COT 4500, Spring 2007
S. Lang (4/11/2007) **Test #2** Print Name:____________________________

Instructions: There are 4 pages, 8 questions, and 100 total points. Formula sheets are provided for the test. Show your work and write your answer in the space provided.

1. (15 pts.) Use the data from the following table and the formulas for Lagrange interpolating polynomials to
   (a) construct the Lagrange interpolating polynomials $L_{1,0}(x)$ and $L_{1,1}(x)$; and
   (b) estimate the error using appropriate error bound if the Lagrange interpolating polynomial of degree one is used to approximate $f(0.1)$ if $f(x) = 2 \cos x - 1$.

x	$f(x)$
0.0	1.00
0.2	0.96

2. (10 pts.) Suppose a natural cubic spine function $S(x)$ is used to approximate the function $f(x) = \sin x$ over the interval [0, 3] using the data points (0, sin 0), (1, sin 1), (2, sin 2), and (3, sin 3). The three spline functions are labeled $S_j(x)$, $j = 0$, 1, and 2, respectively, where $S_j(x) = a_j + b_j (x - x_j) + c_j (x - x_j)^2 + d_j (x - x_j)^3$. Set up the equations (using the matrix form or equations) for solving the unknowns c_0, c_1, c_2, and c_3 (but **do not** solve the equations).

x	$\sin x$
0.0	sin 0 = 0.0000
1.0	sin 1 = 0.8414
2.0	sin 2 = 0.9092
3.0	sin 3 = 0.1411

3. (15 pts.) Suppose $f(x) = 2 \sin x - x$.
   (a) Use appropriate formula to approximate the value $f''(1.2)$ (i.e., the second derivative of f evaluated at 1.2) using the data values given in the following table; and
   (b) Estimate the error of the approximation of Part (a) using appropriate error formula.

x	$f(x)$
1.1	0.6824
1.2	0.6640
1.3	0.6271

4. (15 pts.) Consider the integral $\int_0^2 x \cos x \, dx$.
   (a) Use the composite trapezoidal rule with $n = 2$ to approximate the integral.

   (b) Estimate the error for the approximation in Part (a) using appropriate error formula.

5. (10 pts.) Suppose Romberg's method is used to approximate the integral $\int_0^2 x\cos x\,dx$.

Compute the approximation $R_{3,3}$.

6. (10 pts.) Use the open Newton-Cotes formula with $n = 1$ to approximate the integral $\int_0^2 x\cos x\,dx$.

7. (15 pts.) Use Gaussian quadrature with $n = 2$ to approximate the integral $\int_0^2 x\cos x\,dx$.

8. (10 pts.) Describe **two** approaches that can be used to verify that the degree of precision is one for the mid-point rule in integration, i.e.,
$\int_a^b f(x)\,dx = 2hf(x_0) + \frac{h^3}{3} f^{(2)}(\xi)$ where $h = \frac{b-a}{2}, x_0 = \frac{a+b}{2}, a < \xi < b$.