University of Central Florida
School of Computer Science
COT 4210 Spring 2004

Prof. Rene Peralta
T2: answers to selected problems

1. Put the following grammar in Chomsky Normal Form.

$$
\begin{aligned}
S &\rightarrow SS + AA \\
A &\rightarrow AB + a \\
B &\rightarrow BS + b + \lambda
\end{aligned}
$$

answer:

$$
\begin{aligned}
S &\rightarrow TT + AA \\
T &\rightarrow TT + AA \\
A &\rightarrow AB + a \\
B &\rightarrow BT + TT + AA + b
\end{aligned}
$$

2. Write a context-free grammar for the following language

$$L = \{a^n b^k a^m \mid m = n + k, m, n, k \geq 0\}.$$

answer: Write L as

$$L = \{a^n b^k a^k a^n \mid n, k \geq 0\}.$$

Then it is easy to see the following grammar produces L.

$$
\begin{aligned}
S &\rightarrow aSa + T \\
T &\rightarrow aTb + \lambda
\end{aligned}
$$

3. Let M be the following DFA (?I don't seem to have the picture). Write left-linear and right-linear grammars for the language accepted by M.

4. Eliminate left-recursion from the following grammar

$$\begin{aligned} S &\rightarrow SB + A \\ A &\rightarrow AB + a \\ B &\rightarrow Ba + b \end{aligned}$$

answer: Note

$$\begin{aligned} S &\rightarrow AB^* \\ A &\rightarrow aB^* \\ B &\rightarrow ba^* \end{aligned}$$

Let

$$\begin{aligned} R &\rightarrow BR + \lambda \\ T &\rightarrow aT + \lambda \end{aligned}$$

and

$$\begin{aligned} S &\rightarrow AR \\ A &\rightarrow aR \\ B &\rightarrow bT \end{aligned}$$

5. Consider the following languages

$$\begin{aligned} L &= \{\omega\omega^r\omega \mid \omega \in (a+b)^*\} \\ R &= \{\omega \mid \omega \in (a+b)^* \text{ and } \omega \text{ has exactly 6 b's }\} \\ T &= L \cap R \end{aligned}$$

   (a) Give a string in T of length 12. **answer:** *bbaaaabbbbaa*.
   (b) Find a string $\omega \in b(a+b)^*b$ of length 6 such that $\omega\omega\omega \in T$. **answer:** *baaaab*.

(c) Give a convincing argument that L is not context-free. Your argument should

- use the fact that R is regular;
- use the fact that the intersection between a regular language and a CFL is a CFL;
- define a string $z \in T$ for which the pumping lemma for CFL's does not hold;
- a concise and convincing explanation of why z cannot be "pumped".

answer: Assume L is context-free. Since R is regular, $T = L \cap R$ is context-free. A string z of the form $z = ba^n bba^n bba^n b$ is in T. For n large enough, the pumping lemma for CFL's must hold for z. Since words in T have exactly six b's, z can only be "pumped" at two regions containing only a's. The resulting strings are of the form $ba^k bba^l bba^m b$ with k, l, m not necessarily equal. In the latter case, the word cannot be in T, contradiction. Therefore L cannot be context-free.