - I \tag{19}$$

is a $n \times 1$ vector representing a current level perturbation to the rigid-link dynamics. Due to the simplicity of the direct-current motor dynamics, this term can be effectively viewed as a torque perturbation to the rigid-link dynamics.

The 'fictitious' current control input u_L is designed such that it would provide global exponential stability of the link-tracking error for the rigid-link dynamics alone (assuming it could be applied directly to the links). We will show later in the development that u_L is actually embedded inside an overall corrective control strategy which is designed at u_E, the motor terminal voltage level.

If the current perturbation term in (18) was equal to zero, then u_L could be designed to yield GES for the link tracking error. Since the current perturbation of (19) is not equal to zero in general, we must design a voltage level controller u_E which compensates for η in (18). To accomplish this control objective, the perturbation dynamics are needed. We therefore differentiate (19) to obtain

$$\dot{\eta} = \dot{u}_L - \dot{I} \tag{20}$$

Multiplying (20) by L substituting (9) yields

$$L\dot{\eta} = w_E - K_T r - u_E \tag{21}$$

where

$$w_E = L\dot{u}_L + R(I, \dot{q}) + T_E + K_T r \tag{22}$$

Note, the interconnection term $K_T r$ has been added and subtracted to the right hand side of (21) to facilitate the stability analysis.

Given exact model knowledge and the error dynamics of (18) and (21), we define the controllers u_L and u_E to be

$$u_L = K_T^{-1}[k_L r + w_L] \quad \text{and} \quad u_E = k_E \eta + w_E \tag{23}$$

where k_L and k_E are positive scalar controller gains.

2.4. *Stability analysis of the RLED error system*

We now state a theorem for the RLED error dynamics given exact model knowledge and the proposed controls of (23).

Theorem 2.1: *The filtered tracking error defined in* (15) *is globally exponentially stable (GES) in the following sense:*

$$\|r(t)\| \leqslant \sqrt{\mathcal{B}} e^{-\lambda t/2} \tag{24}$$

where

$$\mathcal{B} = \frac{\lambda_2}{\lambda_1} \|x(0)\|^2 \tag{25}$$

$$x = [r^{\mathrm{T}}(t)\eta^{\mathrm{T}}(t)]^{\mathrm{T}} \tag{26}$$

$$\lambda_1 = \tfrac{1}{2} \min(m_1, l_1) \tag{27}$$

$$\lambda_2 = \tfrac{1}{2} \max(m_2, l_2) \tag{28}$$

$$\lambda_3 = \min(k_L, k_E) \tag{29}$$

and

$$\lambda = \lambda_3/\lambda_2 \tag{30}$$

Proof: Define the following Lyapunov function (Slotine and Li 1991)

$$V = \tfrac{1}{2} r^{\mathrm{T}} M(q) r + \tfrac{1}{2}\eta^{\mathrm{T}} L \eta = \tfrac{1}{2} x^{\mathrm{T}} P x \tag{31}$$

where

$$P = \begin{bmatrix} M(q) & 0_{n\times n} \\ 0_{n\times n} & L \end{bmatrix}$$

Note, $0_{n\times n}$ and $I_{n\times n}$ are used throughout the development to denote the $n \times n$ zero and identity matrices respectively.

Given Property 2.1, we can state that V is upper and lower bounded as shown

$$\lambda_1 \|x\|^2 \leqslant V \leqslant \lambda_2 \|x\|^2 \tag{32}$$

where λ_1 and λ_2 are defined in (27) and (28).

Differentiating (31) with respect to time along the error system given by (18) and (21) yields

$$\dot{V} = r^{\mathrm{T}}[w_L - K_T u_L] + \eta^{\mathrm{T}}[w_E - u_E] \tag{33}$$

where Property 2.2 has been exploited to reduce the expression. We now substitute the controllers of (23) into (33) to obtain

$$\dot{V} = -r^{\mathrm{T}} k_L r - \eta^{\mathrm{T}} k_E \eta = -x^{\mathrm{T}} Q x \tag{34}$$

where

$$Q = \begin{bmatrix} k_L I_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & k_E I_{n\times n} \end{bmatrix}$$

