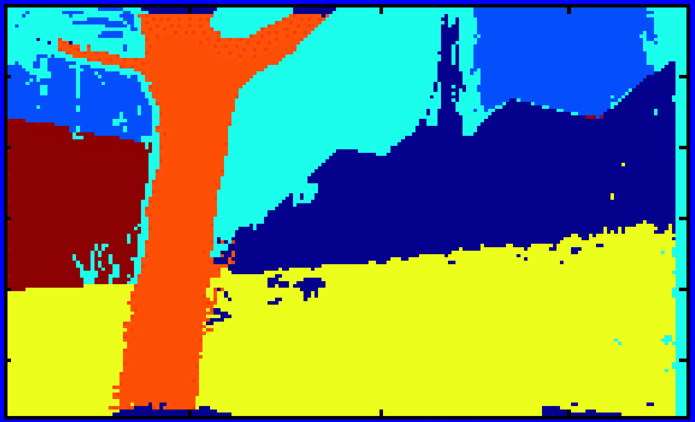
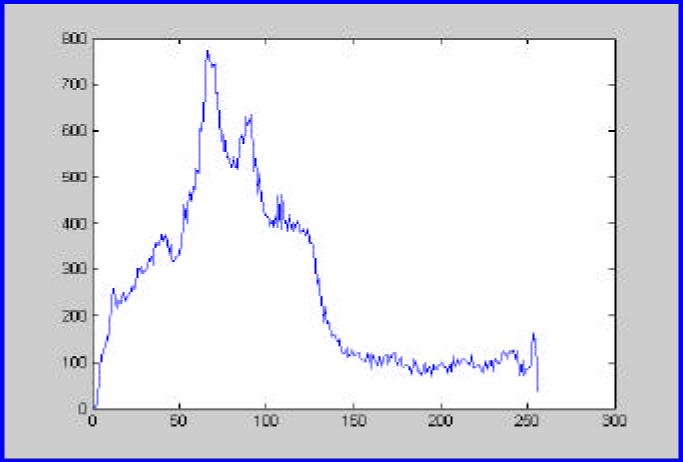
Lecture-11

Region Segmentation -II

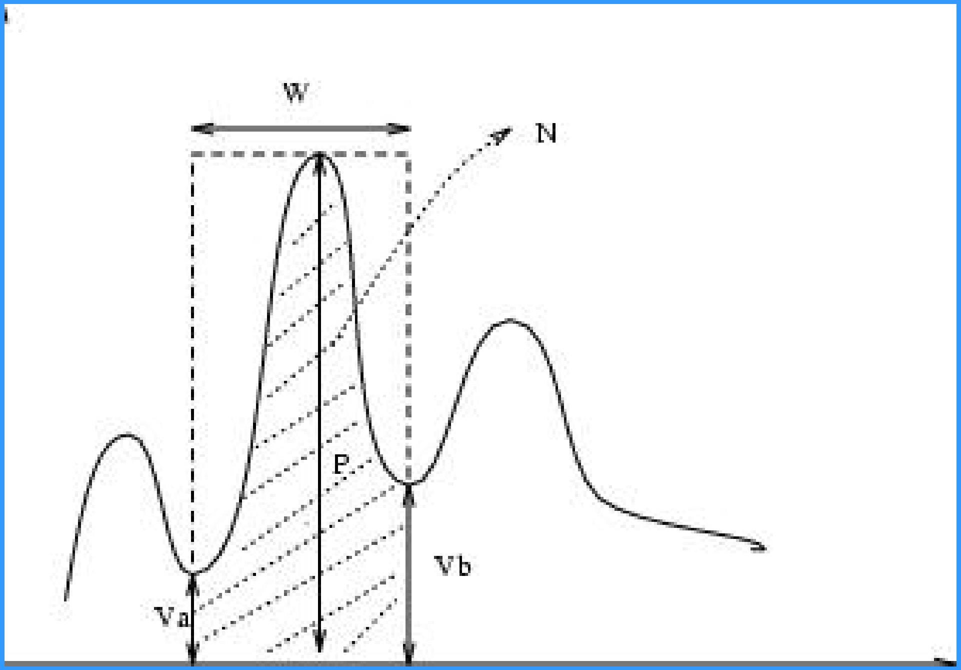
Region Segmentation



Realistic Histogram

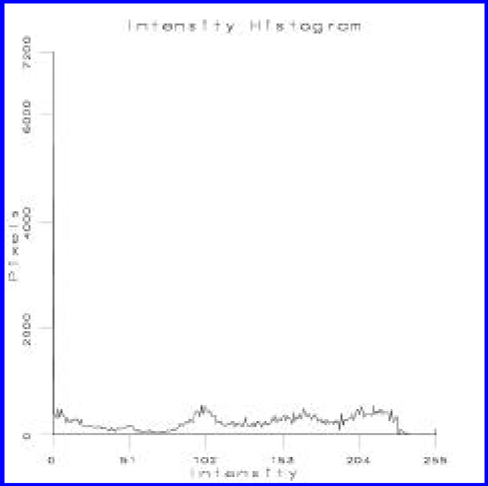


Peakiness Test



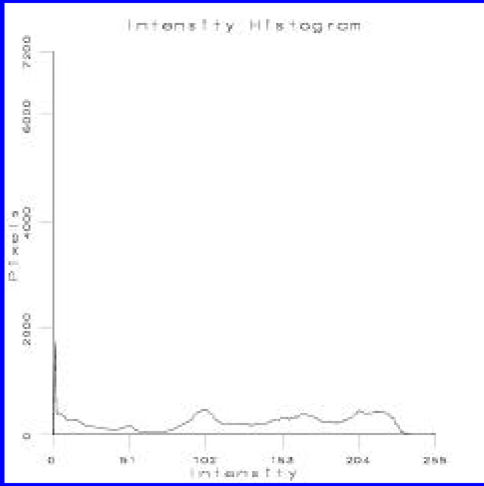
$$Peakiness = \left(1 - \frac{(V_a + V_b)}{2P}\right) \cdot \left(1 - \frac{N}{(W.P)}\right)$$

