Camera Calibration

Computing Camera Parameters

& Pose Estimation

- Section 2.4, and chapter 6 from "Introductory Techniques for 3-D Computer Vision" by Trucco andVerri

Camera Parameters

- Extrinsic parameters
  - Parameters that define the location and orientation of the camera reference frame with respect to a known world reference frame
    - 3-D translation vector
    - A 3 by 3 rotation matrix
- Intrinsic parameters
  - Parameters necessary to link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame
    - Perspective projection
    - Transformation between camera frame coordinates and pixel coordinates

Direct Camera Calibration

Read section 6.2 from Trucco amd Verri's book.

Camera Parameters from Projection Matrix

- Using known 3-D points and corresponding image points, estimate camera matrix employing pseudo inverse method of section 1.6 (Fundamental of Computer Vision).
- Compute camera parameters by relating camera matrix with estimated camera matrix.
- Intrinsic
  - Horizontal and vertical focal lengths
  - Translation o_x and o_y
  - Extrinsic
    - Translation
    - Rotation

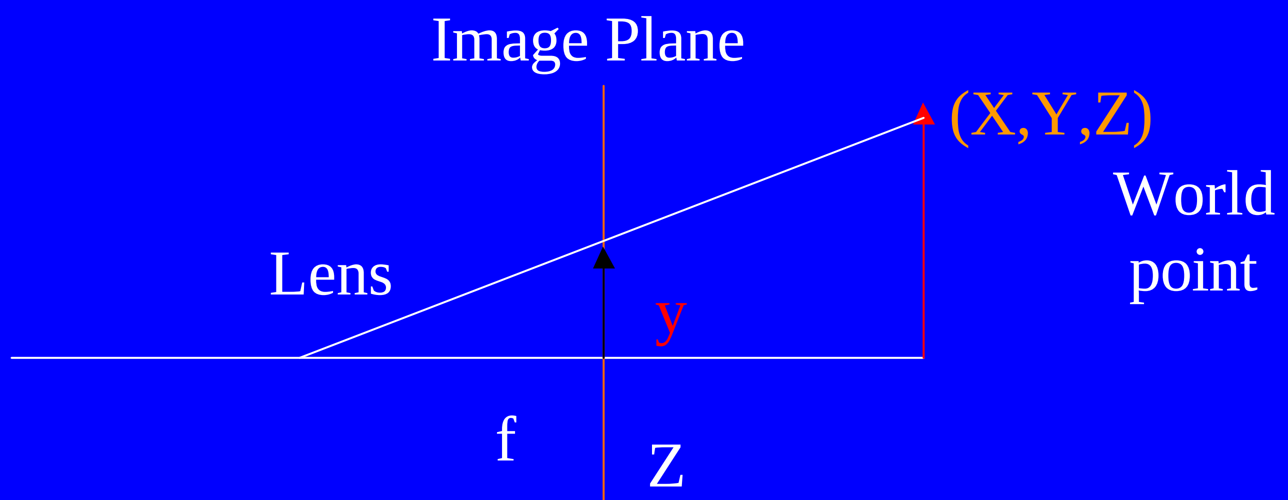
Camera Model Revisited: Rotation & Translation

$$P_c = TRP_w = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Perspective Projection: Revisited