From (34), $\dot{V}$ can now be upper bounded as

$$\dot{V} \leq -\lambda_3 \|x\|^2 \tag{35}$$

where λ_3 is defined as in (29). From (32) and (35), we can apply Lemma 2.1 (with $\varepsilon = 0$) and utilize the fact that $\|r(t)\| \leq \|x(t)\|$ to yield (24). □

Remark 2.1: Given Theorem 2.1, we can apply Lemma 2.2 (with $\mathcal{A} = 0$) to obtain the following upper bound on the link position tracking error

$$\|e(t)\| \leq n\mathrm{e}^{-\alpha t}\|e(0)\| + \frac{2n\sqrt{|\mathcal{B}|}}{2\alpha - \lambda}[\mathrm{e}^{-\lambda t/2} - \mathrm{e}^{-\alpha t}] \tag{36}$$

where α, $\mathcal{B}$, and λ are defined in (15), (25), and (30), respectively. □

Remark 2.2: It should be noted that we can obtain an upper bound on the link velocity tracking error $\dot{e}(t)$ by applying a triangle inequality argument (Vidyasager 1978) to (15). Specifically, we can state that

$$\|\dot{e}(t)\| \leq \alpha\|e(t)\| + \|r(t)\| \tag{37}$$

where $\|e(t)\|$ and $\|r(t)\|$ are bounded by (36) and (24) respectively. □

Remark 2.3: It should be noted that the corrective control u_E given by (23), depends on the calculation of u_L, $\dot{u}_L$ and I. At first it may appear that this requires measurement of q, $\dot{q}$, $\ddot{q}$ and I. However, since we assume exact knowledge of the dynamic model as given by (8) and (9), we can use this information to eliminate the need for measurement of $\ddot{q}$. That is, $\dot{u}_L$ can be written as

$$\dot{u}_L = K_T^{-1}[k_L[(\ddot{q}_\mathrm{d} - \ddot{q}) + \alpha\dot{e}] + \dot{w}_L] \tag{38}$$

where

$$\dot{w}_L = \dot{M}(q)[\ddot{q}_\mathrm{d} + \alpha\dot{e}] + M(q)\left[\frac{\mathrm{d}}{\mathrm{d}t}\ddot{q}_\mathrm{d} + \alpha(\ddot{q}_\mathrm{d} - \ddot{q})\right] + \dot{V}_m(q, \dot{q})[\dot{q}_\mathrm{d} + \alpha e] + V_m(q, \dot{q})[\ddot{q}_\mathrm{d} + \alpha\dot{e}] + \dot{G}(q) + \dot{F}(\dot{q}) + \dot{T}_L \tag{39}$$

and $\ddot{q}$ is found from (8) to be

$$\ddot{q} = M^{-1}(q)[K_T I - V_m(q, \dot{q})\dot{q} - G(q) - F(\dot{q}) - T_L] \tag{40}$$

After substituting (40) for $\ddot{q}$ in (38) and (39), $\dot{u}_L$ will only depend on the measurement of q, $\dot{q}$, and I. To formulate $\dot{u}_L$, we must assume that the robot dynamics of (8) are once differentiable. This implies that terms such as the joint friction $F(\dot{q})$ must be at least once differentiable. □

Remark 2.4: Our assumption that the desired trajectory is sufficiently smooth is motivated by (38) and (39), since the corrective controller requires the first, second, and third time derivatives of the desired link trajectories. Therefore, these trajectories must be bounded for the motor terminal input voltage to be bounded. □

Remark 2.5: The actual control voltage implemented at the motor terminals, u_E, can be found by making the appropriate substitutions into (23)

$$u_E = L\dot{u}_L + R(I, \dot{q}) + K_T r + k_E K_T^{-1}[k_L r - K_T I] + k_E K_T^{-1}[M(q)[\ddot{q}_d + \alpha\dot{e}] + V_m(q, \dot{q})[\dot{q}_d + \alpha e] + G(q) + F(\dot{q}) + T_L] \tag{41}$$

where $\dot{u}_L$ is given by (38). □

Remark 2.6: The Lyapunov based stability analysis given in this section can be readily extended to compensate for uncertainty in the RLED model described by (8) and (9). As shown in § 3, a nonlinear robust corrective tracking controller which compensates for parametric uncertainty and additive disturbances in the RLED model is designed using a similar Lyapunov based approach. □