Example-II

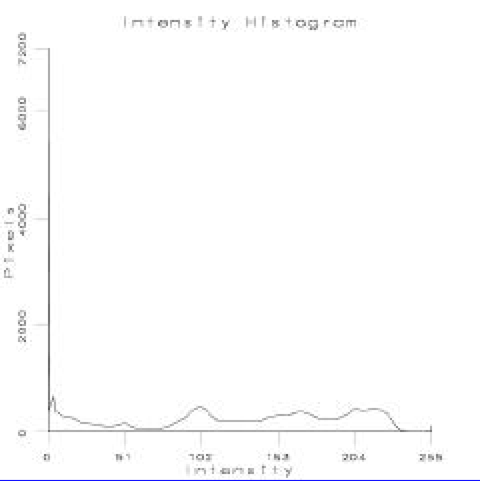


93 peaks

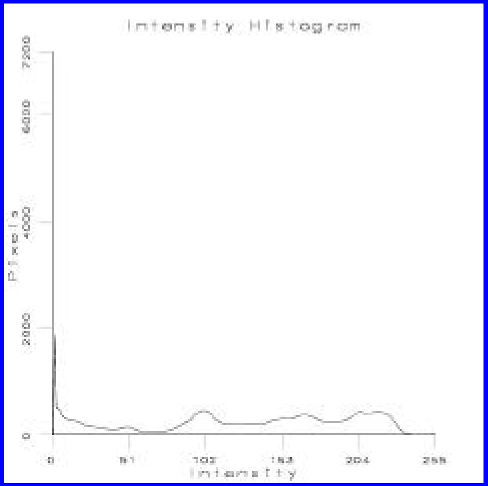
Smoothed Histograms



Smoothed histogram
(averaging using mask
Of size 5, one pass
gives 54 peaks
Peakiness test gives
18 peaks



Twice Smoothed
histogram
21 peaks
After peakiness
Gives 7 peaks



After 3 Smoothings

11 peaks
After peakiness
Gives 4 peaks

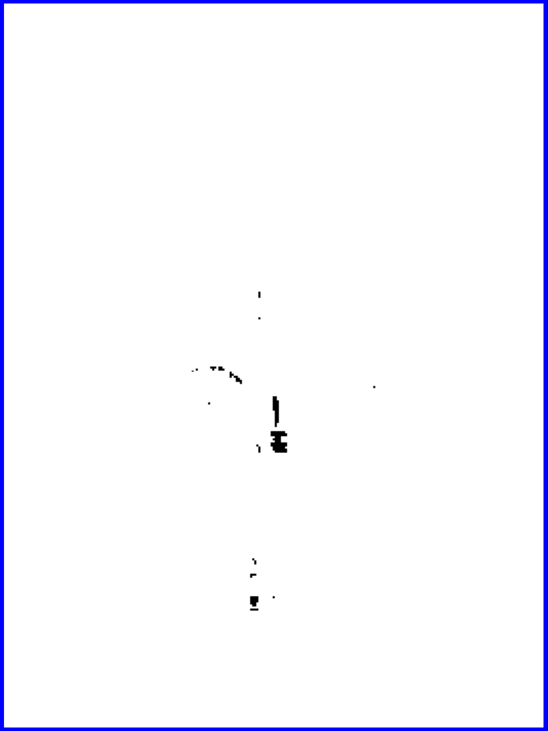
Regions

Regions from peak1
(0,…..,40)

Regions from peak2
(40,….,116)

Regions

Regions from peak 3
(116,….,243)



Regions from peak 4
(243,….,255)

