University of Central Florida
School of Computer Science
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sample questions T1

1. Consider integers written in base 3 with no leading 0s. Let L_1 be the set of such strings representing numbers that are congruent to 3 mod 4.

   (a) Construct a DFA that accepts L_1.

   (b) Construct a left-linear grammar for L_1.

   answer: we have done this type of exercise ad nauseum, so I won't write an answer here

2. Write a regular expression for the set of strings over $\Sigma = \{0, 1\}$ which have an odd number of 1's.

   answer: note you do not need to go through the entire DFA $\rightarrow$ grammar $\rightarrow$ regular expression sequence. "Odd number of 1's" means "one 1 plus an even number of 1's", so $\rightarrow$ $0^*10^*(0^*10^*10^*)^*$. This is not the most compact representation, but we don't care about that here.

3. Describe the four types of grammars in the Chomsky Hierarchy.

   answer: something like i) unrestricted, ii) $|LHS| \leq |RHS|$, iii) $|LHS| = 1$; iv) right-linear.

4. Outline the argument that uses Cantor's diagonalization technique to show the set of subsets of $\aleph$ is not countable.

   answer: It is a proof by contradiction: i) there is a bijection between infinite binary strings and subsets of $\aleph$, so we count the former instead; ii) assume, for a contradiction, that the set of infinite binary strings is countable; iii) the strings can thus be arranged in a list $s(1), s(2), s(3), \ldots$. iv) Let b(i) be the complement of the i^{th} bit of $s(i)$; v) Then $\vec{b}$ is a string which is not in the list, contradiction.

5. Show, using the pumping lemma for regular languages, that the set T consisting of binary string with more $1s$ than $0s$ is not regular.

   answer: By contradiction. Assume the set is regular and let α be the "pumping constant" given by the pumping lemma. Let $\omega = 11^\alpha 0^\alpha$. Then $\omega \in T$. The pumping lemma implies we can "erase" a (non-empty) substring from the prefix 11^α with the resulting string still being in T. But this clearly cannot be done, contradiction.