Remark 2.7: Our development assumes that the arm is revolute. If the manipulator has prismatic joints, Property 2.1 can be rewritten as

$$m_1\|x\|^2 = \lambda_{\min}\{M(q)\}\|x\|^2 \leqslant x^T M(q)x \leqslant \lambda_{\max}\{M(q)\}\|x\|^2 = m_2(q)\|x\|^2 \tag{42}$$

for arbitrary $n \times 1$ vectors x. The term m_1 is a positive scalar constant and $m_2(q)$ is a positive definite scalar function. The controller u_L is then modified to be

$$u_L = K_T^{-1}[k_L \gamma_2(\|q\|)r + w_L] \tag{43}$$

where $\gamma_2(\|q\|)$ is a positive definite, differentiable scalar function given by

$$\gamma_2(\|q\|) \geqslant \tfrac{1}{2}\max(m_2(q), l_2) \tag{44}$$

The controller u_E remains as defined in (23). Lemmas 2.1 and 2.2 can now be applied to yield GES for the link tracking error as shown previously. □

3. Corrective robust tracking in the presence of RLED model uncertainty

In this section, we design a corrective robust tracking controller to compensate for uncertainties in the RLED robot dynamics. A GUUB stability result for the link tracking error is given for the proposed controller. The controller is robust with regard to parametric uncertainties and additive bounded disturbances in the RLED dynamics presented in § 2.

3.1. *Formulation of the corrective robust tracking controller*

Given the RLED error dynamics of (18) and (21), we define the robust controllers u_L and u_E to be

$$u_L = k_1^{-1}[k_L r + v_L] \quad \text{and} \quad u_E = k_E \eta + v_E \tag{45}$$

where k_L and k_E are positive scalar controller gains and k_1 is defined in (10). The auxiliary control terms v_L and v_E are defined as (Corless 1989)

$$v_L = \frac{r\rho_L^2}{\|r\|\rho_L + \varepsilon_L} \quad \text{and} \quad v_E = \frac{\eta\rho_E^2}{\|\eta\|\rho_E + \varepsilon_E} \tag{46}$$

where ε_L and ε_E are positive scalar control gains which are adjusted to achieve a desired link tracking performance. The terms ρ_L and ρ_E are scalar functions used to 'bound' the uncertainty in the RLED model. These bounding functions are defined as follows

$$\rho_L(e, \dot{e}) \geq \|w_L\| \quad \text{and} \quad \rho_E(e, \dot{e}, \eta) \geq \|w_E\| \tag{47}$$

where w_L and w_E are defined in (17) and (22) respectively.

The procedure for calculating the bounding functions will be discussed later in the development. For now, we simply assume their existence. It should be emphasized that these bounding functions depend only on measurements of I, q and $\dot{q}$. This is due to the fact that the proposed electrical dynamics are a set of first-order differential equations.

3.2. *Stability analysis of RLED error system*

We now show that the corrective robust controllers given in (45) with the auxiliary control terms given in (46) result in GUUB stability of the link tracking error. It should be emphasized that the proposed robust controller does not require exact knowledge of the dynamics as described by (8) and (9). Since w_L and w_E in (17) and (22) contain unknown parameters and additive disturbances, there will be some uncertainty associated with these terms. Using bounds on the uncertainty given by (47), the robust controllers of (45) ensure 'good' link tracking performance.

A theorem for the stability of the RLED error dynamics given parametric uncertainty and bounded additive disturbances under the proposed controls of § 3.1 are now stated.

Theorem 3.1: *The filtered tracking error defined in* (15) *is GUUB in the following sense*:

$$\|r(t)\| \leq [\mathcal{A} + \mathcal{B}e^{-\lambda t}]^{1/2} \tag{48}$$

where

$$\mathcal{A} = \frac{\varepsilon}{\lambda_1 \lambda} \tag{49}$$

$$\mathcal{B} = \frac{\lambda_2}{\lambda_1}\|x(0)\|^2 - \frac{\varepsilon}{\lambda_1 \lambda} \tag{50}$$

$$\varepsilon = \varepsilon_L + \varepsilon_E \tag{51}$$

and x, λ_1, λ_2, λ_3 *and* λ *are defined in* (26)–(30), *respectively*.