$$\frac{y}{Y} = \frac{f}{Z}$$

$$y = \frac{fY}{Z} \qquad x = \frac{fX}{Z}$$

Camera Model Revisited: Perspective

$$C_h = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

Origin at the lens

Image plane in front of the lens

Camera Model Revisited: Image and Camera coordinates

$$x = -(x_{im} - o_x)s_x$$

$$y = -(y_{im} - o_y)s_y$$

$$x_{im} = -\frac{x}{s_x} + o_x$$

$$y_{im} = -\frac{y}{s_y} + o_y$$

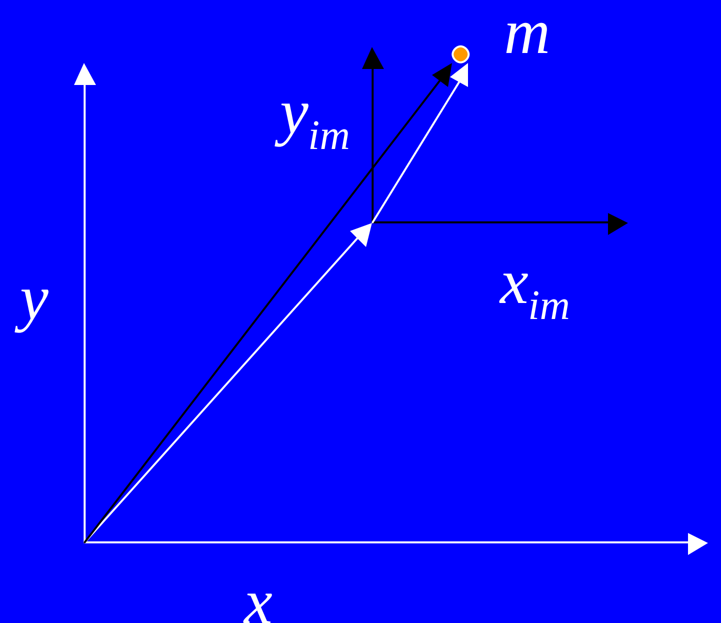
$$\begin{bmatrix} x_{im} \\ y_{im} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_x} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(x_{im}, y_{im}) image coordinates

(x, y) camera coordinates

(o_x, o_y) image center (in pixels)

(s_x, s_y) effective size of pixels (in millimeters) in the horizontal and vertical directions.



Camera Model Revisited

$$C_h = C'P'T'R'W_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_x} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M_{\text{int}} M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

f_x effective focal length expressed in effective horizontal pixel size

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Equation 6.18, pp 134

We have seen earlier how elements of matrix M can be estimated from knowledge of real world coordinates coordinates in camera reference frame. Let the estimate of M be

$$\hat{M} = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix}$$

Comparison

$$\hat{M} = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix}$$

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Computing Camera Parameters: Estimating scale

estimated → $\hat{M} = gM$

Since M is defined up to a scale factor

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = g \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$\sqrt{\hat{m}_{31}^2 + \hat{m}_{32}^2 + \hat{m}_{33}^2} = |g| \sqrt{r_{31}^2 + r_{32}^2 + r_{33}^2} = |g|$$

Divide each entry of $\hat{M}$ by $|g|$.

Computing Camera Parameters: estimating third row of rotation matrix and translation in depth

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = g \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$T_z = s\hat{m}_{34}, \quad s = \pm 1$$

$$r_{3i} = s\hat{m}_{3i}, \quad i = 1,2,3$$

Since we can determine $T_z > 0$ *(origin of world reference is in front)*
Or $T_z < 0$ *(origin of world reference is in back)*
we can determine sign.

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = g \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Let

$$q_1 = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} \end{bmatrix}$$
$$q_2 = \begin{bmatrix} \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} \end{bmatrix}$$
$$q_3 = \begin{bmatrix} \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} \end{bmatrix}$$

Computing Camera Parameters: origin of image

$$q_1^T q_3 = \hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33}$$

$$\hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33} =$$

$$\begin{pmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x \end{pmatrix} . \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$= \begin{pmatrix} -f_x r_{11} & -f_x r_{12} & -f_x r_{13} \end{pmatrix} . \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix} +$$

$$\begin{pmatrix} r_{31} o_x & r_{32} o_x & r_{32} o_x \end{pmatrix} . \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$= \begin{pmatrix} r_{31} o_x & r_{32} o_x & r_{32} o_x \end{pmatrix} . \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$= (r_{31}{}^2 o_x + r_{32}{}^2 o_x + r_{33}{}^2 o_x)$$

$$= o_x (r_{31}{}^2 + r_{32}{}^2 + r_{33}{}^2)$$

$$q_1^T q_3 = o_x$$

Therefore:

