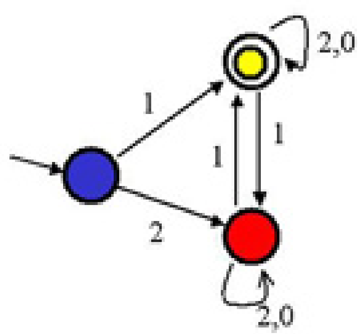
University of Central Florida
School of Computer Science
COT 4210 Spring 2004

Prof. Rene Peralta
Answers to T1

1. Consider integers written in base 3 with no leading 0s. Let L_1 be the set of such strings which represent odd numbers.

   (a) Construct a DFA that accepts L_1.

   answer:



   (b) Construct a left-linear grammar for L_1.

   answer: Let Y, B, R denote the yellow, blue, and red states resp. Associate with each state the set of strings whose computation <u>ends</u> at that state. Then the language is composed of those strings associated with Y. The derivation rules of the grammar are

$$
\begin{aligned}
B &\rightarrow \lambda \\
Y &\rightarrow B1 + R1 + Y(0+2) \\
R &\rightarrow B2 + Y1 + R(0+2)
\end{aligned}
$$

   The initial non-terminal of the grammar is Y.

2. Consider the language L_2 generated by the following grammar

$$\begin{array}{rcl} S & \rightarrow & AB + C \\ A & \rightarrow & aB + C \\ B & \rightarrow & Ab + C \\ C & \rightarrow & b + aaaC \end{array}$$

Characterize L_1 using a combination of set notation and regular expressions.

answer: (This problem was harder than intended because of a typo - mea culpa).

- C generates the regular language $(aaa)^*b$; From now on we use C to denote this language.
- substituting the rules for B into the rules for A we obtain

  $$A- > a(Ab + C) + C = aAb + (a + \lambda)C.$$

  Thus, A generates the language

  $$\{a^n(a + \lambda)Cb^n \mid n \geq 0\}.$$

- Similarly, B generates the language

  $$\{a^nC(b + \lambda)b^n \mid n \geq 0\}.$$

- Finally, A generates the concatenation of A and B, union the language C:

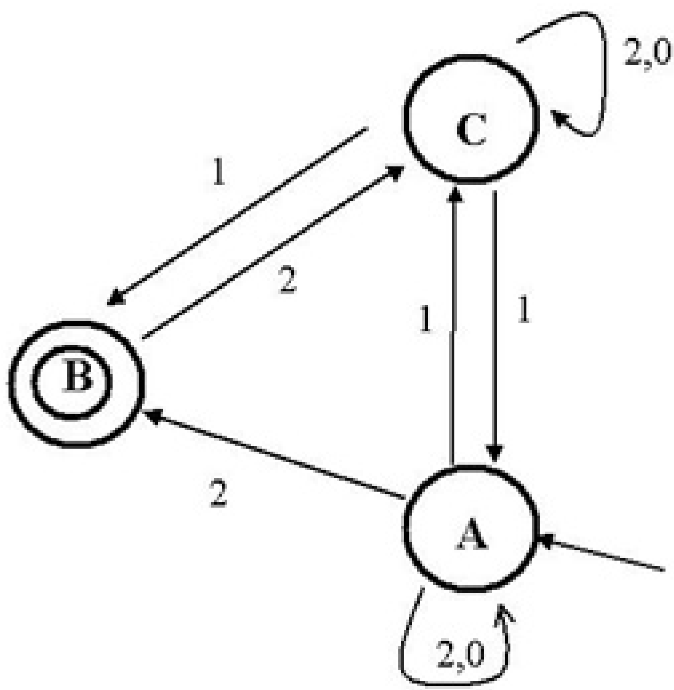
  $$\{a^n(a + \lambda)Cb^n \mid n \geq 0\}\{a^nC(b + \lambda)b^n \mid n \geq 0\} + C$$

  where $C = (aaa)^*b$.

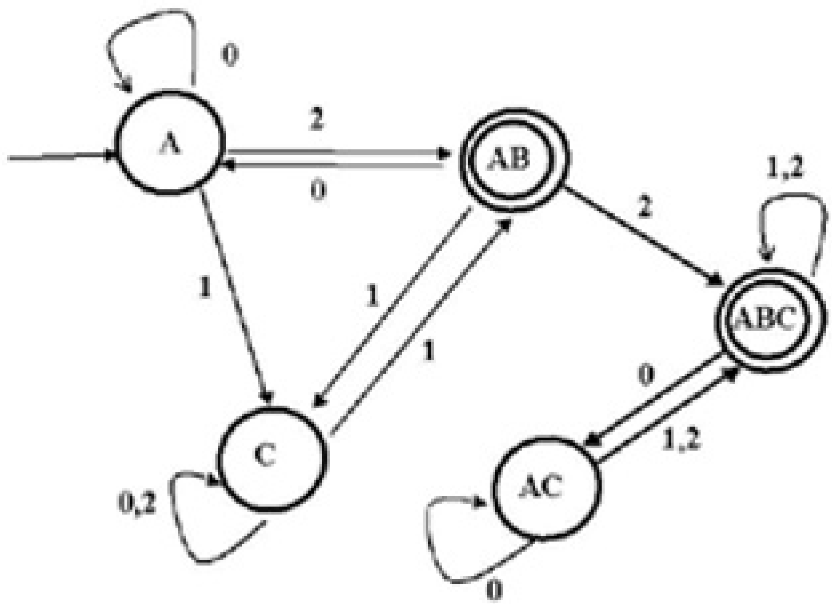
3. What does it mean for an infinite set to be "countable"?

   answer: An infinite set S is countable if and only there exists a bijection between S and the natural numbers.

4. Construct a DFA equivalent to the following NFA.



answer:



5. Consider the language over $\Sigma = \{a, b, c\}$ consisting of strings with more occurrences of the pattern "abc" than occurrences of the pattern "abb". Is this a regular language? Justify your answer.

   answer: It is not a regular language. This can be proven using the Pumping Lemma:

   - the word $x = (abc)^{n+1}(abb)^n$ is in the language for any n;
   - for large enough n, $x = uvw$ such that
     - uv is a prefix of $(abc)^{n+1}$ with v not equal to λ;
     - uv^iw is in the language for all $i \geq 0$;
   - in particular, the word $uv^0w = uw$ is in the language;
   - but this erases at least one symbol from a prefix of $(abc)^{n+1}$;
   - the resulting word does not contain more substrings of the form abc than abb and hence is not in the language, contradiction.