Proof: Starting from (33) in the Proof of Theorem 2.1, we have

$$\dot{V} = r^{\mathrm{T}}[w_L - K_T u_L] + \eta^{\mathrm{T}}[w_E - u_E] \tag{52}$$

Substituting (45) for u_L and u_E into (52) yields

$$\dot{V} = -r^{\mathrm{T}} K_T k_L r / k_1 - \eta^{\mathrm{T}} k_E \eta + [r^{\mathrm{T}} w_L - r^{\mathrm{T}} K_T v_L / k_1] + [\eta^{\mathrm{T}} w_E - \eta^{\mathrm{T}} v_E] \tag{53}$$

Substituting (46) for v_L and v_E into (53) yields

$$\dot{V} = -x^{\mathrm{T}} Q x + \left[r^{\mathrm{T}} w_L - r^{\mathrm{T}} K_T \left[\frac{r\rho_L^2}{\|r\|\rho_L + \varepsilon_L}\right] / k_1\right] + \left[\eta^{\mathrm{T}} w_E - \eta^{\mathrm{T}} \left[\frac{\eta \rho_E^2}{\|\eta\|\rho_E + \varepsilon_E}\right]\right] \tag{54}$$

where

$$Q = \begin{bmatrix} K_T k_1^{-1} k_L & 0_{n\times n} \\ 0_{n\times n} & k_E I_{n\times n} \end{bmatrix} \tag{55}$$

From (10) and (54), $\dot{V}$ can be upper bounded as

$$\dot{V} \leq -\lambda_3\|x\|^2 + \left[\|r\|\|w_L\| - \left[\frac{\|r\|^2\rho_L^2}{\|r\|\rho_L + \varepsilon_L}\right]\right] + \left[\|\eta\|\|w_E\| - \left[\frac{\|\eta\|^2\rho_E^2}{\|\eta\|\rho_E + \varepsilon_E}\right]\right] \tag{56}$$

where λ_3 is defined as in (29). Combining the bracketed terms on the right hand side of (56) under common denominators and applying (47) yields the new upper bound on $\dot{V}$

$$\dot{V} \leq -\lambda_3\|x\|^2 + \varepsilon \tag{57}$$

where ε is defined in (51). From (32) and (57), we can apply Lemma 2.1 and utilize the fact that $\|r(t)\| \leq \|x(t)\|$ to yield (48). □

Remark 3.1: Given (48) through (51), we can obtain a new upper bound for the filtered tracking error as

$$\|r(t)\| \leq \sqrt{\mathscr{A}} + \sqrt{|\mathscr{B}|}\mathrm{e}^{-\lambda t/2} \tag{58}$$

Applying Lemma 2.2 with (15) and (58) yields the link position tracking error bound

$$\|e(t)\| \leq n\mathrm{e}^{-\alpha t}\|e(0)\| + n\frac{\sqrt{\mathscr{A}}}{\alpha}[1 - \mathrm{e}^{-\alpha t}] + \frac{2n\sqrt{|\mathscr{B}|}}{2\alpha - \lambda}[\mathrm{e}^{-\lambda t/2} - \mathrm{e}^{-\alpha t}] \tag{59}$$

where α, λ, $\mathscr{A}$, and $\mathscr{B}$ are defined in (15), (30), (49), and (50), respectively. Note that the control parameters can be arbitrarily adjusted to give a desired transient response and an ultimate bound for the link position tracking error. □

Remark 3.2: It should be noted that we can obtain an upper bound on the link velocity tracking error $\dot{e}(t)$ by applying a triangle inequality argument as shown in (37) with $\|e(t)\|$ and $\|r(t)\|$ defined by (59) and (48), respectively. □