Steps in Seed Segmentation Using Histogram

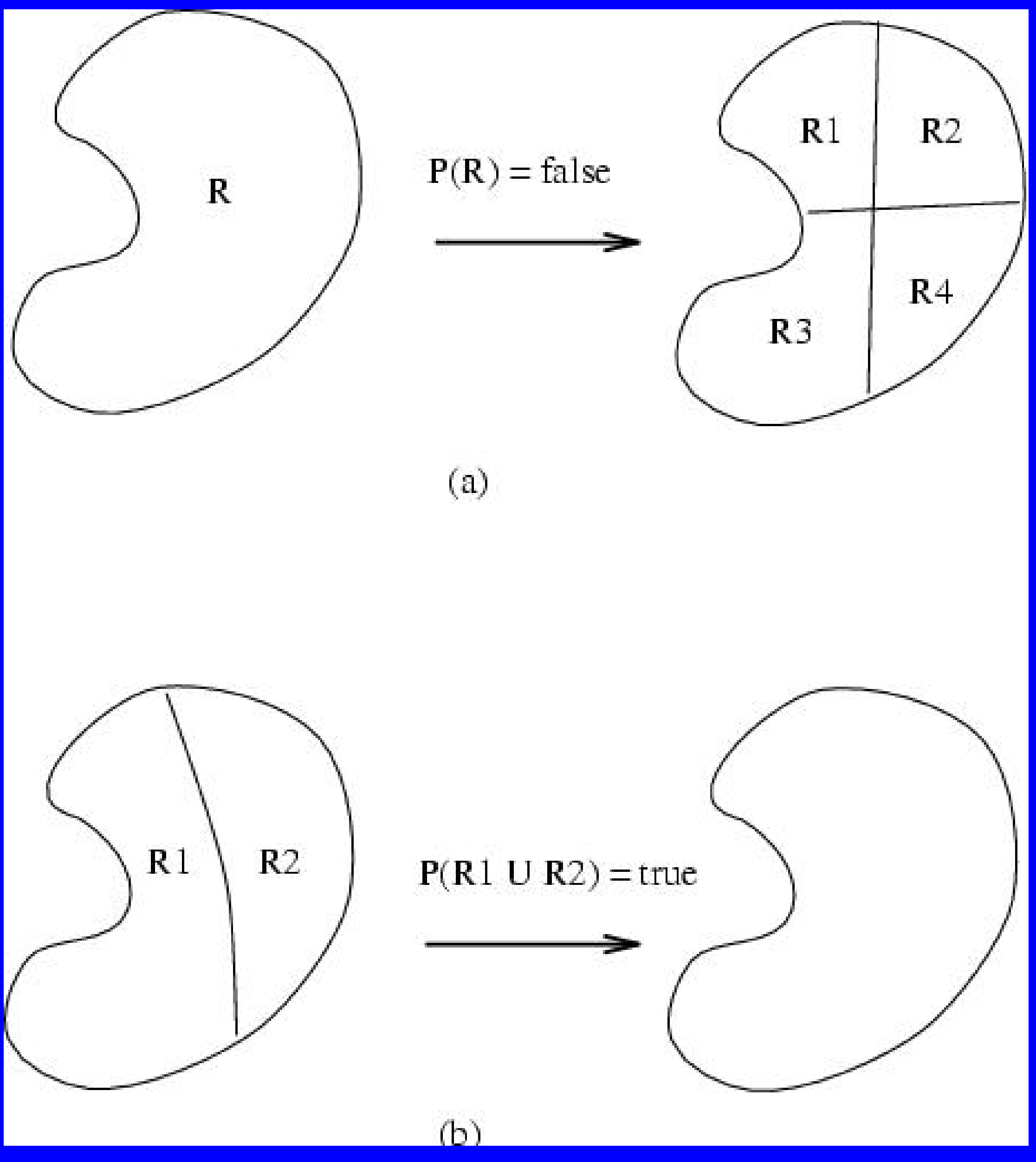
1. Compute the histogram of a given image.
2. Smooth the histogram by averaging peaks and valleys in the histogram.
3. Detect good peaks by applying thresholds at the valleys.
4. Segment the image into several binary images using thresholds at the valleys.
5. Apply connected component algorithm to each binary image find connected regions.

Improving Seed Segmentation

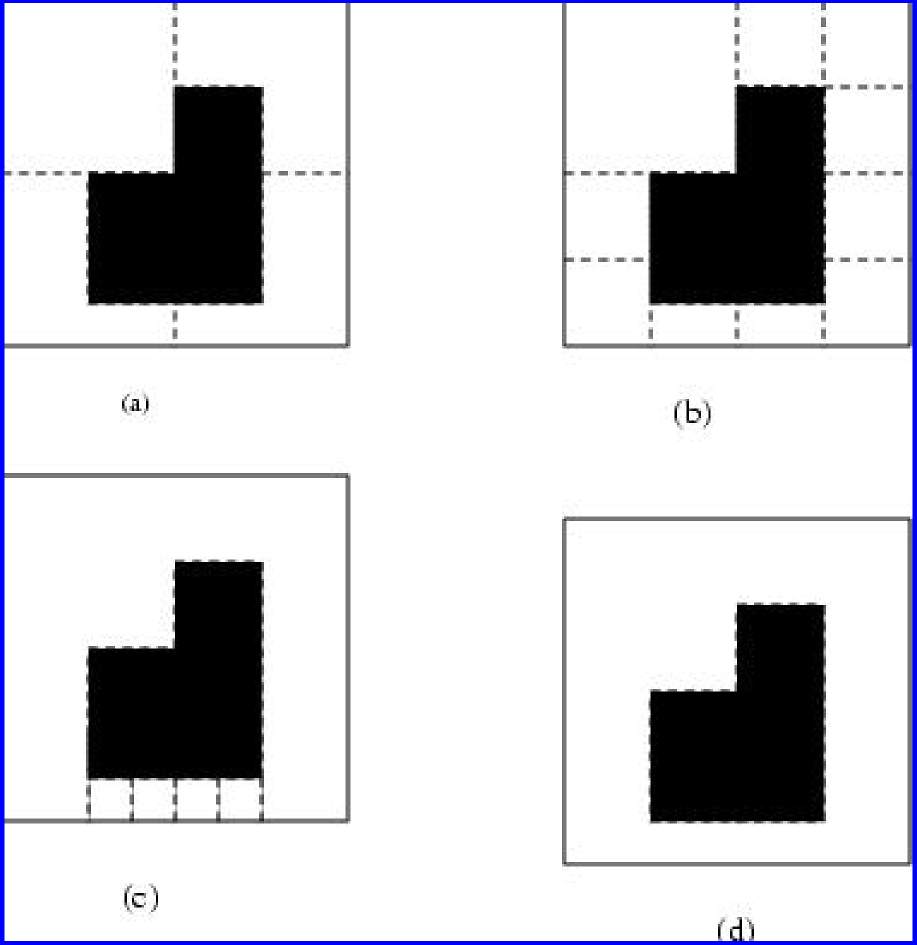
- Merge small neighboring regions
- Split large regions
- Remove weak boundaries between adjacent regions

Split and Merge

1. Split region R into four adjacent regions (quadrants) if $Predicate(R) = false$.
2. Merge any two adjacent regions R_1 and R_2 if $R_1 \; U \; R_2 = true$.
3. Stop when no further merging and splitting are possible.