$$o_x = q_1^T q_3$$

$$o_y = q_2^T q_3$$

Computing Camera Parameters: vertical and horizontal focal lengths

$$q_1^T q_1 = \hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13}$$

$$\hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13} =$$

$$\left(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x\right) \cdot \left(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x\right)$$

$$= (-f_x r_{11} + r_{31} o_x)^2 + (-f_x r_{12} + r_{32} o_x)^2 + (-f_x r_{13} + r_{33} o_x)^2$$

$$= (f_x^2 r_{11}^2 + r_{31}^2 o_x^2) + (f_x^2 r_{12}^2 + r_{32}^2 o_x^2) + (f_x^2 r_{13}^2 + r_{33}^2 o_x^2)$$

$$= f_x^2 (r_{11}^2 + r_{12}^2 + r_{13}^2) + o_x^2 (r_{31}^2 + r_{32}^2 + r_{33}^2)$$

$$= f_x^2 + o_x^2$$

$$q_1^T q_1 = f_x^2 + o_x^2$$

$$\sqrt{q_1^T q_1 - o_x^2} = f_x$$

Therefore:

$$f_x = \sqrt{q_1^T q_1 - o_x^2}$$

$$f_y = \sqrt{q_2^T q_2 - o_y^2}$$

Computing Camera Parameters: remaining rotation and translation parameters

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$r_{1i} = s\,(0_x \hat{m}_{3i} - \hat{m}_{1i}) / f_x, \quad i = 1,2,3$$

$$r_{2i} = s\,(0_y \hat{m}_{3i} - \hat{m}_{2i}) / f_y, \quad i = 1,2,3$$

$$T_x = s\,(0_x T_z - \hat{m}_{14}) / f_x$$

$$T_y = s\,(0_y T_z - \hat{m}_{24}) / f_y$$

Pose Estimation for Object Recoginition

Object Recognition

Same Object appears differently in different poses.

From an imge , we can hypothesize an object, and estimate its pose

(variation from original object position).

3D World points project to different positions on the 2D image.

The difference in image coordinates can serve as clue to 3D changes

(Rotation and Translation).

Pose estimation: Object Recognition

- To determine the orientation and position of an object which would result in the projection of given 3-D points into a given set of image points.
  - 3-D points are given from the model X’,Y’,Z’
  - 2-D points are given from the image x, y
  - Determine 3-D rotation(orientation), and 3-D translation (position)

Pose Estimation

$$(X',Y',Z') \xrightarrow{\text{Rotation}} (X,Y,Z) \xrightarrow{\text{trans}} (X+T_X, Y+T_Y, Z+T_Z) \xrightarrow{\text{perspect}} (x',y')$$

$$(x',y') = \left(\frac{f(X+T_X)}{Z+T_Z}, \frac{f(Y+T_Y)}{Z+T_Z} \right)$$

Image displacements instead of
3-D translations

T Replaced by D in Z direction and using $c = \dfrac{1}{Z+D_Z}$

$$(x',y') = \left(\frac{fX}{Z+D_Z} + D_X, \frac{fY}{Z+D_Z} + D_Y \right) = (fXc + D_X, fYc + D_Y)$$

x' , y' are projected points on image . These are compared with actual image points and error determined.

Error in x' image coordinate can be related to errors in various terms using First order Taylor Series expansion

$$E_{x'} = \frac{\partial x'}{\partial D_X}\Delta D_X + \frac{\partial x'}{\partial D_Y}\Delta D_Y + \frac{\partial x'}{\partial D_Z}\Delta D_Z + \frac{\partial x'}{\partial f_X}\Delta f_X + \frac{\partial x'}{\partial f_Y}\Delta f_Y + \frac{\partial x'}{\partial f_Z}\Delta f_Z$$