Remark 3.3: It is important to note that the corrective robust tracking controller does not require that the robot dynamics of (8) be differentiable, as was the case for the controllers designed using exact knowledge in § 2. This is due to the fact that the robust controller u_E depends only on a bound of $\dot{u}_L$, rather than $\dot{u}_L$ itself. □

3.3. *Formulation of bounding functions*

We now illustrate how the bounding functions given in (47) can be found. With regard to $\rho_L(e, \dot{e}) \geq \|w_L\|$, it has been shown (Dawson *et al.* 1990) that

$$\rho_L = \zeta_2\|\underline{e}(t)\|^2 + \zeta_1\|\underline{e}(t)\| + \zeta_0 \tag{60}$$

where $\underline{e}(t) = [e^{\mathrm{T}}(t)\ \dot{e}^{\mathrm{T}}(t)]^{\mathrm{T}}$, and ζ_2, ζ_1, and ζ_0 are positive scalar constants that depend on estimates of the upper bounds on parametric quantities such as the largest payload mass.

Although we do not provide a general expression for ρ_E, a procedure for finding this function is outlined. It is easy to establish that the dynamics of (17) and (22) can be bounded by combinations of constants and functions of the measurable quantities q, $\dot{q}$ and I. That is, we can compute $\dot{u}_L$ from its partial derivatives as

$$\dot{u}_L = \frac{\partial u_L(t)}{\partial t} + \frac{\partial u_L(\underline{e})}{\partial \underline{e}}[\underline{\dot{e}}] \tag{61}$$

Therefore, $\|\dot{u}_L\|$ can be written in terms of combinations of constants and functions of q, $\dot{q}$, I. This implies that it is possible to generate a $\rho_E(e, \dot{e}, \eta)$ such that $\rho_E \geqslant \|w_E\|$.

4. Simulation

In this section, we give simulation results of the theoretical developments presented in § 2 and 3. The simulated rigid-link dynamics are of a planar two-link revolute manipulator which can be found in Craig (1986). The actuator dynamics are assumed to be the permanent magnet direct-current motor (Tarn *et al.* 1991) (i.e. first-order dynamics)

$$L\dot{I} + RI + k_T\dot{q} = u_E \tag{62}$$

The RLED system parameters are assumed identical for each link and are given as

$$L = 0{\cdot}01\,\mathrm{H},\ R = 1{\cdot}0\,\Omega,\ K_T = 1{\cdot}0\,\mathrm{Nm\,A^{-1}} \tag{63}$$

and

$$D = 1{\cdot}0\,\mathrm{m},\ M = 1{\cdot}0\,\mathrm{kg},\ Fd = 0{\cdot}1\,\mathrm{Nm\,m\,s^{-1}},\ \text{and } G = 9{\cdot}81\,\mathrm{kg\,m\,s^{-1}} \tag{64}$$

which represent the motor armature winding inductance, armature winding resistance, torque coupling/back-e.m.f. coefficient, link length, link mass, coefficient of dynamic friction, and the acceleration of gravity respectively. The desired joint trajectories are defined to be

$$q_d(t) = \sin(t)\,\mathrm{rad} \tag{65}$$

The initial joint position errors, joint velocity errors, and motor current perturbations are set to zero. In addition, all controller gains are set to 10. For control purposes, all parameters are assumed to be off by +50%. The position and velocity tracking errors for the corrective tracking controller under these assumptions are shown in Fig. 1(a), and the associated motor torques and voltages are shown in Fig. 1(b).

To simulate the corrective robust tracking controller, all RLED parameters are set to the nominal values given above. For control purposes, all parameters are upper bounded at +50% of the nominal values. In addition, additive disturbances are injected into the RLED dynamics as shown in (8) and (9) of the form

$$T_L = T_E = 0{\cdot}1[\sin(t)\sin(3t/2)]\,\mathrm{Nm\ and\ V,\ respectively} \tag{66}$$