R
P(R) = false
R1
R2
R3
R4
(a)
R1
R2
P(R1 U R2) = true
(b)

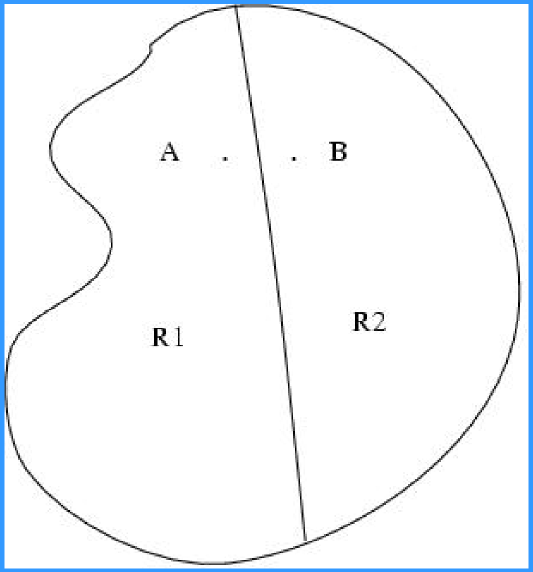
Split and Merge



Phagocyte Algorithm: Weakness of Boundaries

$$W(A,B)=\begin{cases}1 & \text{if} \quad S(A,B)<T_1\\ 0 & \text{Otherwise}\end{cases}$$

$$W(Boundary)=\sum_{\forall A,B} W(A,B)$$



Phagocyte Algorithm

1. Merge two regions if

Phagocyte

Where P_1 and P_2 are the perimeters of regions R_1 and R_2.
if threshold T_2 > 1/2 then the resulting boundary must shrink, and
If threshold T_2 <1/2 then the boundary may grow

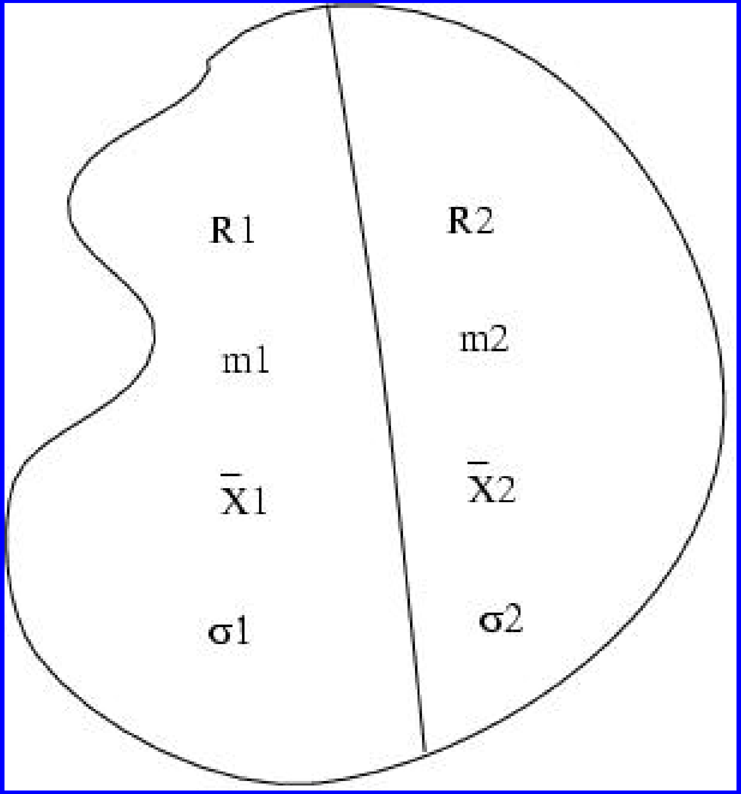
$$\frac{W(Boundary)}{\min(P_1, P)_2} > T_2, \quad 0 \leq T_2 \leq 1$$

Weakness

2. Merge regions if

$$\frac{W(Boundary)}{\text{Total number of points on the border}} > T_3, \quad 0 < T_3 \leq 1$$

Merging Using Likelihood Ratio Test



Merging Using Likelihood Ratio Test

H_1: There are two regions

H_2: There is one region

$$p(x) = \frac{1}{\sqrt{2\hat{\lambda} s}} e^{-\frac{(x-\bar{x})^2}{2s^2}}$$

$$p(x_1, \ldots, x_{m_1}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_1}} \right)^{m_1} e^{-\frac{m_1}{2}} \qquad p(x_{m_1+1}, x_{m_1+2}, \ldots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_2}} \right)^{m_2} e^{-\frac{m_2}{2}}$$

$$p(x_1, x_{2,\ldots}, x_{m_1}, x_{m_1+1}, x_{m_1+2}, \ldots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_0}} \right)^{m_1+m_2} e^{-\frac{m_1+m_2}{2}}$$

$$P(H_2) = P(x_1, x_{2,\ldots}, x_{m_1}, x_{m_1+1}, x_{m_1+2}, \ldots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_0}} \right)^{m_1+m_2} e^{-\frac{m_1+m_2}{2}}$$

$$P(H_1) = p(x_1, \ldots, x_{m_1}).p(x_{m_1+1}, x_{m_1+2}, \ldots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_1}} \right)^{m_1} e^{-\frac{m_1}{2}} . \left(\frac{1}{\sqrt{2\hat{\lambda} s_2}} \right)^{m_2} e^{-\frac{m_2}{2}}$$

