

COT 4500 Quiz #1 Solutions

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$$1) \quad f(x) = c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + \dots$$

$$f(x) = \ln(x), \quad f'(x) = x^{-1}, \quad f''(x) = -x^{-2}, \quad f'''(x) = 2x^{-3}$$

$$f^{(4)}(x) = -6x^{-4}, \quad f^{(5)}(x) = 24x^{-5}$$

$$\Rightarrow f(1) = 0, \quad f'(1) = 1, \quad f''(1) = -1, \quad f'''(1) = 2$$

$$f^{(4)}(1) = -6, \quad f^{(5)}(1) = 24$$

$$c_n = \frac{f^{(n)}(1)}{n!}, \quad \text{thus, we have}$$

$$p_5(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5$$

2)

$$\text{sign} = -$$

$$\text{exponent} = 10000000111_2 = 1024 + 4 + 2 + 1 = 1031$$

$$\text{value} = 2^{1031-1023} = 2^8 = 256$$

$$\text{mantissa} = 1001010\dots 0$$

$$= \frac{1}{2} + \frac{1}{16} + \frac{1}{64}$$

$$\text{Number} = -2^8 \left(1 + \frac{1}{2} + \frac{1}{16} + \frac{1}{64} \right)$$

$$= - \left(2^8 + 2^7 + 2^4 + 2^2 \right)$$

$$= -256 - 128 - 16 - 4$$

$$= \boxed{-404}$$

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$$3) \quad \frac{3}{7} = .4285 \text{ (using 4-digit chopping)}$$

$$\frac{7}{9} = .7777 \text{ (using 4-digit chopping)}$$

$$.4285 + .7777 = 1.2062$$

$$\text{Actual value} = \frac{3}{7} + \frac{7}{9} = \frac{27+49}{63} = \frac{76}{63} \sim 1.206349206$$

$$\text{Difference/Absolute Error} \approx 1.492063492 \times 10^{-4}$$

$$3.492063492 \times 10^{-4}$$

4) double calcE(int n, double x) {

double num = 1, den = 1;

double ans = 0;

int i = 0;

for (i = 0; i <= n; i++) {

ans = ans + num/den;

num = num * x;

den = den * (i+1);

}

return ans;

}

$$5) \text{ Let } x = \lim_{n \rightarrow \infty} \frac{A_{n+1}}{A_n}.$$

$$x = \lim_{n \rightarrow \infty} \frac{A_{n+1}}{A_n} = \lim_{n \rightarrow \infty} \frac{A_n + 2A_{n-1}}{A_n}$$

$$x = \lim_{n \rightarrow \infty} \frac{A_n}{A_n} + \lim_{n \rightarrow \infty} \frac{2A_{n-1}}{A_n}$$

$$x = 1 + \frac{2}{x}$$

$$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$\boxed{x=2}, \text{ since } x > 0.$$