These disturbances are also assumed to be upper bounded by +50%. The desired joint trajectories are once again assumed to be

$$q_d(t) = \sin(t)\,\mathrm{rad} \tag{67}$$

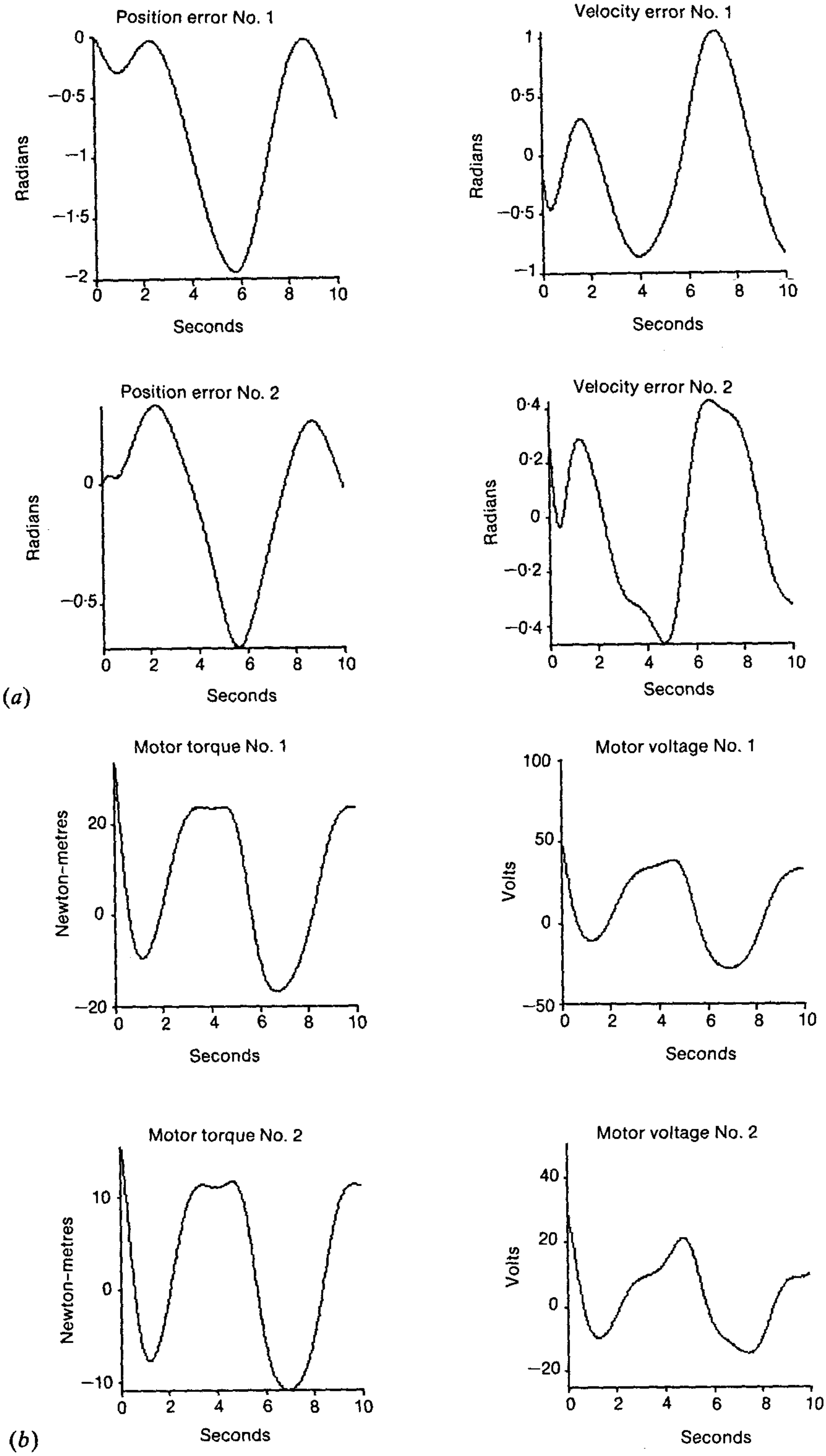

Figure 1. (*a*) Link tracking errors for the corrective tracking controller; (*b*) motor torques and voltages for the corrective tracking controller.

and the initial joint position errors, joint velocity errors, and motor current perturbations are set to zero. In addition, all controller gains are set to the values assumed for the corrective tracking controller given above. As shown in Figs 2(*a*) and 2(*b*), the corrective robust tracking controller out performs the corrective tracking controller despite the greater degree of system uncertainty.