Merging Using Likelihood Ratio Test

$$P(H_2) = P(x_1, x_{2,\ldots}, x_{m_1}, x_{m_1+1}, x_{m_1+2}, \ldots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_0}}\right)^{m_1+m_2} e^{-\frac{m_1+m_2}{2}}$$

$$P(H_1) = p(x_1, \ldots, x_{m_1}).p(x_{m_1+1}, x_{m_1+2}, \ldots, x_{m_1+m_2}) = \left(\frac{1}{\sqrt{2\hat{\lambda} s_1}}\right)^{m_1} e^{-\frac{m_1}{2}} . \left(\frac{1}{\sqrt{2\hat{\lambda} s_2}}\right)^{m_2} e^{-\frac{m_2}{2}}$$

$$LH = \frac{P(H_1)}{P(H_2)} = \frac{(s_0)^{m_1+m_2}}{(s_1)^{m_1}(s_2)^{m_2}}$$

Merge regions if $LH < T$.

Region Adjacency Graph

- Regions are nodes
- Adjacent regions are connected by an arc

2	2	2	2	4
1	1	2	2	4
1	1	2	4	4
2	2	2	4	3
5	5	5	4	3

(a)

4 -> 3

2	2	2	2	3
1	1	2	2	3
1	1	2	3	3
2	2	2	3	3
5	5	5	3	3

(b)

3 -> 2

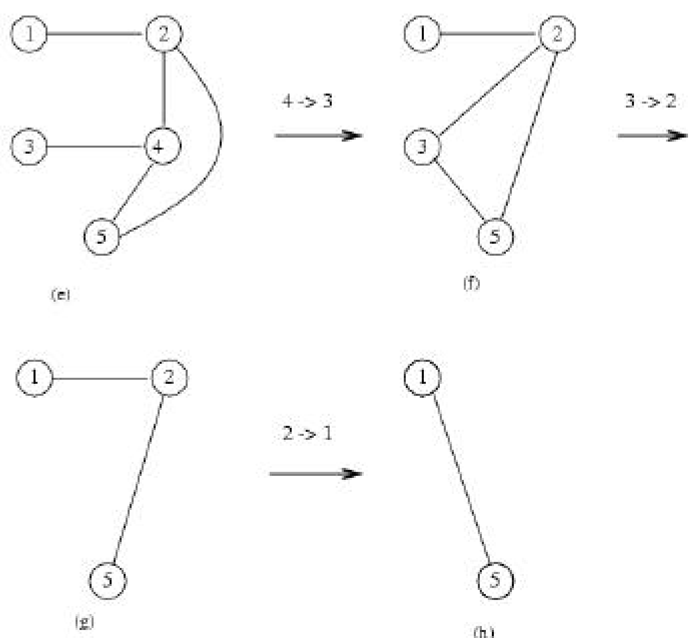
2	2	2	2	2
1	1	2	2	2
1	1	2	2	2
2	2	2	2	2
5	5	5	2	2

(c)

2 -> 1

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
5	5	5	1	1

(d)



Issues in Region Growing

- The number of thresholds used in the algorithm.
- The order of merging is very important.
- Seed segmentation is important.

Edge Detection Vs Region Segmentation

- Region segmentation results in closed boundaries, while the boundaries obtained by edge detection are not necessarily closed.
- Region segmentation can be improved by using multi-spectral images (e.g. color images), however there is not much an advantage in using multi-spectral images in edge detection.
- The position of a boundary is localized in edge detection, but not necessarily in region segmentation.

Geometrical Properties

Area

$$A = \sum_{x=0}^{m} \sum_{y=0}^{n} B(x, y)$$

Centroid

$$\bar{x} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y)}{A}, \quad \bar{y} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)}{A}$$

Moments

General Moments

$$m_{pq} = \int\int x^p y^q B(x,y)\,dxdy$$

Discrete

$$M_x^1 = \sum_{x=0}^{m}\sum_{y=0}^{n} xB(x,y),\quad M_y^1 = \sum_{x=0}^{m}\sum_{y=0}^{n} yB(x,y)$$

$$M_x^2 = \sum_{x=0}^{m}\sum_{y=0}^{n} x^2B(x,y),\quad M_y^2 = \sum_{x=0}^{m}\sum_{y=0}^{n} y^2B(x,y),\quad M_{xy}^2 = \sum_{x=0}^{m}\sum_{y=0}^{n} xyB(x,y)$$

Moments

Central Moments (Translation Invariant)

$$m_{pq} = \int\int (x-\bar{x})^p (y-\bar{y})^q B(x,y)\ d(x-\bar{x})d(y-\bar{y})$$

$$\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}}$$ Centroid

Geometrical Properties

Area

$$A = \sum_{x=0}^{m}\sum_{y=0}^{n} B(x,y)$$

Centroid

$$\bar{x} = \frac{\sum_{x=0}^{m}\sum_{y=0}^{n} xB(x,y)}{A}, \quad \bar{y} = \frac{\sum_{x=0}^{m}\sum_{y=0}^{n} yB(x,y)}{A}$$

Moments

$$M_x^1 = \sum_{x=0}^{m}\sum_{y=0}^{n} xB(x,y), \quad M_x^1 = \sum_{x=0}^{m}\sum_{y=0}^{n} xB(x,y), \quad M_y^1 = \sum_{x=0}^{m}\sum_{y=0}^{n} yB(x,y)$$

$$M_x^2 = \sum_{x=0}^{m}\sum_{y=0}^{n} x^2 B(x,y), \quad M_y^2 = \sum_{x=0}^{m}\sum_{y=0}^{n} xB(x,y), \quad M_y^1 = \sum_{x=0}^{m}\sum_{y=0}^{n} y^2 B(x,y)$$

