University of Central Florida
School of Computer Science
COT 4210 Spring 2004

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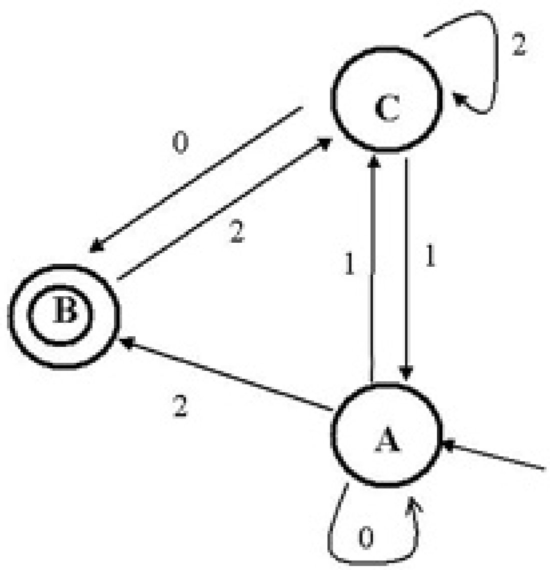
sample questions T2

1. Construct a left-linear grammar for the following DFA.



2.
   - State De Morgan's laws for sets.
   - Prove or disprove "context-free languages are closed under intersection".

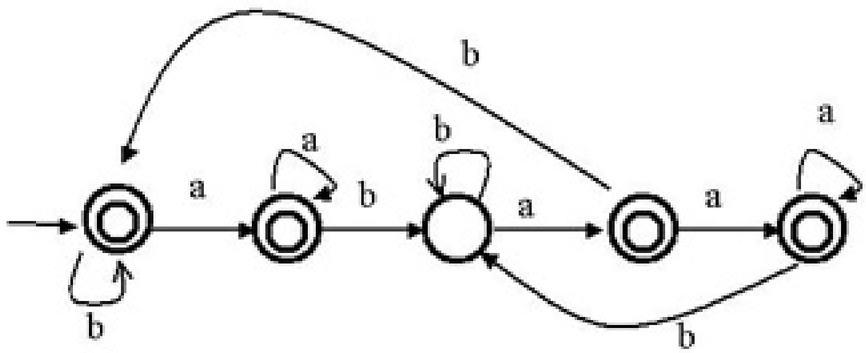
   answer: covered in class (see chapter 8, section 5 of textbook).

3. Let L be the language generated by the following grammar.

$$\begin{array}{lcl} S & \rightarrow & Sa + Ab \\ A & \rightarrow & Bb + Ab + a \\ B & \rightarrow & Sb + \lambda \end{array}$$

   Find a regular expression for L.

4. Use the state-minimization algorithm learned in class to find a smallest DFA equivalent to the following DFA.



answer: I get a machine with 3 states.

5. Show that the set of palindromes over $\Sigma = \{a, b, c\}$ is not regular.

6. Write a Chomsky-Normal Form grammar for the set of palindromes over $\Sigma = \{a, b\}$.

answer:

$$\begin{array}{rcl} S & \rightarrow & AT + BR + \lambda \\ W & \rightarrow & AT + BR + a + b \\ T & \rightarrow & WA + a \\ R & \rightarrow & WB + b \\ A & \rightarrow & a \\ B & \rightarrow & b \end{array}$$

7. Consider the following grammar G

$$\begin{array}{rcl} S & \rightarrow & VW + \lambda \\ V & \rightarrow & XY + b \\ W & \rightarrow & YX + a \\ X & \rightarrow & ZR + c \\ Y & \rightarrow & RZ \\ R & \rightarrow & RV + d \\ Z & \rightarrow & a \end{array}$$

find <u>specific</u> strings u, v, w, x, y, such that

- $vx \neq \lambda$; and
- uv^iwx^iy belongs to $L(G)$ for all $i \geq 0$.

answer: An easy way to get such a string is to draw a derivation tree that repeats a variable along a path from the root. Then see what happens to the string when the derivation tree rooted at the lower of the repeated variables is exchanged for the derivation tree rooted at the higher one. This is how I got the following

$$u = \lambda \qquad v = ad \qquad w = b \qquad x = da \qquad w = a$$

i.e.

$$(ad)^i b(da)^i a \in L(G) \ \text{ for all } i \geq 0.$$

8. For the following grammar, write an equivalent grammar that is not left-recursive.

$$\begin{array}{rcl} S & \rightarrow & SS + AA \\ A & \rightarrow & Ab + a \\ B & \rightarrow & BS + \lambda \end{array}$$

9. Construct a PDA for the language

$$L = \{\omega \mid \#(w)_a \equiv 1 \bmod 3\} \cap \{a^n b^n \mid n \geq 0\}.$$

answer: The straight-forward construction yields a machine with 4 states.