5. Conclusion

In this paper, we developed a design approach that can be used to design robot tracking controllers which compensate for actuator dynamics in RLED robots. Under the assumption of exact model knowledge, the design approach yields a corrective tracking controller which is globally exponentially stable with respect to the link tracking error. The design approach is intuitively simple in that it is based on concepts that most control engineers can readily identify with. The approach was then extended to the RLED robot dynamics with parametric uncertainty and additive disturbances. A corrective robust tracking controller was then developed which gave a globally ultimately uniformly bounded stability result for the link tracking errors despite the uncertainty in the dynamic model. Simulation results show that the corrective robust tracking controller effectively compensates for uncertainty in the rigid-link and joint actuator dynamics.

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Appendix

Proof of Lemma 2.1: Given (1) through (3), we multiply (2) by $-\lambda = -\lambda_3/\lambda_2$ to obtain

$$-\lambda_3\|x(t)\|^2 \leqslant -\lambda V(x, t) \leqslant -\lambda_1\lambda\|x(t)\|^2 \tag{A 1}$$

We can use (A 1) to place the new upper bound on $\dot{V}$ in (3) as

$$\dot{V}(x, t) \leqslant -\lambda V(x, t) + \varepsilon \tag{A 2}$$

Multiplying (A 2) by $e^{\lambda t}$ and rearranging terms yields

$$\frac{d}{dt}[e^{\lambda t}V(x, t)] = e^{\lambda t}\dot{V}(x, t) + e^{\lambda t}\lambda V(x, t) \leqslant \varepsilon e^{\lambda t} \tag{A 3}$$

or equivalently

$$\int_0^t d[e^{\lambda t}V(x, t)] \leqslant \int_0^t \varepsilon e^{\lambda t}\,dt \tag{A 4}$$

Evaluating (A 4) and rearranging terms yields the new upper bound on V

$$V(x, t) \leqslant e^{-\lambda t}V(x(0), 0) + \frac{\varepsilon}{\lambda}[1 - e^{-\lambda t}] \tag{A 5}$$

To bound V in (A 5), we apply (2) to obtain

$$\lambda_1\|x(t)\|^2 \leqslant \lambda_2\|x(0)\|^2 e^{-\lambda t} + \frac{\varepsilon}{\lambda}[1 - e^{-\lambda t}] \tag{A 6}$$

Solving (A 6) for $\|x(t)\|$ shows that the state $x(t)$ is GUUB in the sense of (4). □