Compactness

$$C = 4\lambda \frac{A}{P^2}$$

Moments

Binary image

General Moments

$$m_{pq} = \int\int x^p y^q r(x, y) dxdy$$

Central Moments (Translation Invariant)

$$m_{pq} = \int\int (x - \bar{x})^p (y - \bar{y})^q r(x, y) \; d(x - \bar{x}) d(y - \bar{y})$$

$$\bar{x} = \frac{m_{10}}{m_{00}}, \bar{y} = \frac{m_{01}}{m_{00}}$$

centroid

Central Moments

$$m_{00} = m_{00} \equiv m$$

$$m_{01} = 0$$

$$m_{10} = 0$$

$$m_{20} = m_{20} - m\bar{x}^2$$

$$m_{11} = m_{11} - m\overline{xy}$$

$$m_{02} = m_{02} - m\bar{y}^2$$

$$m_{30} = m_{30} - 3m_{20}\bar{x} + 2m\bar{x}^3$$

$$m_{21} = m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2m\bar{x}^2 y$$

$$m_{12} = m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2m\bar{x}y^2$$

$$m_{03} = m_{03} - 3m_{02}\bar{y} + 2m\bar{y}^3$$

Moments

Hu Moments: translation, scaling and rotation invariant

$$u_1 = m_{20} + m_{02}$$

$$u2 = (m_{20} - m_{02})^2 + m_{11}{}^2$$

$$u_3 = (m_{30} - 3m_{12})^2 + (3m_{12} - m_{03})^2$$

$$u_4 = (m_{30} + m_{12})^2 + (m_{21} + m_{03})^2$$

$$\vdots$$

Half size, mirror
Rotated 2, rotated 45

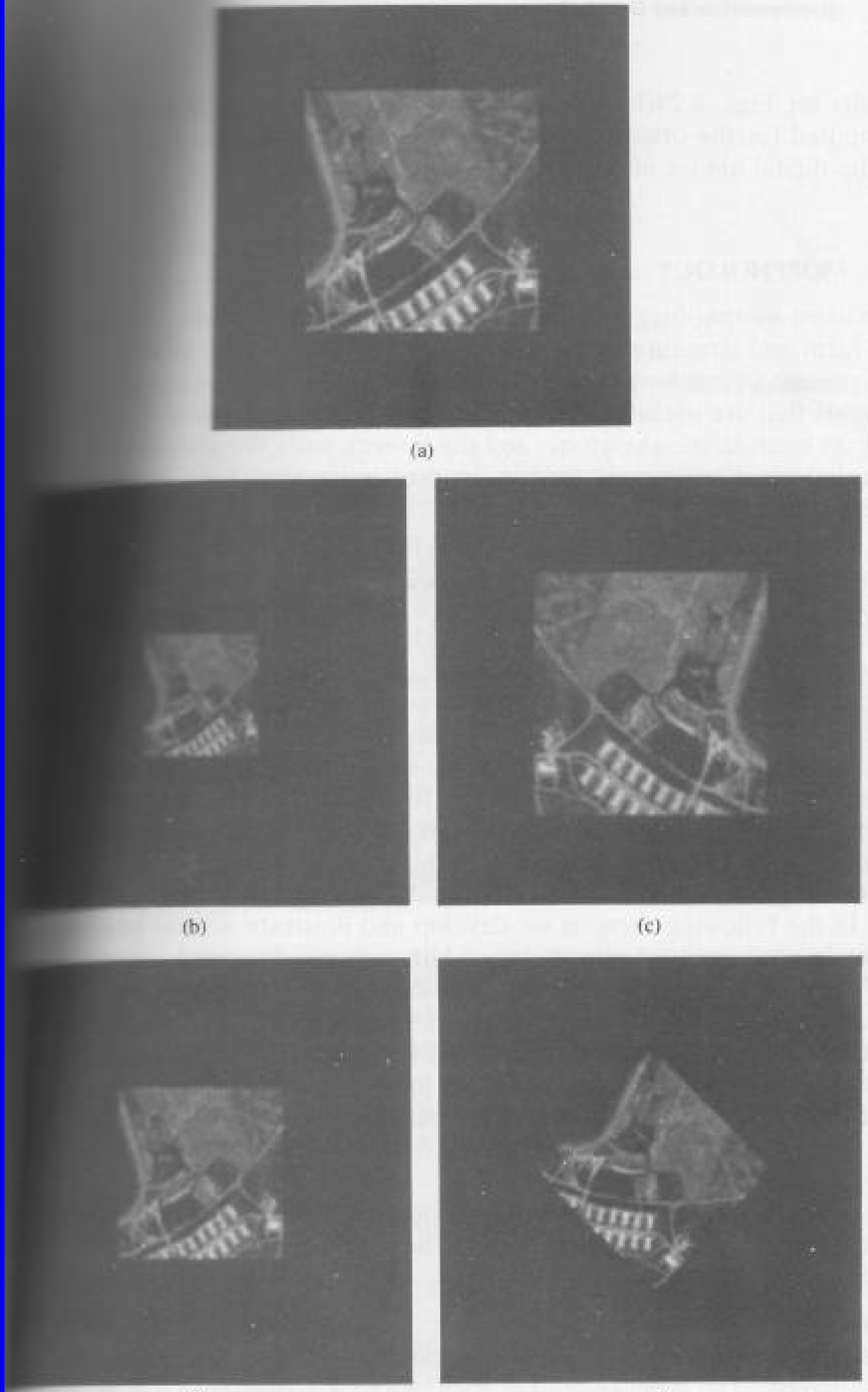


Table 8.2 Moment Invariants for the Images in Figs. 8.24(a)–(e)

