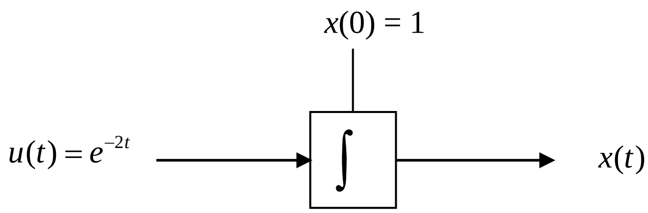
EEL 4890 Exam 1 Name__________________

Fall 2004

SHOW ALL WORK!

Problem 1 (30 pts)

A continuous integrator with initial condition is shown below.



Fill in the table below. Choose $T = 0.05$ for each integrator. Round all answers to 4 places after the decimal point.

n	$x_A(n)$ Explicit Euler	$x_A(n)$ Trapezoidal
0		
1		
2		
3		

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Problem 2 (35 pts)

A second order system is modeled by the differential equation

$$\frac{d^2w}{dt^2} + \frac{dw}{dt} + 2w = \frac{d^2u}{dt^2}.$$

The initial conditions $w(0) = \frac{dw}{dt}(0) = 0.$

a) Draw a simulation diagram for the system.

b) Find matrices A, B, C, D in the state variable model form

$$\dot{\underline{x}} = A\underline{x} + Bu$$
$$y = C\underline{x} + Du$$

The single output is $y = w.$

c) The input $u(t) = t,\ t \geq 0.$ Use explicit Euler integration with step size $T = 0.1$ to find $y_A(1), y_A(2)$ and $y_A(3).$ Round answers to 4 places after the decimal point.

Hint: First find $\underline{x}_A(1),\ \underline{x}_A(2)$ and $\underline{x}_A(3).$

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Problem 3 (35 pts)

An exponential population growth model

$$\frac{dP}{dt} = -kP, \quad (k > 0)$$

is to be simulated in order to approximate the population $P(t)$ for a period of time. The difference equation for $P_A(n)$ intended to approximate $P(t)$ is

$$P_A(n+1) + \alpha P_A(n) = 0$$

a) Find an expression for α in terms of k and the step size T using

   i) Implicit Euler

   ii) Trapezoidal

   iii) Improved Euler

b) Evaluate α for each integrator and round answers to 6 places after decimal point.

c) Fill in the Table comparing the three numerical integrators and the exact solution. $P(0)$ = 5 million and $T = 2$ yr. Round all answers in millions to 6 places after the decimal point. For the exact, enter $P(T)$ and $P(2T)$.

	$P_A(0)$	$P_A(1)$	$P_A(2)$
Implicit Euler	5.000000		
Trapezoidal	5.000000		
Improved Euler	5.000000		
Exact	5.000000		