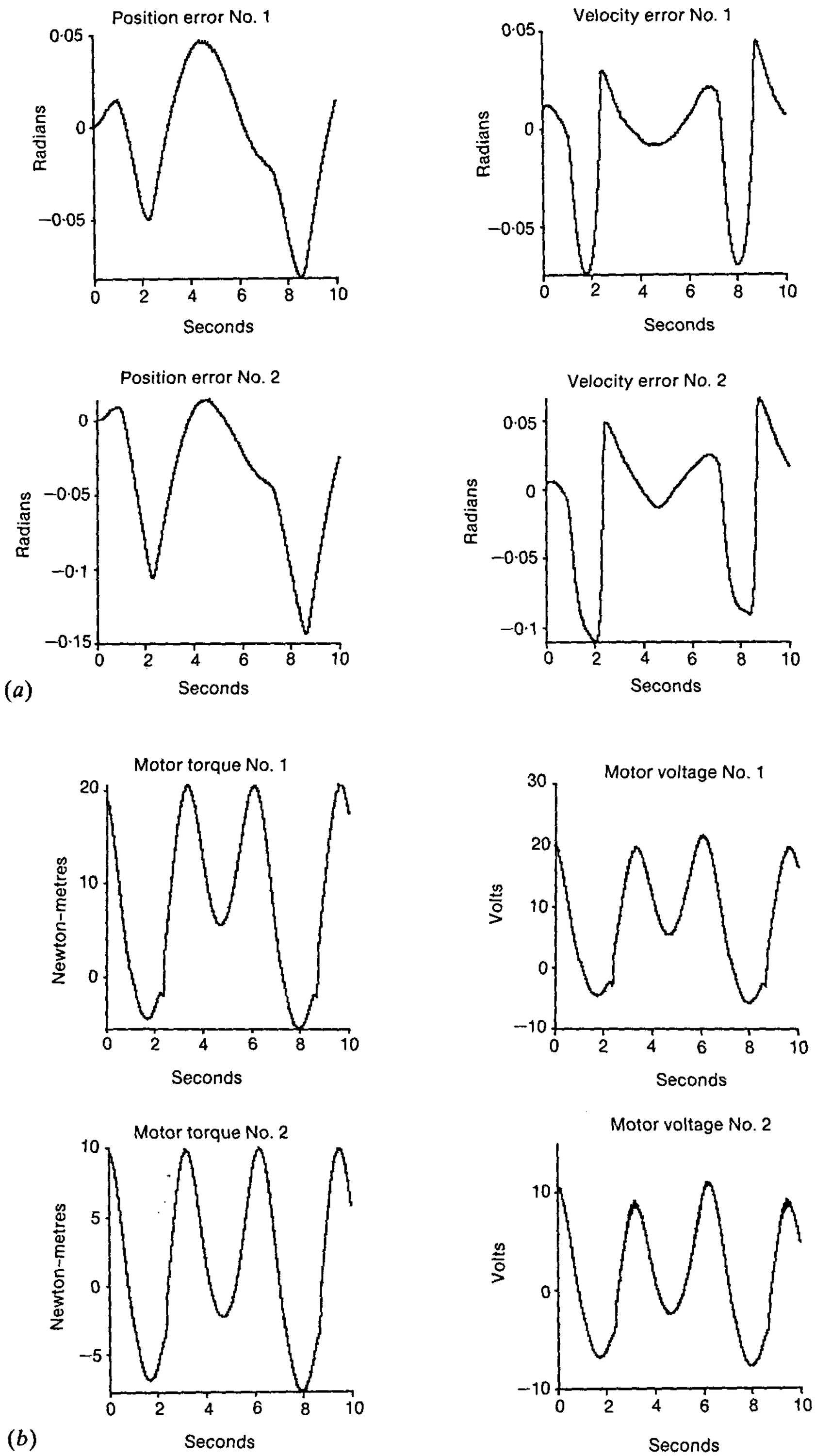

Figure 2. (*a*) Link tracking errors for the corrective robust tracking controller; (*b*) motor torques and voltages for the corrective robust tracking controller.

Proof of Lemma 2.2: Given (5), we can state that the ith component of the $y(t)$ and $z(t)$ vectors have the following frequency domain relationship

$$y_i(s) = [sz_i(s) - z_i(0)] + \alpha z_i(s) \quad \text{(A 7)}$$

where s denotes a frequency domain variable. Solving (A 7) for $z_i(s)$ and transforming the expression back to the time domain yields

$$z_i(t) = z_i(0)e^{-\alpha t} + \int_0^t e^{-\alpha(t-\sigma)} y_i(\sigma)\, d\sigma \quad \text{(A 8)}$$

Therefore the ith component of the vector $z(t)$ can be upper bounded in terms of (A 8) as

$$|z_i(t)| \leqslant |z_i(0)|e^{-\alpha t} + \int_0^t e^{-\alpha(t-\sigma)} |y_i(\sigma)|\, d\sigma \quad \text{(A 9)}$$

Applying $\|z(t)\| \leqslant \sum_{i=1}^{n} |z_i(t)|$ to the left-hand side of (A 9), and $|z_i(t)| \leqslant \|z(t)\|$ and $|y_i(t)| \leqslant \|y(t)\|$ to the right-hand side of (A 9) yields an upper bound on $z(t)$

$$\|z(t)\| \leqslant n\|z(0)\|e^{-\alpha t} + n\int_0^t e^{-\alpha(t-\sigma)} \|y(\sigma)\| d\sigma \quad \text{(A 10)}$$

Substituting the upper bound for $\|y(t)\|$ given by (6) into (A 10) and evaluating the integral yields (7). □

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