Invariant (Log)	*Original*	*Half Size*	*Mirrored*	*Rotated 2°*	*Rotated* [illegible]
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

Hu moments

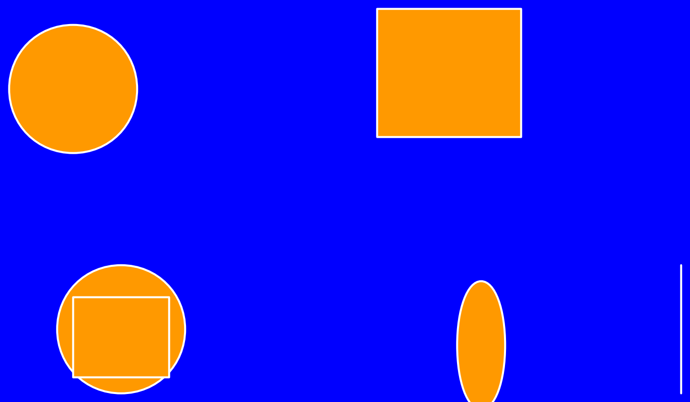
Perimeter & Compactness

Perimeter: The sum of its border points of the region. A pixel which has at least one pixel in its neighborhood from the background is called a border pixel.

Compactness

$$C = \frac{P^2}{4pA}$$

Circle is the most compact, has smallest value



Area decreases

Orientation of the Region

Least second moment

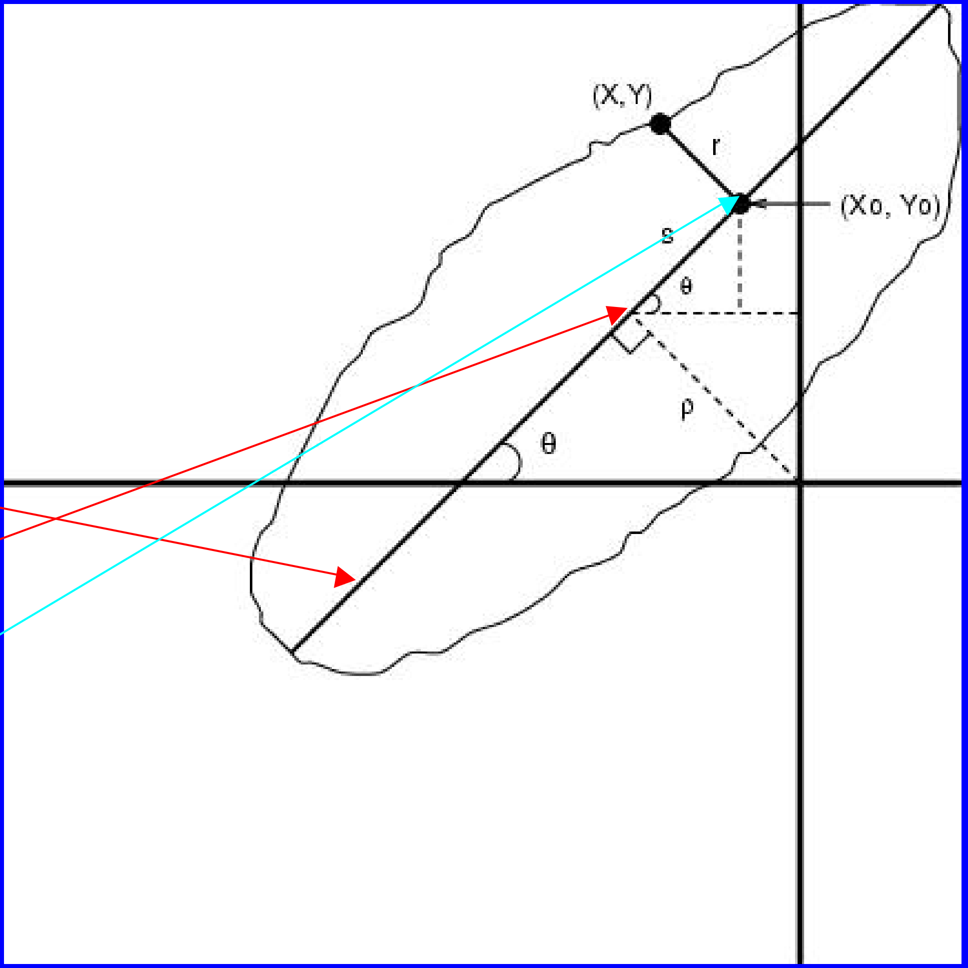
Minimize

$$E = \iint r^2 B(x, y)\,dx\,dy$$

$$x \sin q - y \cos q + r = 0$$

$$(-r \sin q, r \cos q)$$

$$x_0 = -r \sin q + s \cos q$$
$$y_0 = r \cos q + s \sin q$$



Orientation of the Region

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

$$x_0 = -r \sin q + s \cos q$$

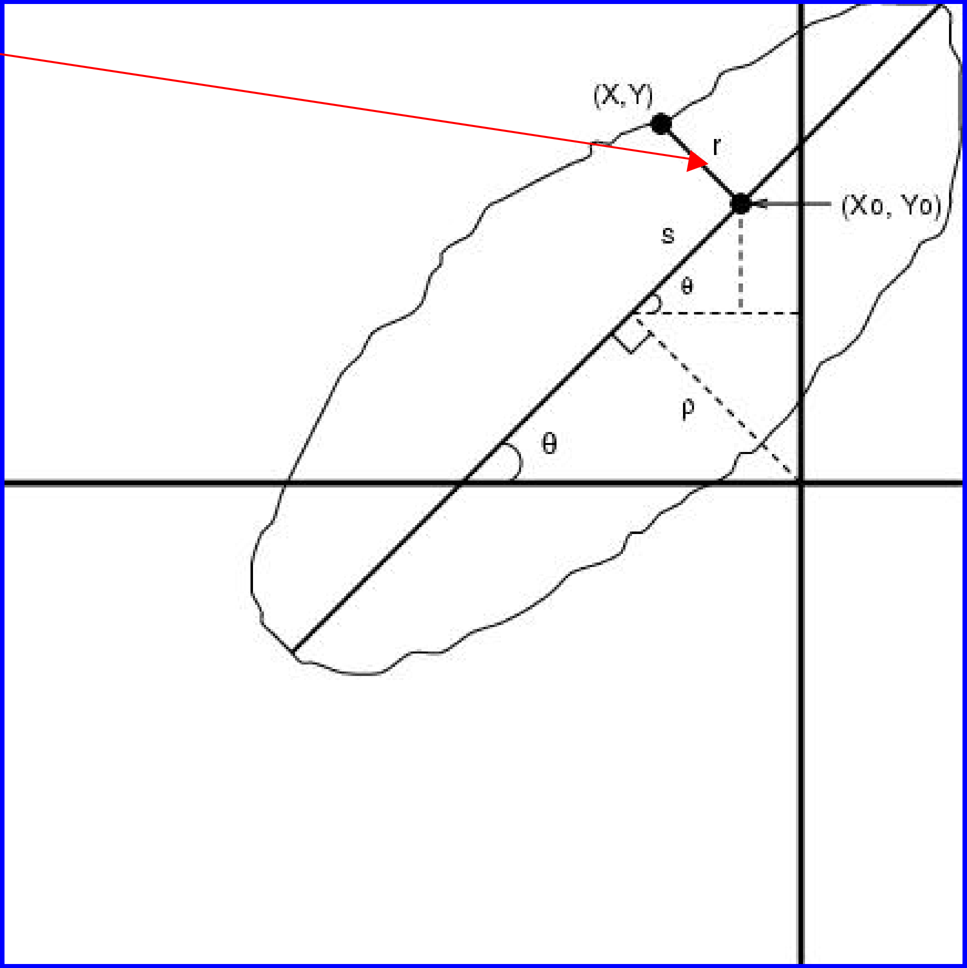
$$y_0 = r \cos q + s \sin q$$

Substituting (x_0, y_0) in r^2
And differentiating:

$$s = x \cos q + y \sin q$$

Substitute s in (x_0, y_0), then r:

$$r^2 = (x \sin q - y \cos q + r)^2$$



Orientation of the Region

$$r^2 = (x \sin q - y \cos q + r)^2$$

$$E = \iint r^2 B(x, y)\, dx dy$$

$$E = \iint (x \sin q - y \cos q + r)^2 B(x, y)\, dx dy$$

Substitute r in E and differentiate
Wrt to r and equate it to zero

$$A(\bar{x} \sin q - \bar{y} \cos q + r) = 0$$

$(\bar{x}, \bar{y})$ is the centroid

$$x' = x - \bar{x}, \quad y' = y - \bar{y}$$

$$a = \iint x'^2 B(x, y)\, dx' dy'$$

$$b = \iint x' y' B(x, y)\, dx' dy'$$

$$c = \iint y'^2 B(x, y)\, dx' dy'$$