Pose Estimation

Derivatives

$$(x', y') = \left(\frac{fX}{Z + D_Z} + D_X , \frac{fY}{Z + D_Z} + D_Y \right) = (fXc + D_X , fYc + D_Y)$$

$$\frac{\partial x'}{\partial D_X} = 1$$

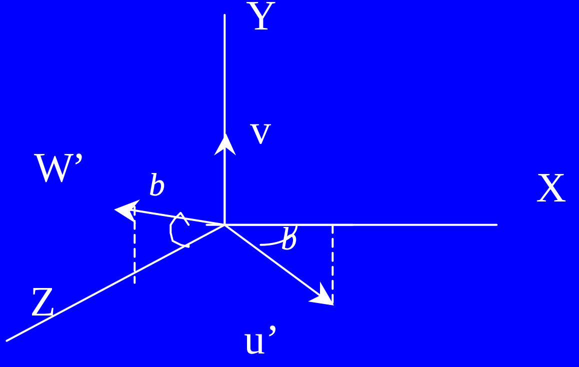
$$\frac{\partial x'}{\partial f_Y} = \frac{\partial x'}{\partial X} \frac{\partial X}{\partial f_Y} + \frac{\partial x'}{\partial Z} \frac{\partial Z}{\partial f_Y}$$

Chain rule

$$\frac{\partial x'}{\partial f_Y} = \frac{\partial X}{\partial f_Y} \frac{f}{Z + D_Z} - \frac{fX}{(Z + D_Z)^2} \frac{\partial Z}{\partial f_Y}$$

Pose Estimation

$$R^{Y}_{-b} = \begin{bmatrix} \cos b & 0 & -\sin b \\ 0 & 1 & 0 \\ \sin b & 0 & \cos b \end{bmatrix}$$



$$R^{Y}_{f_Y} = \begin{bmatrix} \cos f_Y & 0 & \sin f_Y \\ 0 & 1 & 0 \\ -\sin f_Y & 0 & \cos f_Y \end{bmatrix}$$

$$\frac{\partial X}{\partial f_Y} = -X'\sin f_Y + Z'\cos f_Y = Z$$

$$X = X'\cos f_Y + Z'\sin f_Y$$

$$Y = Y'$$

$$Z = -X'\sin f_Y + Z'\cos f_Y$$

$$\frac{\partial Y}{\partial f_Y} = 0$$

$$\frac{\partial Z}{\partial f_Y} = -X'\cos f_Y - Z'\sin f_Y = -X$$

Pose Estimation

$$E_{x'} = \frac{\partial x'}{\partial D_X}\Delta D_X + \frac{\partial x'}{\partial D_Y}\Delta D_Y + \frac{\partial x'}{\partial D_Z}\Delta D_Z + \frac{\partial x'}{\partial f_X}\Delta f_X + \frac{\partial x'}{\partial f_Y}\Delta f_Y + \frac{\partial x'}{\partial f_Z}\Delta f_Z$$

$$E_{y'} = \frac{\partial y'}{\partial D_X}\Delta D_X + \frac{\partial y'}{\partial D_Y}\Delta D_Y + \frac{\partial y'}{\partial D_Z}\Delta D_Z + \frac{\partial y'}{\partial f_X}\Delta f_X + \frac{\partial y'}{\partial f_Y}\Delta f_Y + \frac{\partial y'}{\partial f_Z}\Delta f_Z$$

$$\begin{bmatrix} \frac{\partial x'}{\partial D_X} & \frac{\partial x'}{\partial D_Y} & \frac{\partial x'}{\partial D_Z} & \frac{\partial x'}{\partial f_X} & \frac{\partial x'}{\partial f_Y} & \frac{\partial x'}{\partial f_Z} \\ & & & & \vdots & \\ \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} & \frac{\partial y'}{\partial D_X} \\ & & & & \vdots & \end{bmatrix} \begin{bmatrix} \Delta D_X \\ \Delta D_Y \\ \Delta D_Z \\ \Delta f_X \\ \Delta f_Y \\ \Delta f_Z \end{bmatrix} = \begin{bmatrix} E_{x'} \\ \\ E_{y'} \\ \\ \end{bmatrix}$$

$$A\Delta = E$$

$$\Delta = (A^T A)^{-1} A^T E$$

Least squares fit

Home Work will be posted on the webpage