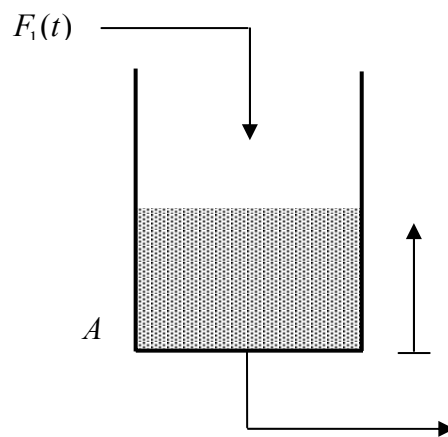


SHOW ALL WORK!

Problem 1 (25 pts)

The flow out of the tank shown below is given by $F_0 = cH^{\frac{1}{2}}$. The cross-sectional area of the tank $A = 50 \text{ ft}^2$ and the constant $c = 2 \text{ ft}^3/\text{min per ft}^{1/2}$. The initial level in the tank is $H(0) = 16 \text{ ft}$.



- The flow into the tank is $F_1(t) = \bar{F}_1 = 10 \text{ ft}^3$ per min, $t \geq 0$. Find the steady-state height of liquid in the tank, $H(\infty)$.
- Use explicit Euler integration with a step size T and find the equation for updating the state $H_A(n)$, i.e. the equation with $H_A(n+1)$ only on the left hand side. Leave your answer in terms of c , A , and T .
- Use the result from Part B) to find $H_A(1)$ and $H_A(2)$ when the step size $T = 0.25 \text{ min}$. Express your answers to 4 places after the decimal point.

EEL 4890

Exam 1

Name _____

Fall 2002

SS # _____

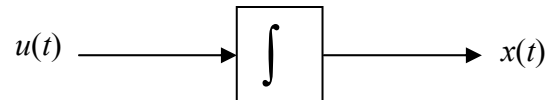
Problem 2 (25 pts)

A first order system is modeled by the differential equation $\frac{dy}{dt} = ky$.

- a) Find the equation for updating $y_A(n)$, the approximation to $y(nT)$, using trapezoidal integration with step size T . Leave your answer in terms of k and T .
- b) Suppose $k = -0.4$, $y(0) = 1$ and the step size $T = 0.1$. Find $y_A(2)$.

Problem 3 (25 pts)

The input to the integrator shown below is the continuous signal $u(t) = 1, t \geq 0$



- Find the equation for updating the state $x_A(n)$ recursively when the Improved Euler integrator with a step size T is used.
- Find $x_A(1)$ if $x(0) = 0$ and $T = 0.5$.
- Compare $x_A(1)$ to the exact value $x(T)$.

EEL 4890
Fall 2002

Exam 1

Name _____
SS # _____

Problem 4 (25 pts)

For the first order system with input $u = u(t)$ and output $y = y(t)$ modeled by

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 3y = 6 \frac{du}{dt}$$

- a) Draw a simulation diagram
- b) Write the equations for the system in state variable form

$$\dot{\underline{x}} = A\underline{x} + Bu$$

$$y = C\underline{x} + Du$$