$$E = a \sin^2 q - b \sin q \cos q + c \cos^2 q$$

Substitute value of r

$$E = \frac{1}{2}(a + c) - \frac{1}{2}(a - c) \cos 2q - \frac{1}{2} b \sin 2q$$

Orientation of the Region

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2q - \frac{1}{2} b \sin 2q$$

Differentiating this wrt q

$$\tan 2q = \frac{b}{a-c}$$

$$\sin 2q = \pm \frac{b}{\sqrt{b^2 + (a-c)^2}}$$

$$\cos 2q = \pm \frac{a-c}{\sqrt{b^2 + (a-c)^2}}$$

$$a = \iint x'^2 B(x,y)\,dx'dy'$$

$$b = \iint x'y' B(x,y)\,dx'dy'$$

$$c = \iint y'^2 B(x,y)\,dx'dy'$$

$$x' = x - \bar{x}, \quad y' = y - \bar{y}$$

$$a = \sum \sum x^2 B(x,y) - A\bar{x}^2$$

$$b = 2\sum \sum xyB(x,y) - A\bar{x}\bar{y}$$

$$c = \sum \sum y^2 B(x,y) - A\bar{y}^2$$

Applications of Segmentation

- Object recognition
- MPEG-4 video compression

Object Recognition Using Region Properties

- Training
  - For all training samples of each model object
    - Segment the image
    - Compute region properties (features)
  - Compute mean feature vector for each model object
- Recognition
  - Given an image of unknown object,
    - segment the image
    - compute its feature vector
    - match the vector to all possible models to determine its identity.

Object-Based Compression (MPEG-4)

- Advantages of OBC
  - large increase in compression ratio
  - allows manipulation of compressed video (inserting, deleting and modifying objects)
- How does it work?
  - Find objects (*Object Segmentation*)
  - code objects and their locations separately
      - through masks or splines
  - Build mosaics of globally static objects
  - Render scene at receiver