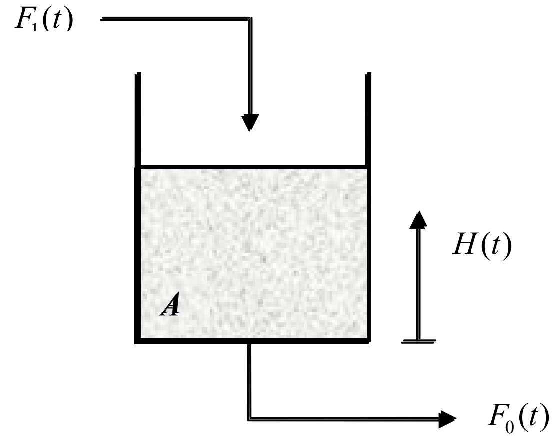
EEL 5890 Exam 1 Name_________________

Fall 2001 SS # __________________

Problem 1 (25 pts)

The flow out of the tank shown below is given by $F_0 = cH^{\frac{1}{2}}$. The cross-sectional area of the tank A = 50 ft^2 and the constant c = 2 ft^3/min per ft$^{1/2}$. The initial level in the tank is $H(0)$ = 16 ft.



A) The flow in to the tank is $F_1(t) = \overline{F}_1 = 10$ ft^3 per min, $t \geq 0$.
Find the steady-state height of liquid in the tank, $H(\infty)$.

B) The flow in to the tank is $F_1(t) = 4 + \dfrac{t}{10}, \quad t \geq 0$.

Use forward Euler integration with a step size T and find the equation for updating the state $H_A(n)$, i.e. the equation with $H_A(n+1)$ on the left hand side. Leave your answer in terms of c, A, and T.

C) Use the result from Part B) to find $H_A(1)$ and $H_A(2)$ when the step size T = 0.5 min.
Express your answers to 4 places after the decimal point.

EEL 5890 Exam 1 Name__________________

Fall 2001 SS # ___________________

Problem 2 (25 pts)

The population of a country $P(t)$ is modeled by the differential equation $\frac{dP}{dt} = kP$.

A) Find the equation for updating $P_A(n)$, the approximate population at the end of year nT, using trapezoidal integration with step size T. Leave your answer in terms of k and T.

B) Suppose $k = 0.01$ people/year per person, the initial population is 1 million people and the step size $T = 1$ yr. Find $P_A(1)$ and $P_A(2)$ to the nearest person.

C) Find the general solution for $P_A(n)$ and use it to find $P_A(100)$.

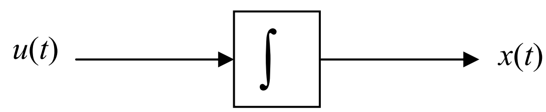
D) Compare the result from Part C) to thee exact value $P(100)$.

EEL 5890 Exam 1 Name_________________

Fall 2001 SS # __________________

Problem 3 (25 pts)

The input to the integrator shown below is the continuous signal $u(t) = \frac{1}{t+1}, \quad t \geq 0$



A) Find the equation for computing the state $x_A(n)$ recursively when backward Euler integration with a step size T is used.

B) Find $x_A(1)$, $x_A(2)$ and $x_A(3)$ if $T = 0.1$.

C) Compare your answer for $x_A(3)$ to the exact value $x(3T)$.

Note: $\int_0^t \frac{1}{t'+1} dt' = \ln(1+t)$